Performance analysis of Vehicular Platoon considering V2V Communication

基于车车通信的车辆队列性能分析

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Outline

- Background
- Problem Statements
- Performance Analysis considering V2V communication
- Conclusions

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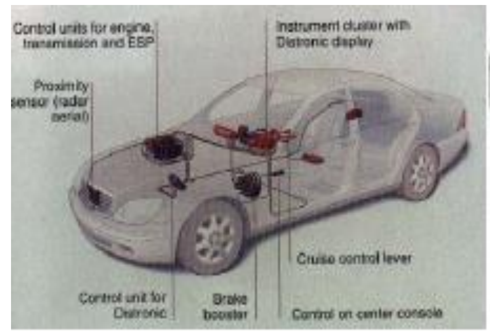
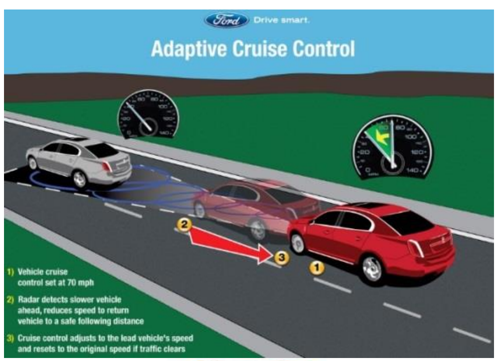
Background

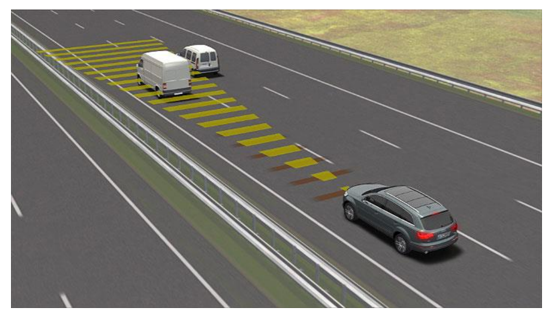
- History of Adaptive Cruise Control (ACC）
  - Bosch Ltd, developed the first prototyping ACC
  - Mercedes Benz first to offer ACC, called Distronic system

BOSCH



S-class
Distronic





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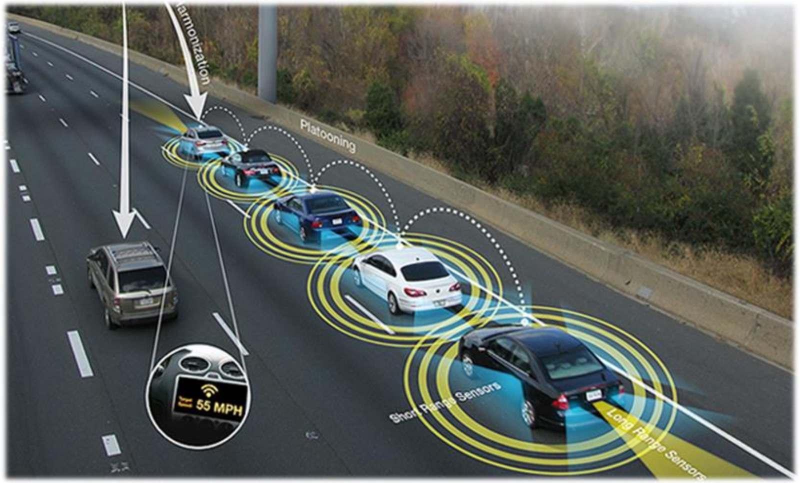
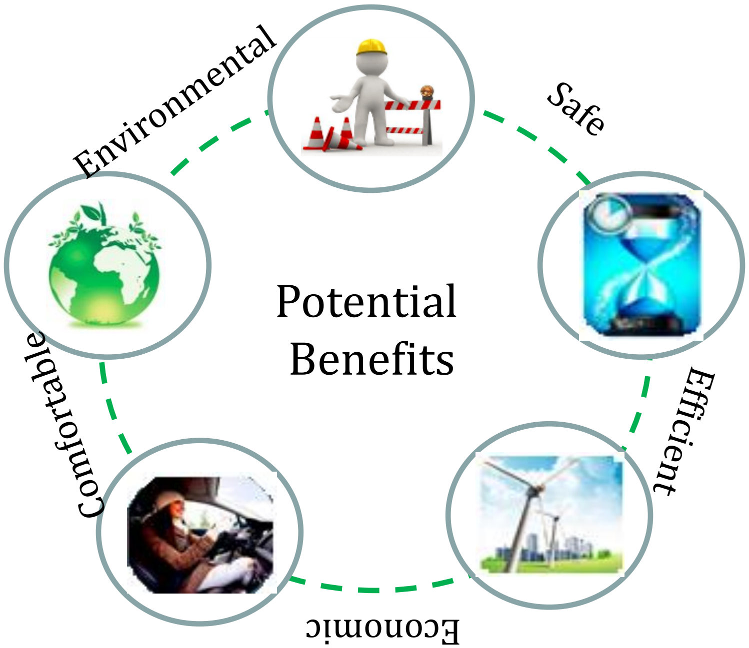
Background

- Next Generation of ACC
  - V2V/V2I communication
  - Multi-vehicle cooperative control (e.g., Vehicular Platoon, CACC)



- Potential Benefits



- Short following distance for better traffic capacity
- Faster reaction to leader fluctuation to reduce potential congestion
- Reduced aerodynamic resistance for better fuel economy and less emission
- More reliable automation for comfortable mobility

Background

- Platoon: Topics and Researchers
  - Research topics

    1) Selection of spacing policies; 2) Communication delay; 3) Vehicle dynamic uncertainty; etc.

  - Researchers

    1) **U.S.:** J. Hedrick, P. Seiler, S. Darbha, Huei Peng etc.
    2) **Europe:** Jeroen Ploeg, N. van de Wouw, H. Nijmeijer, etc.
    3) **China:** Keqiang Li, Feng Gao, etc.

- Platoon: Experiments

US: PATH

Europe: SARTRE

Japan: Energy ITS

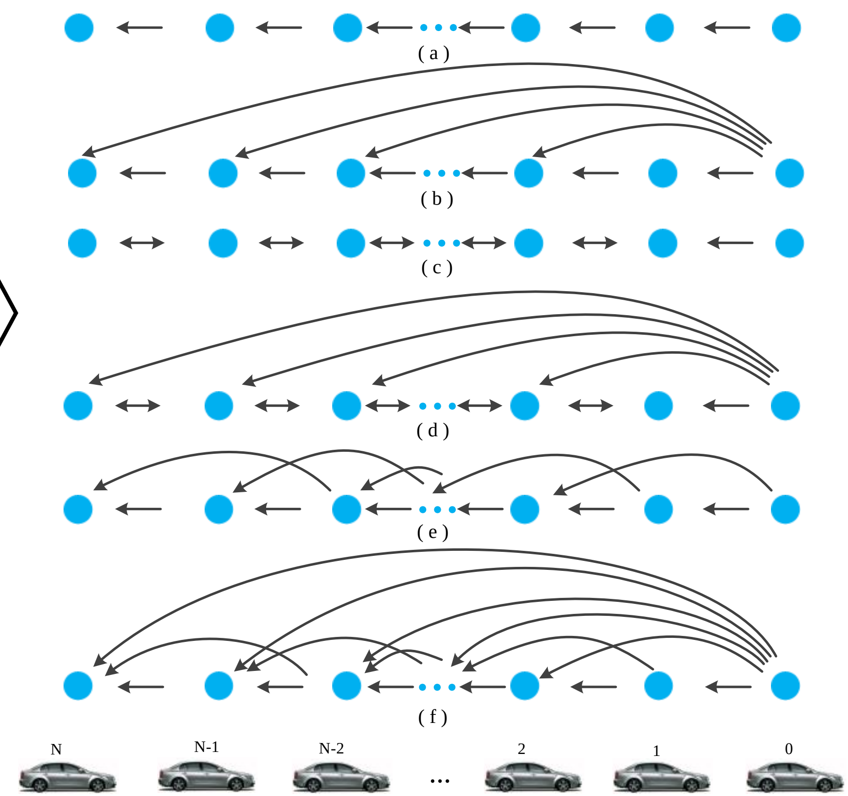
Background

Platoon: Emerging Topics

Typical Information Flow

Connected Vehicle by V2V

Different Information Flow Topologies



Platoon: Key Tasks

1. Model and design of a platoon system with a broad types of topologies.
2. Performance analysis of different topologies on platoons.

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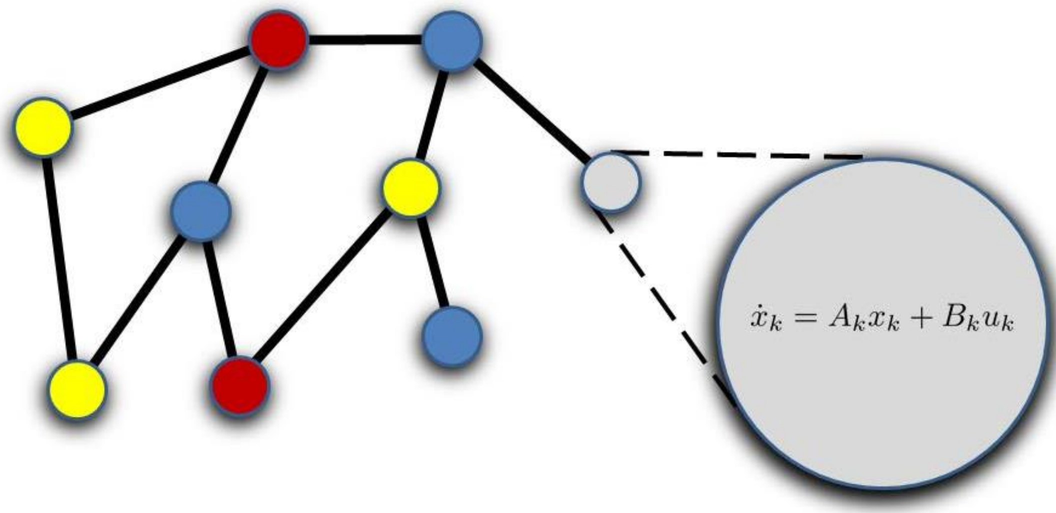
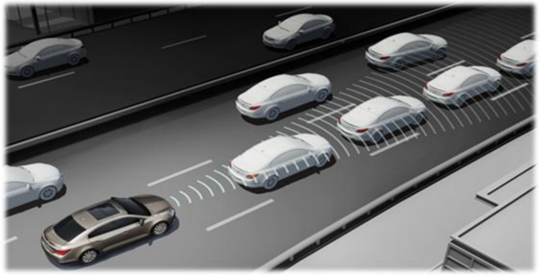
Problem Statements

- Modeling of Platoons from the viewpoint of **Networks of Dynamical Systems**



- From Control Perspective
  1. Dynamics + Communication
  2. Control Theory + Graph Theory
- Research topics
  1. **Dynamic**: single integrator, double integrator, linear dynamic, nonlinear dynamic
  2. **Communication**: data rate, switching topology, time-delay

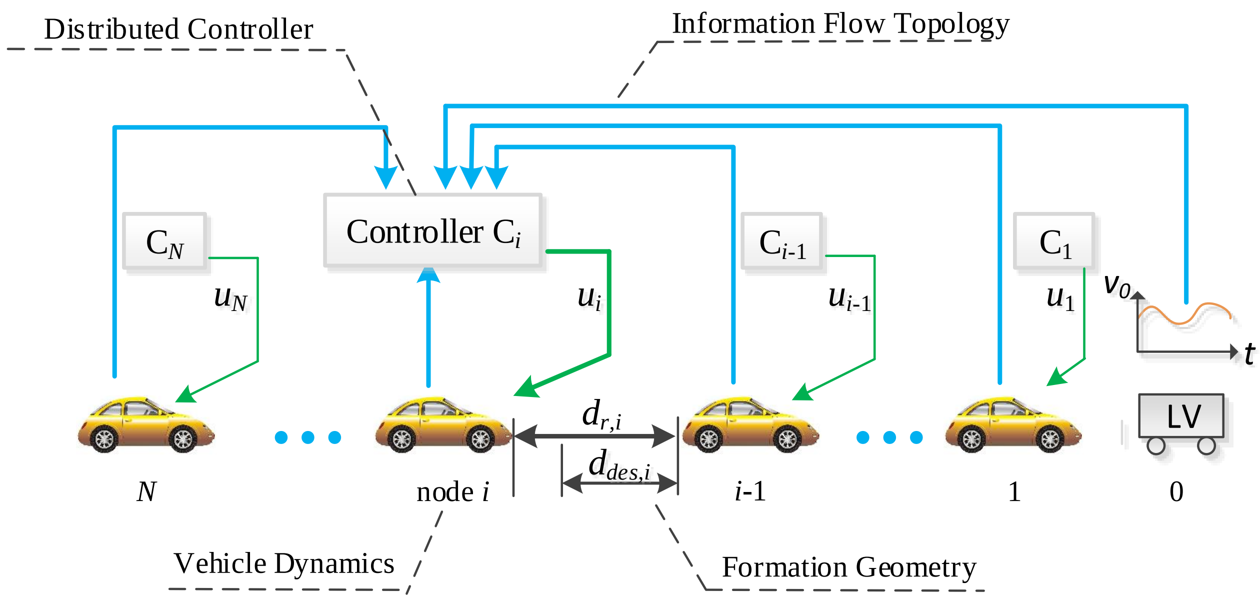
- Applications



A vehicular platoon can be viewed as a one-dimensional network of dynamical system

Problem Statements

- Modeling of Platoons under the Four-component Framework



- **Vehicle Dynamics:** The vehicle dynamics describe the behavior of each node;
- **Formation Geometry:** The formation geometry dictates the desired distance between any two successive nodes.
- **Distributed Controller:** The distributed controller implements feedback control for each vehicle;
- **Information Flow Topology**: The information flow topology defines how nodes exchange information with each other;

Problem Statements

- Model for Vehicle Longitudinal Dynamics
  - Linear Dynamics

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t)$$

$$x_i(t) = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix}$$

  - Feedback linearization technique is used to convert the nonlinear model into a linear one;
  - The vehicle dynamics is assumed to be homogeneous.

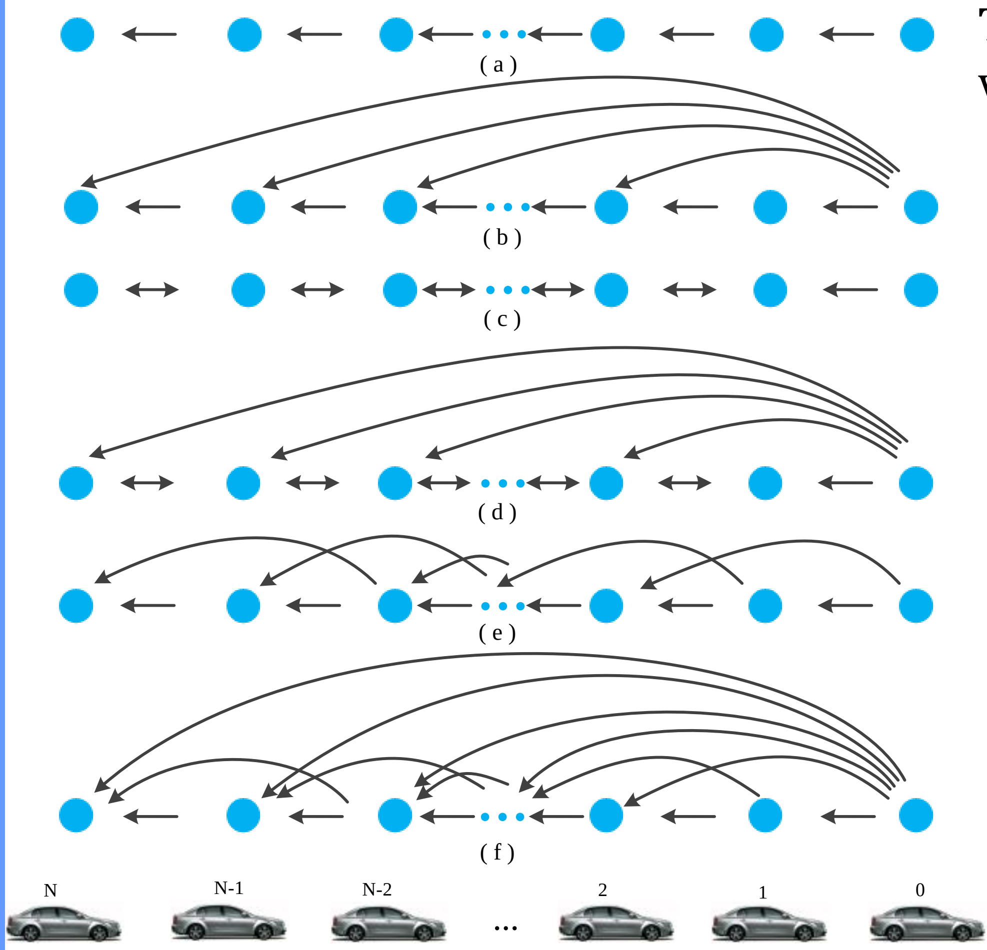
- Model for Formation Geometry

$$\begin{cases} \lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0, & i = 1,2,\cdots N \\ \lim_{t\to\infty} \left\|p_{i-1}(t) - p_i(t) - d_{i-1,i}\right\| = 0 & \end{cases},$$

  - Constant distance (CD) policy
  - Constant time headway (CTH) policy
  - Nonlinear distance policy

Problem Statements

- Model for information flow topology



The V2V communication can generates various information flow topologies.

a) Predecessor following topology;
→ PF topology

b) Predecessor-leader following topology;
→ PLF topology

c) Bidirectional topology;
→ BD topology

d) Bidirectional-leader topology;
→ BDL topology

e) Two predecessors following topology
→ TPF topology

f) Two predecessor-leader following topology
→ TPLF topology

Problem Statements

- Model for information flow topology
  - Algebraic Graph Theory
    - ✓ Viewed as a directed graph G, and use Pinning matrix, Adjacent matrix and Laplacian matrix to model the connections.
    - ✓ The communication is assumed as to be perfect. There is no delay, data loss etc.
  - Definitions

- Pinning Matrix

To model the information flow from the leader to followers

$$\mathcal{P} = \begin{bmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{bmatrix}$$

$$p_i = 1 \, , if \ \{\alpha_0, \alpha_i\} \in E$$

- Adjacent Matrix

To model the information flow among followers

$$\mathcal{A}_N = [a_{ij}] \in \mathbb{R}^{N\times N}$$

$$\begin{cases} a_{ij} = 1, if \ \{\alpha_j, \alpha_i\} \in E \\ a_{ij} = 0, if \ \{\alpha_j, \alpha_i\} \notin E \end{cases}$$

- Laplacian Matrix

An induced matrix from adjacent matrix

$$\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$$

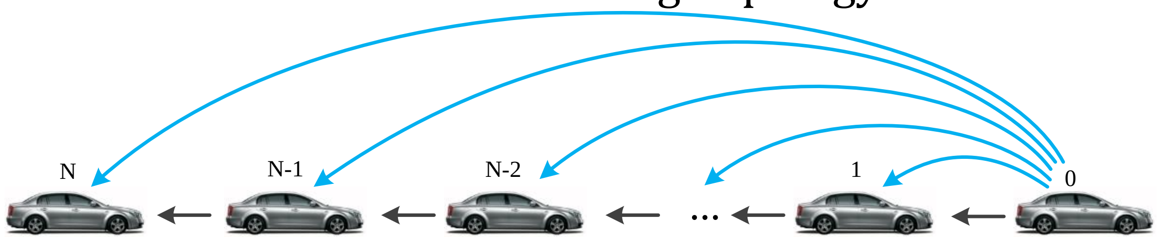
$$l_{ij} = \begin{cases} -a_{ij} \quad , & i \neq j \\ \sum_{k=1}^{N} a_{ik} \, , & i = j \end{cases}.$$

Problem Statements

- Model for information flow topology
  - Example 1: Bidirectional Topology

$$\mathcal{P} = \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix} \qquad \mathcal{A}_N = \begin{array}{cc} \begin{matrix} 1 & 2 & 3 & \cdots & N \end{matrix} & \\ \begin{bmatrix} 0 & 1 & & & \\ 1 & 0 & 1 & & \\ & 1 & 0 & \ddots & \\ & & \ddots & \ddots & 1 \\ & & & 1 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ N \end{matrix} \end{array} \qquad \mathcal{L} = \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & \ddots & \\ & & \ddots & \ddots & -1 \\ & & & -1 & 1 \end{bmatrix}$$

  - Example 2: Predecessor-leader following Topology



$$\mathcal{P} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \qquad \mathcal{A}_N = \begin{bmatrix} 0 & & & & \\ 1 & 0 & & & \\ & 1 & 0 & & \\ & & \ddots & \ddots & \\ & & & 1 & 0 \end{bmatrix} \qquad \mathcal{L} = \begin{bmatrix} 0 & & & & \\ -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix}$$

Problem Statements

- Model for Distributed Controller
  - Linear State Feedback Controller

$$u_i(t) = -\sum_{j\in \mathbb{I}_i}\left[k_p\left(p_i - p_j - d_{i,j}\right) + k_v\left(v_i - v_j\right) + k_a\left(a_i - a_j\right)\right]$$

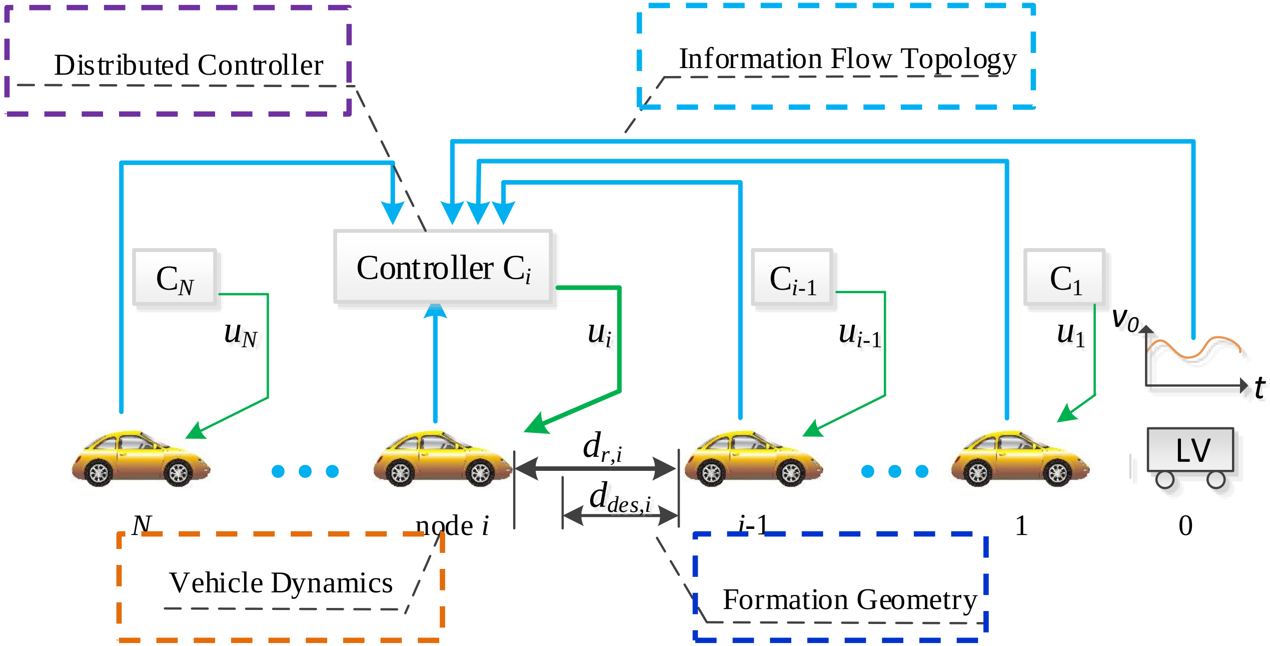
  - The local controller in node i only uses its neighborhood information specified by $\mathbb{I}_i$.
  - The controller is assumed to be linear for the convenience on theoretical analysis.
  - The controller in each node is assumed to be homogeneous.

- Formulation for the Closed-loop Dynamics of Platoons
  - Tracking error $\tilde{x}_i(t) = x_i(t) - x_0(t) - \tilde{d}_i, \quad \tilde{d}_i = \left[d_{0,i}, 0,0\right]^T$
  - Collective state vector $X = [\tilde{x}_1^T, \tilde{x}_2^T, \cdots, \tilde{x}_N^T]^T \in \mathbb{R}^{3N\times 1}$,
  - Collective input vector $U = [u_1, u_2, \cdots, u_N]^T \in \mathbb{R}^{N\times 1}$,
  - Controller $U = -(\mathcal{L} + \mathcal{P}) \otimes k^T \cdot X \qquad k = \left[k_p, k_v, k_a\right]^T$, $\otimes$ is Kronecker product.
  - Closed-loop Dynamics of platoon $\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} \cdot X$

Problem Statements

- Unified Closed-loop Dynamics of Platoons

$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} \cdot X$$



- Questions

Q1. What' the stabilizing region of controller gain k under different information flow topologies?

Q2. How to choose topology and design controller to improve stability?

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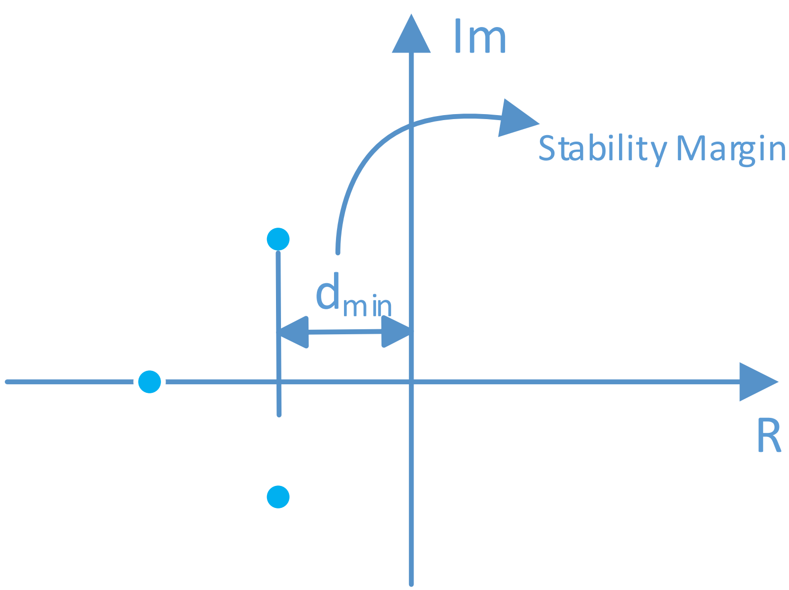
Performance Analysis considering V2V communication

Performance Definition

- **Closed-loop Stability:** A platoon with linear time-invariant dynamics is said to be closed-loop stable if and only if the closed-loop system has eigenvalues with **strictly negative real parts**
- **Stability Margin**: The stability margin of a platoon is defined as the absolute value of the real part of the least stable eigenvalue, which characterizes the convergence speed.



Performance Analysis considering V2V communication

1. Stability Region Analysis

- Unified Closed-loop Dynamics for vehicular platoons

$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} X$$

$$\boxed{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T \in \mathbb{R}^{3N \times 3N}}$$

- Requirements of Closed-loop Stability

$$\mathrm{Re}(\sigma_i(I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T)) < 0$$

✓ large-scale matrix, which is difficult to analyze directly.

- Dynamic decouple by using similarity transformation

✓ Key step: to decompose the large-scale vehicular platoon into multiple subsystems, which is easier to handle the closed-loop stability

$$S(I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T) = \boxed{\bigcup_{i=1}^{N} \{S(A - \lambda_i B k^T)\}}$$

$S(\cdot)$ is the spectrum of a matrix.
λ_i is the eigenvalue of $\mathcal{L} + \mathcal{P}$

✓ Multiple small-scale matrix, whose size is equal to that of node dynamics (n=3).

Performance Analysis considering V2V communication

1. Stability Region Analysis

- Dynamic decouple by using similarity transformation

$$S(I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes Bk^T) = \bigcup_{i=1}^{N} \{S(A - \lambda_i Bk^T)\}$$

- Routh–Hurwitz stability criterion

$$|sI - (A - \lambda_i Bk^T)| = s^3 + \frac{\lambda_i k_a + 1}{\tau} s^2 + \frac{\lambda_i k_v}{\tau} s + \frac{\lambda_i k_p}{\tau}.$$

$$\begin{array}{lcc}
s^3 & 1 & \dfrac{\lambda_i k_v}{\tau} \\[2ex]
s^2 & \dfrac{\lambda_i k_a + 1}{\tau} & \dfrac{\lambda_i k_p}{\tau} \\[2ex]
s^1 & \dfrac{\lambda_i k_v(\lambda_i k_a + 1) - \lambda_i k_p \tau}{\tau(\lambda_i k_a + 1)} & \\[2ex]
s^0 & \dfrac{\lambda_i k_p}{\tau} &
\end{array}$$

- ✓ Real coefficient polynomials.
- ✓ It needs λ_i to be positive real number.

$$\begin{cases} k_p > 0 \\ k_v > k_p \tau / (\lambda_i k_a + 1) \\ k_a > -1/\lambda_i \end{cases}$$

20

Performance Analysis considering V2V communication

1. Stability Region Analysis

Consider a homogeneous platoon with linear controllers given by

$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} X$$

(1.1) If graph G satisfies certain conditions (all the eigenvalues of $\mathcal{L} + \mathcal{P}$ are positive real numbers), the platoon is asymptotically stable if and only if

$$\begin{cases} k_p > 0 \\ k_v > k_p \tau / \min(\lambda_i k_a + 1) \\ k_a > -1/\max(\lambda_i) \end{cases}$$

Remarks

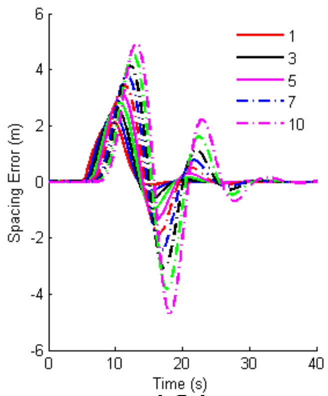
- λ_i is the eigenvalue of $\mathcal{L} + \mathcal{P}$ (λ_i need to be positive real number).
- This result can cover a lot of information flow topologies, including all the aforementioned topologies
- The influence of information flow topology on stability is mainly reflected by λ_i.

Performance Analysis considering V2V communication

- **1. Stability Region Analysis**– Simulation Results

$$\boxed{k_p = 1, k_v = 2, k_a = 1, \tau{=}0.5}$$

$$\begin{cases} k_p > 0 \\ k_v > k_p\tau/\min(\lambda_i k_a + 1) \\ k_a > -1/\max(\lambda_i) \end{cases}$$ **Stable**



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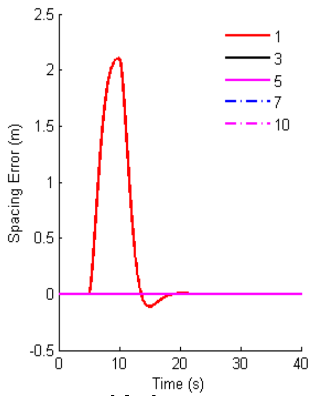
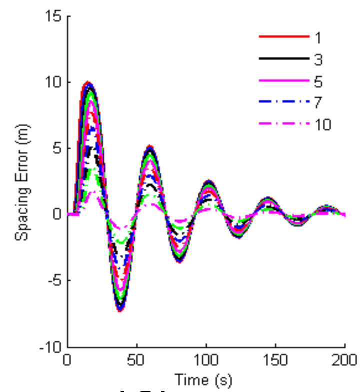
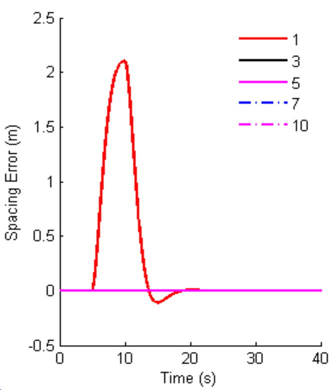


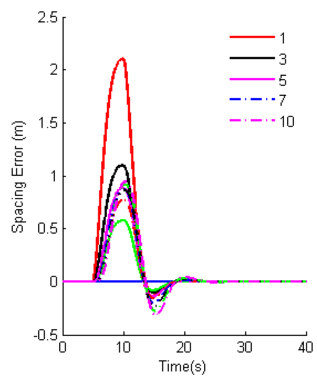
(b)



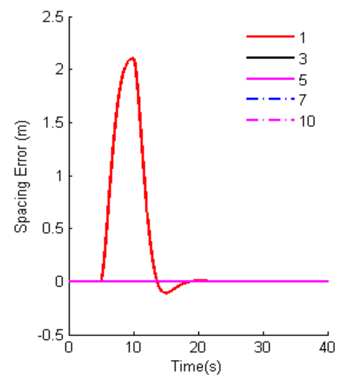
(c)



(d)



(e)



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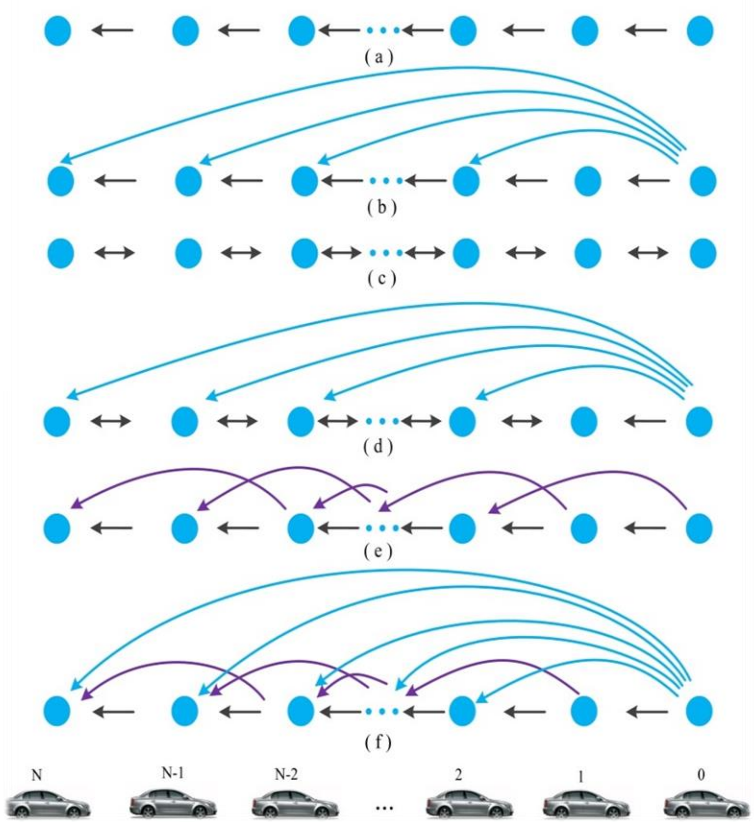


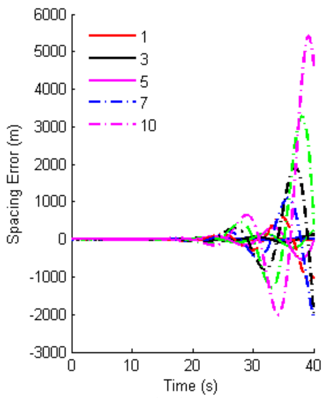
Performance Analysis considering V2V communication

- **1. Stability Region Analysis**– Simulation Results

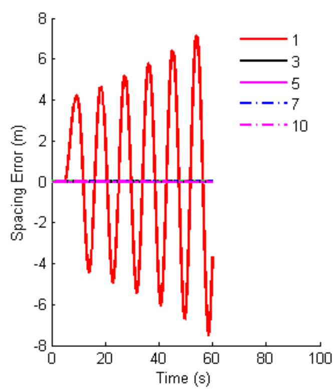
$$\boxed{k_p = 1, k_v = 0.2, k_a = 1, \tau{=}0.5}$$

$$\begin{cases} k_p > 0 \\ k_v > k_p \tau / \min(\lambda_i k_a + 1) \\ k_a > -1/\max(\lambda_i) \end{cases}$$

Unstable



(a)



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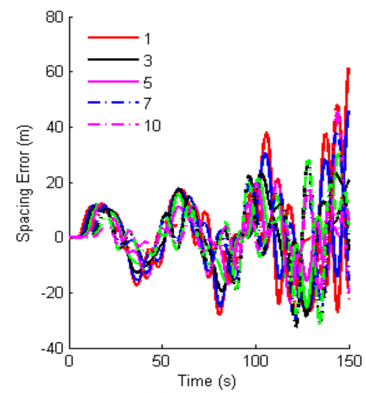

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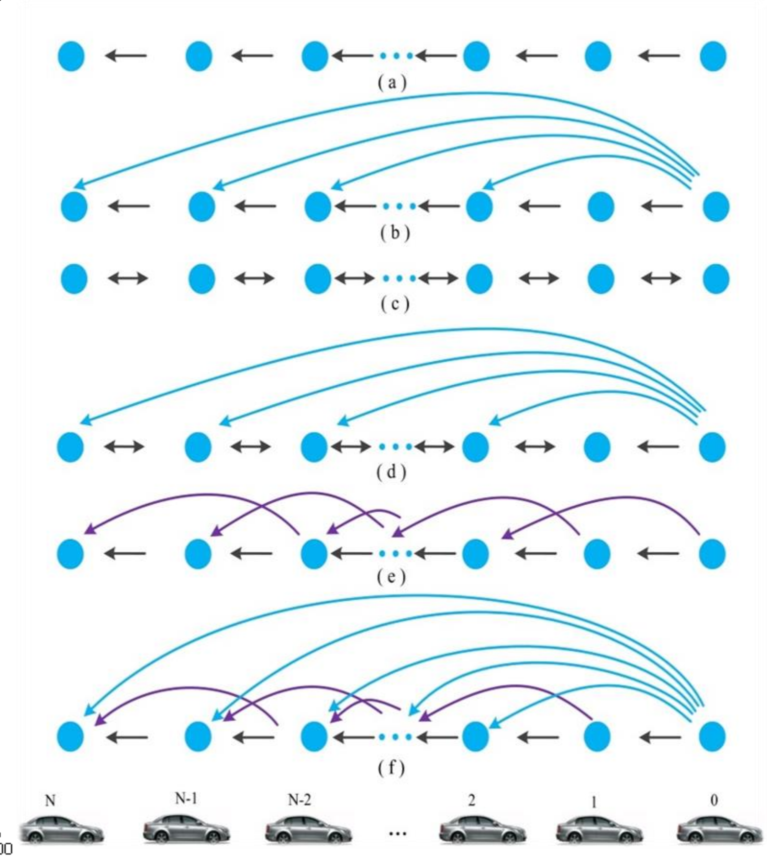

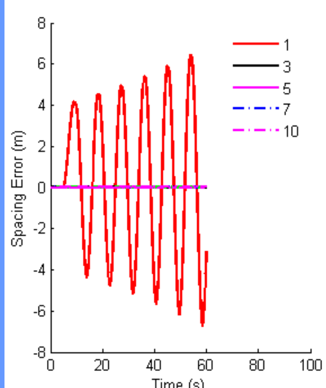

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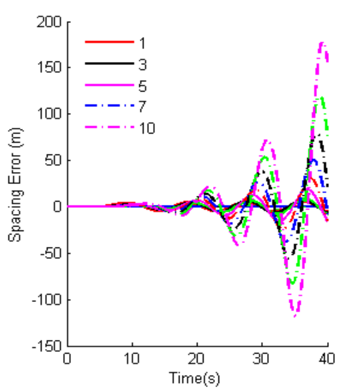

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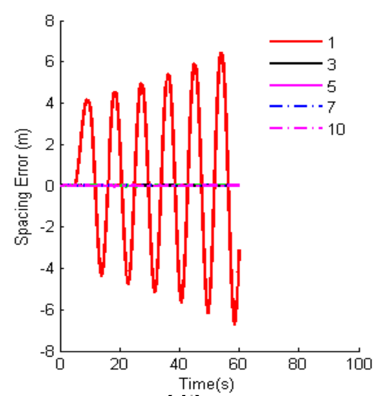


(f)

Performance Analysis considering V2V communication

2. Scaling trend of Stability Margin with increasing platoon size

Consider a homogeneous platoon with linear controllers given by

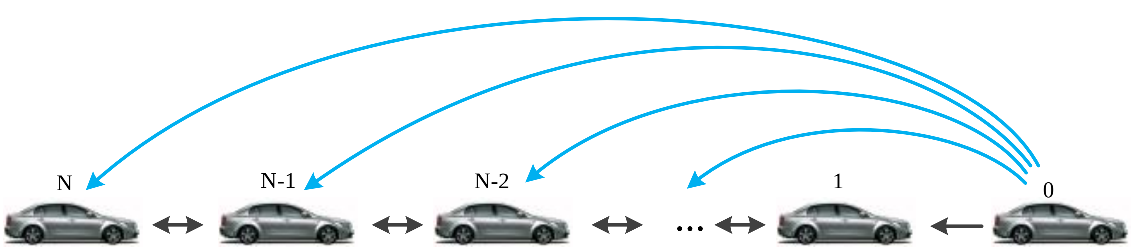
$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} X$$

(2.1) if the graph G is in Bidirectional topology, then the stability margin of platoons decays to zero as $O(1/N^2)$

(2.2) if the graph G is in BDL topology, then the stability margin of platoons is always bounded away from zero.

Bidirectional (BD) topology

- Decay to zero with increasing size;
- Independent with controller gains.



Bidirectional-leader (BDL) topology

- Will not decay to zero as platoon size increases.

Performance Analysis considering V2V communication

2. Scaling trend of Stability Margin with increasing platoon size - Proof

Step. 1 Closed-loop Dynamics

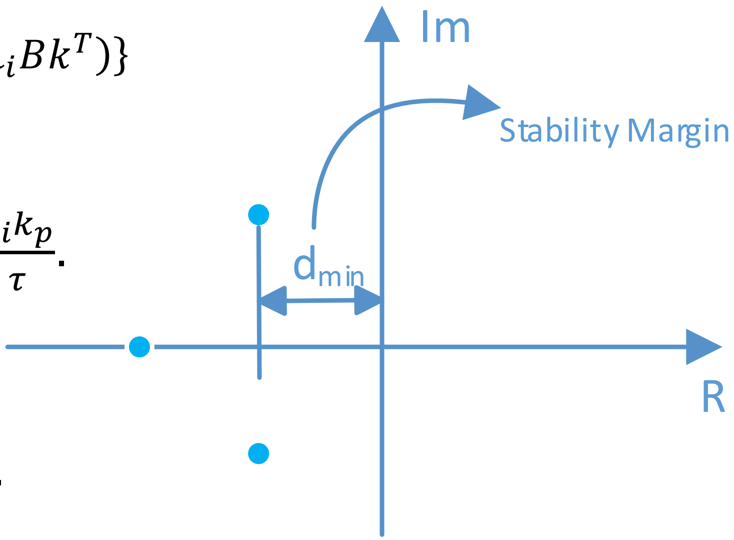
$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} X \qquad I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T \in \mathbb{R}^{3N \times 3N}$$

Step. 2 Similarity transformation

$$S(I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T) = \bigcup_{i=1}^{N} \{S(A - \lambda_i B k^T)\}$$

Step. 3 Eigenvalue Analysis

$$|sI - (A - \lambda_i B k^T)| = s^3 + \frac{\lambda_i k_a + 1}{\tau} s^2 + \frac{\lambda_i k_v}{\tau} s + \frac{\lambda_i k_p}{\tau}.$$



Stability Margin $\quad d_{\min} = |\mathrm{Re}(s_{\min})| = O(\lambda_{\min}).$

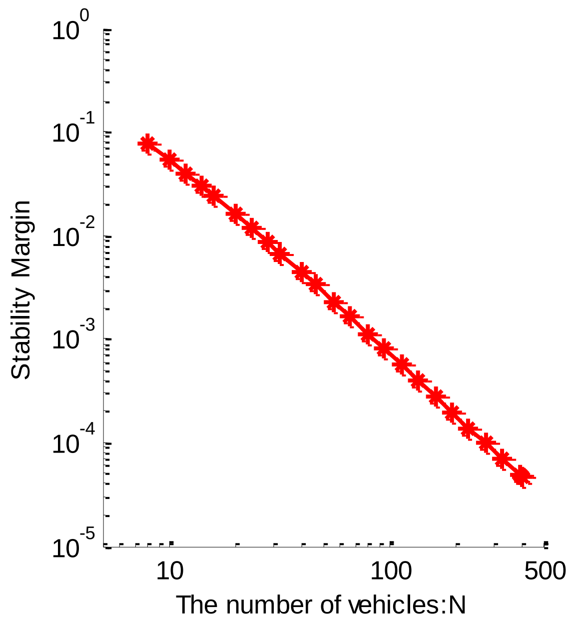
- For Bidirectional topology $\quad d_{\min} = O(\lambda_{\min}) = O(1/N^2)$ $\quad$ decay to zero
- For Bidirectional-leader topology $\quad d_{\min} = O(\lambda_{\min})$ =constant number $\quad$ Independent with size

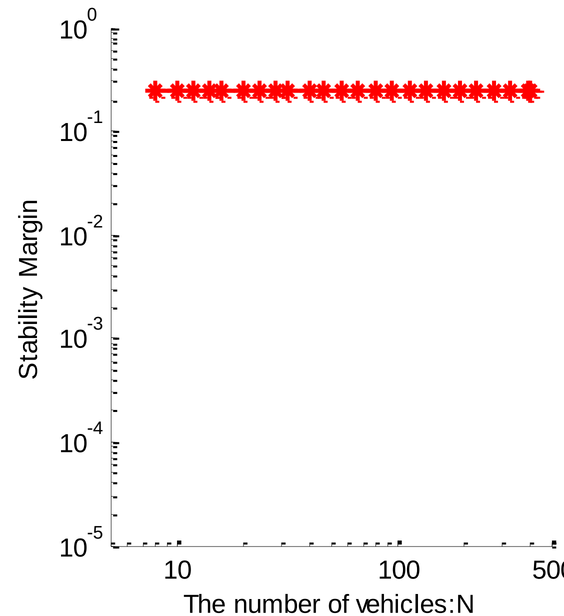
Performance Analysis considering V2V communication

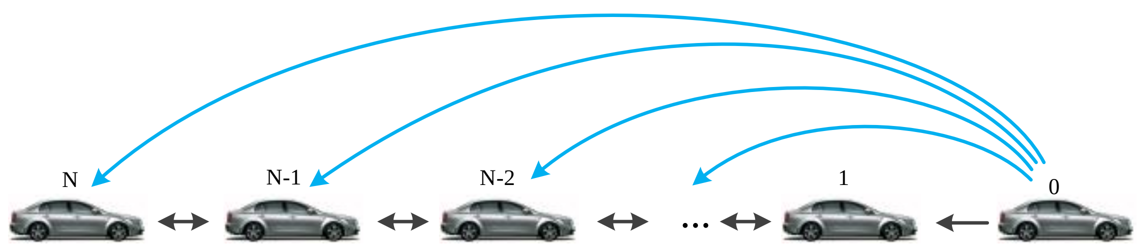
- **2. Scaling trend of Stability Margin with increasing platoon size** - Simulations

$k_p = 1, k_v = 2, k_a = 1, \tau=0.5$

Bidirectional topology







Bidirectional-leader topology

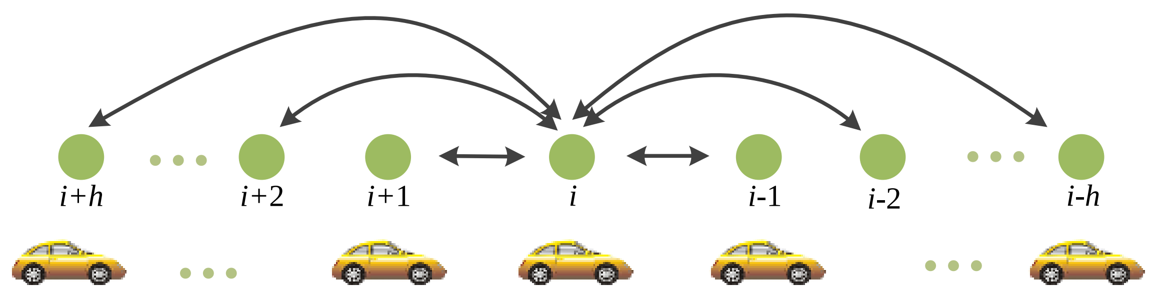
Performance Analysis considering V2V communication

- **3. Stability Margin Improvement** – Topology Selection

Consider a platoon with linear controllers given by

$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} X$$

(3.1) if the graph G is undirected, to maintain bounded stability margin, it needs at least lots of followers (i.e. $\Omega(N) = O(N)$) to obtain the leader's information.



- Remarks

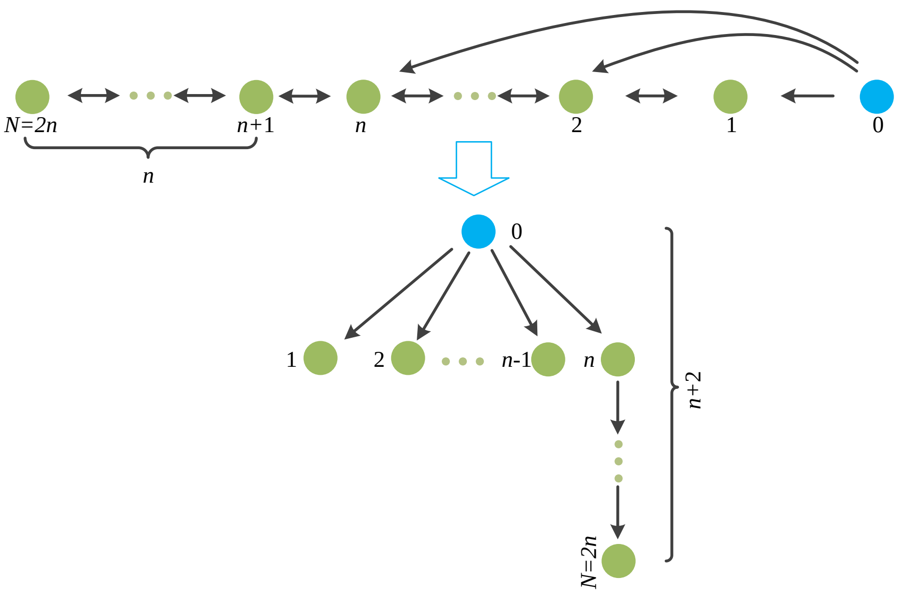
➢ BD topology is a special case, i.e., $\Omega(N)=1$, for which the stability margin decays to zero as the platoon size increases;

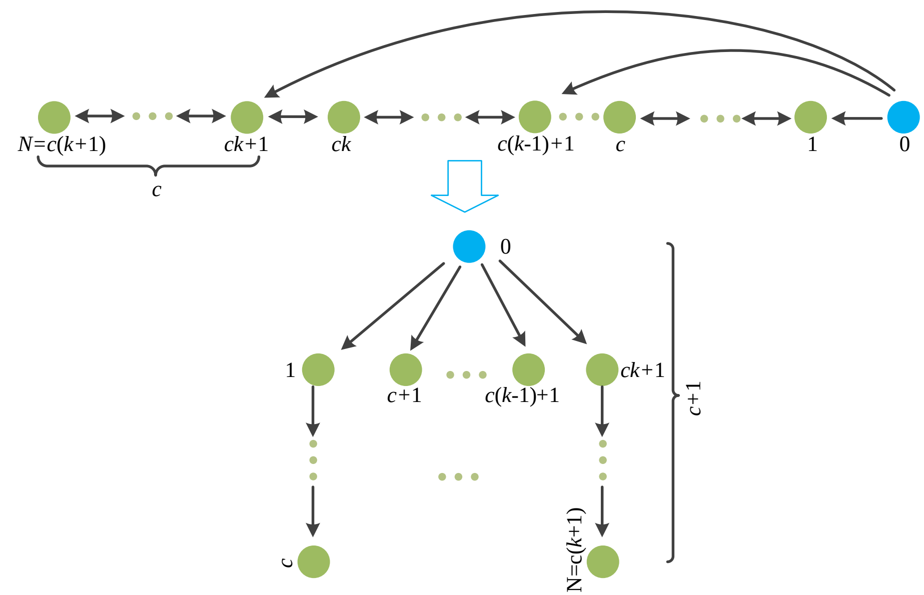
➢ It implies that the information flow from the leader is more important than that among the followers.

Performance Analysis considering V2V communication

3. Stability Margin Improvement – Topology Selection

- ✓ To maintain bounded stability margin, the tree depth of graph G should be a constant number and independent of the platoon size N.
  - Tree dpeth $c = \max\{n_1, n_2 - n_1, \cdots, n_p - n_{p-1}, N - n_p + 1\}$, where $\{n_1, n_2, \cdots, n_p\}, 1 \le n_1 \le \cdots \le n_p \le N$ is the set of followers pinned to the leader



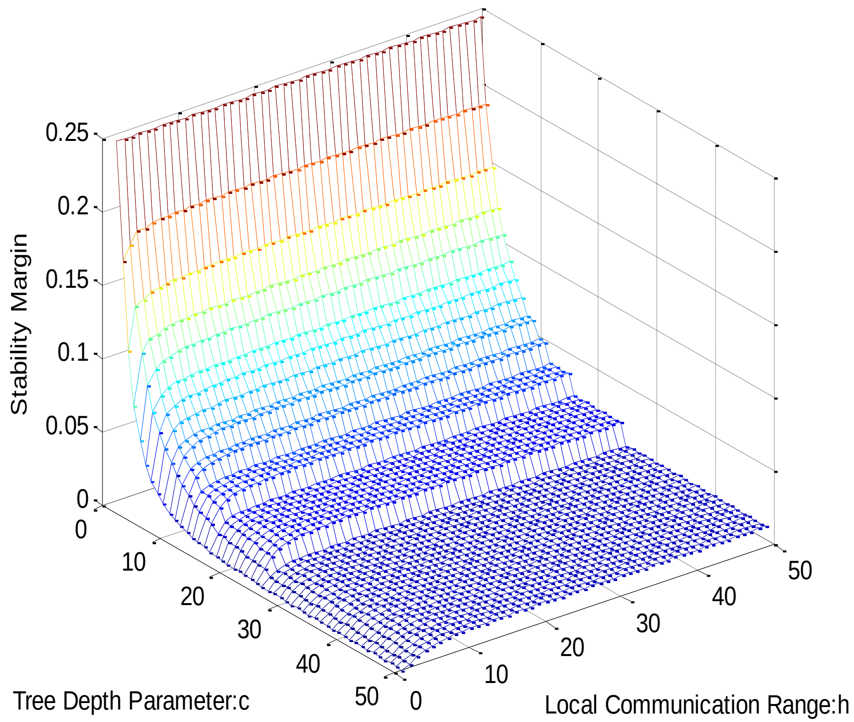


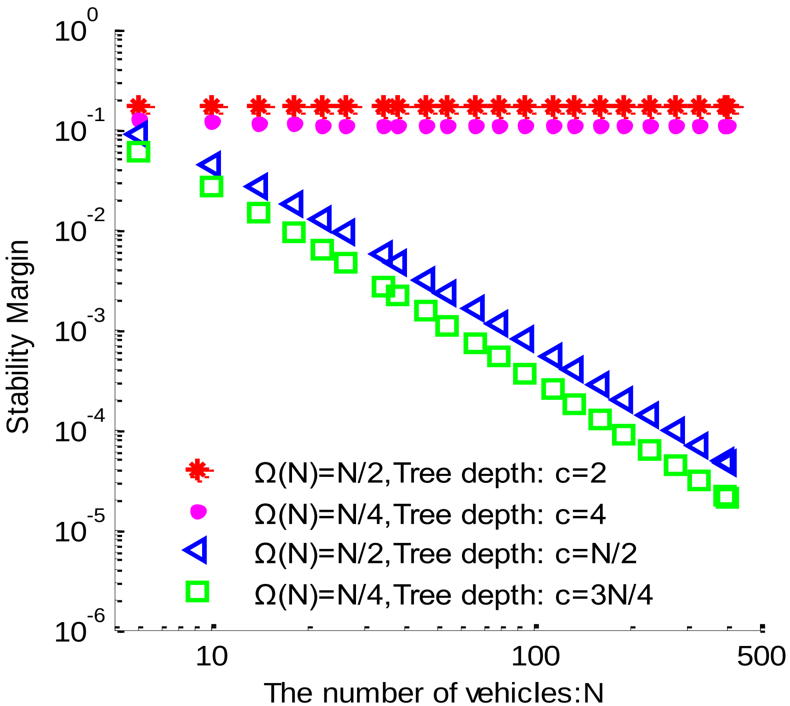
- ✓ Tree depth is $N/2$, increasing with platoon size;
- ✓ Stability margin decay to zero;

- ✓ Tree depth is c, a constant number, independent with platoon size;
- ✓ Stability margin can be bounded away from zero

Performance Analysis considering V2V communication

3. Stability Margin Improvement – Topology Selection





- It is the tree depth *c* rather than local communication range *h* that dominates the stability margin.
- Extending information flow to reduce the tree depth is one major way to guarantee a bounded stability margin.

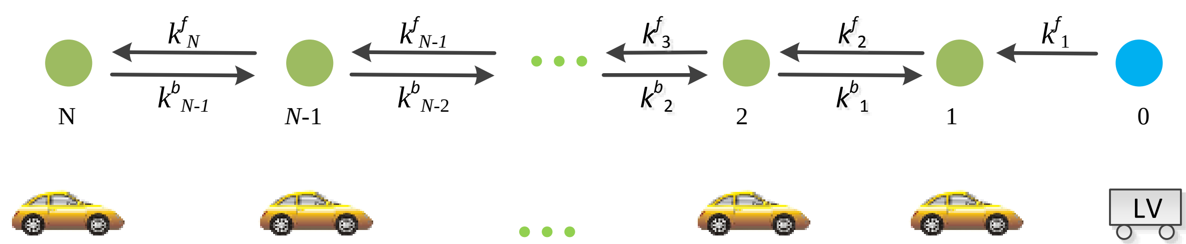
Performance Analysis considering V2V communication

3. Stability Margin Improvement – Asymmetric Control

Consider a homogeneous platoon under the BD topology with the asymmetric controller architecture given by

$$\dot{X} = \{I_N \otimes A - (\mathcal{L}_{BD} + \mathcal{P}_{BD})_\epsilon \otimes B k^T\} X$$

(3.1) For any fixed $\epsilon \in (0,1)$, the stability margin is bounded away from zero and independent of the platoon size N (N can be any finite integer).



Asymmetric control

The controller is called asymmetric, if

$$\begin{cases} k_i^f = (1+\epsilon)k, k_i^b = (1-\epsilon)k & i = 1, \cdots, N-1 \\ k_N^f = (1+\epsilon)k, & \end{cases}$$

where $\epsilon \in (0,1)$ is called the asymmetric degree. Note that if $\epsilon = 0$, then it is reduced to the symmetric case.

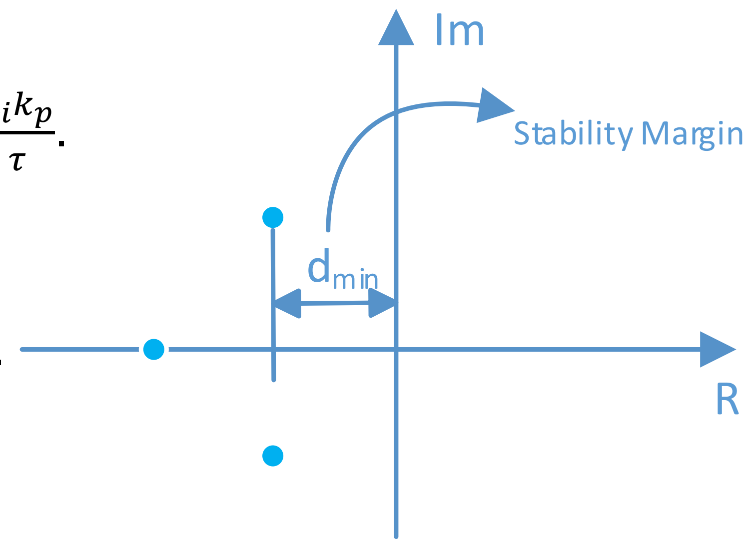
Performance Analysis considering V2V communication

- **3. Stability Margin Improvement** – Asymmetric Control- **Proof**

Eigenvalue Analysis

$$|sI - (A - \lambda_i B k^T)| = s^3 + \frac{\lambda_i k_a + 1}{\tau} s^2 + \frac{\lambda_i k_v}{\tau} s + \frac{\lambda_i k_p}{\tau}.$$

Stability Margin $\quad \mathrm{d_{min}} = |\mathrm{Re}(\mathrm{s_{min}})| = \mathrm{O}(\lambda_{\mathrm{min}}).$



- For asymmetric control

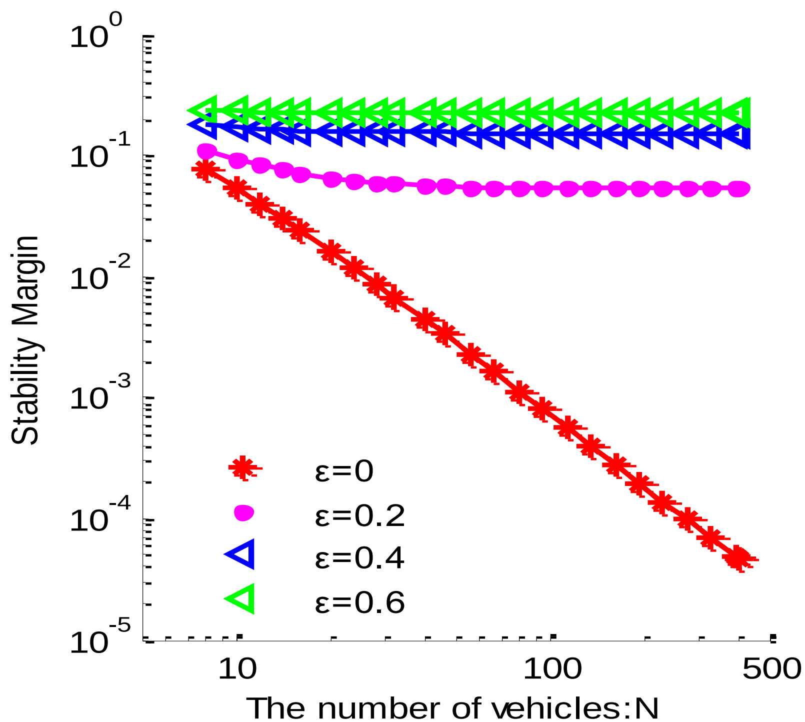
$\mathrm{d_{min}} = \mathrm{O}(\lambda_{\mathrm{min}}) = \text{constant number}, \text{ For any fixed } \epsilon \in (0,1)$

Independent with size

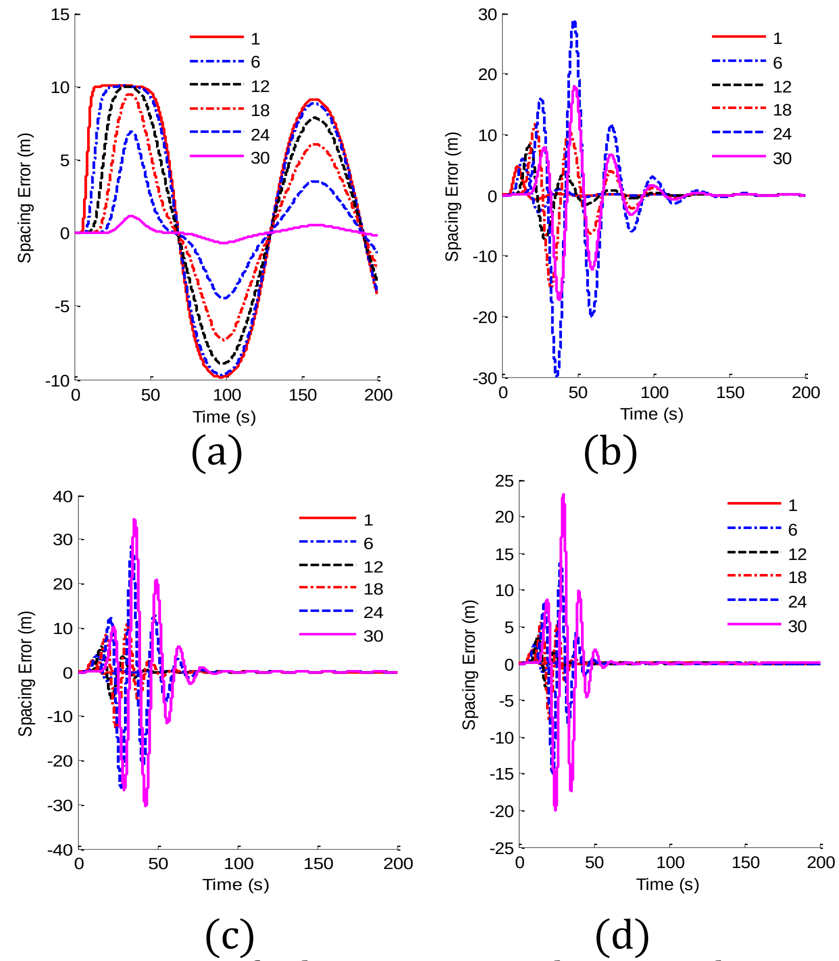
- Tradeoff between Convergence Speed and Transient Performance
  - Benefit: bounded stability margin, → good for convergence speed
  - Cost: overshooting phenomena in transient process.

Performance Analysis considering V2V communication

3. Stability Margin Improvement – Asymmetric Control



- The stability margin of a platoon with asymmetric controllers is indeed bounded away from zero and independent of with the platoon size.



Space errors for homogeneous platoon under BD topology with different asymmetric degree ϵ. (a) ϵ=0 (symmetric); (b) ϵ=0.2; (c) ϵ=0.4; (d) ϵ=0.6

Performance Analysis considering V2V communication

- **4. Linear Stable Controller Design -** solving a Riccati equation

Consider a homogeneous platoon with linear controllers given by

$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} X$$

- Stability Region

$$\begin{cases} k_p > 0 \\ k_v > k_p \tau / \min(\lambda_i k_a + 1) \\ k_a > -1/\max\boxed{(\lambda_i)} \end{cases}$$

How to choose a specific controller gain?

- Controller design by solving a Riccati equation

$$A^T P_\varepsilon + P_\varepsilon A - P_\varepsilon B B^T P_\varepsilon + \varepsilon I_3 = 0,$$

Choosing the controller gain as $k^T = \alpha B^T P_\varepsilon$

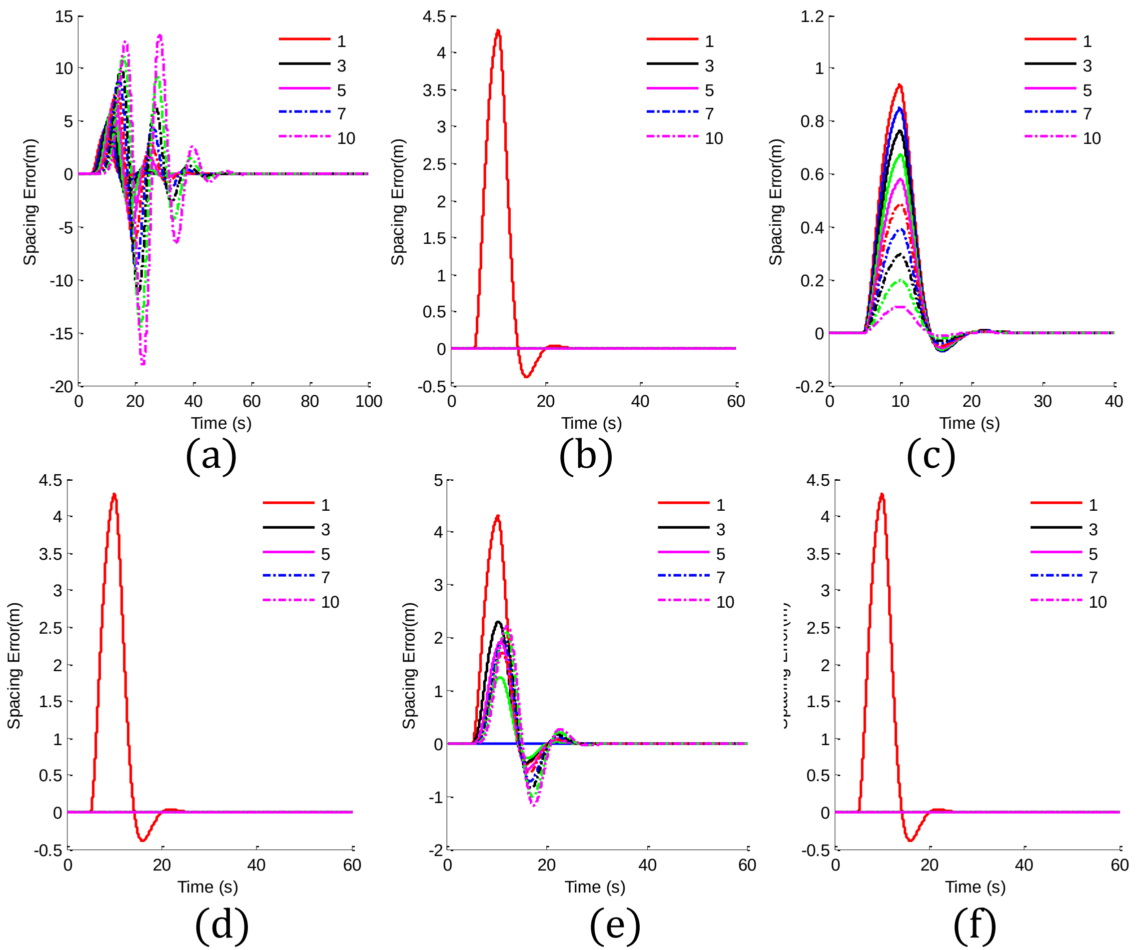
- Closed-loop Stability requirements

$$\alpha \geq \frac{1}{2 \min_{i \in \mathcal{N}} \boxed{(\lambda_i)}}$$

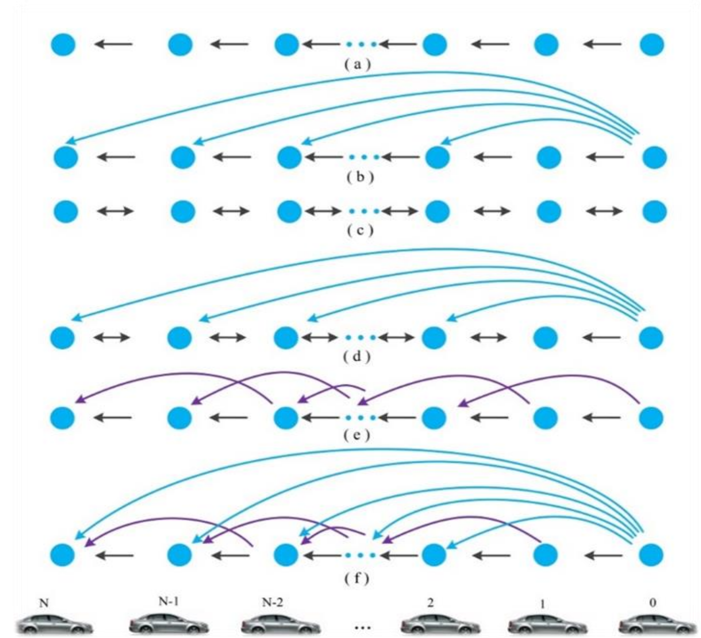
Influence of Information flow topology

Performance Analysis considering V2V communication

4. Linear Stable Controller Design - Simulation



Topology	λ_{min}	α	ε
PF	1	0.5	1
PLF	1	0.5	1
BD	0.022	22.5	1
BDL	1	0.5	1
TPF	1	0.5	1
TPLF	1	0.5	1



Outline

- Background
- Problem Statements
- Performance Analysis considering V2V communication
- Conclusions

Conclusions

- Vehicular platoon can bring many potential benefits, e.g.,
  - Improving traffic capacity; Enhancing highway safety; Reducing road congestion
- V2V communication can generate various types of topologies for platoon.
- For vehicular platoons under “homogeneity + linear feedback”
  - **1) Stability Region Analysis**
    - Explicitly established the stabilizing thresholds of linear controller gains for platoons under different information flow topologies.
  - **2) Stability Margin Scaling Trend**
    - Obtained stability margin scaling trend for platoons under two typical topologies, i.e., Bidirectional Topology and Bidirectional-leader Topology.
  - **3) Stability Margin Improvement**
    - Proposed two basic ways to improve the stability margin, i.e., topology selection and controller adjustment.
  - **4) Linear Stable Controller Design**
    - Converted the platoon control problem to a parametric algebraic Riccati equation. The designed controllers can guarantee the internal stability for a variety of topologies

Thanks for your attention

Q & A?

Appendix: Robust issue

Robust Performance

Disturbance with Finite Energy

$$\|w_i(t)\|_{\mathcal{L}_2} = \int_0^{+\infty} (w_i(t))^2 dt < \infty$$

1. First-to-last amplification factor (AF_{f2l})

$$AF_{f2l} = \sup \frac{\|\tilde{p}_N\|_{\mathcal{L}_2}}{\|w_1\|_{\mathcal{L}_2}} = \left\|G_{f2l}(s)\right\|_{\mathcal{H}_\infty}$$

2. All-to-all amplification factor(AF_{a2a})

$$AF_{a2a} = \sup \frac{\|\tilde{p}\|_{\mathcal{L}_2}}{\|W\|_{\mathcal{L}_2}} = \|G_{a2a}(s)\|_{\mathcal{H}_\infty}$$

Dynamics in Frequency domain

Time domain

$$\dot{X} = \{I_N \otimes A - (\mathcal{L} + \mathcal{P}) \otimes B k^T\} \cdot X + B \cdot W \qquad Y = C \cdot X$$

Frequency domain

$$G(s) = \frac{\mathcal{L}(Y)}{\mathcal{L}(W)} = C(sI_{3N} - A_{cl})^{-1} B$$

$$= \left[I_N \otimes (\tau s^3 + s^2) + (\mathcal{L} + \mathcal{P}) \otimes \left(k_p + k_v s + k_a s^2\right)\right]^{-1}$$

- It's very difficult to analysis the $\mathcal{H}_\infty$ norm of transfer function $G(s)$ under general information flow topology.

Appendix: Robust issue

1. Scaling Trend of $\mathcal{H}_\infty$ norms under PF topology

Consider a homogeneous platoon under the PF topology with linear controllers given by

$$G(s) = \left[I_N \otimes (\tau s^3 + s^2) + (\mathcal{L} + \mathcal{P}) \otimes \left(k_p + k_v s + k_a s^2\right)\right]^{-1}$$

(2.1) the amplification factors AF_{f2l} and AF_{a2a} satisfy the following conditions

$$\beta_1 \alpha^{N-1} \le AF_{f2l} \le \beta_2 \alpha^{N-1} \qquad \beta_1 \alpha^{N-1} \le AF_{a2a} \le \frac{\beta_2(\alpha^N - 1)}{\alpha - 1}$$

where, β_1, β_2 is constant real number, $\alpha > 1$.

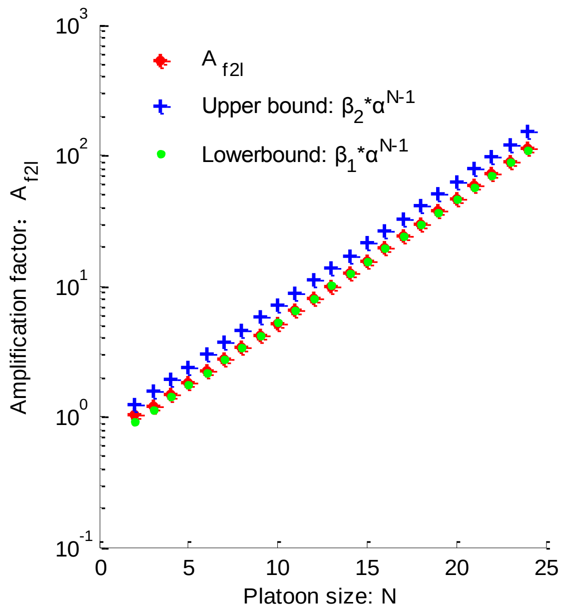
Fig. Predecessor-following (PF) topology

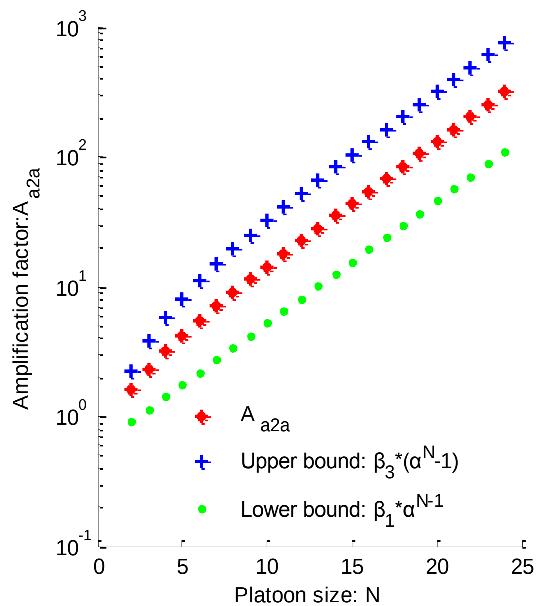
Remarks

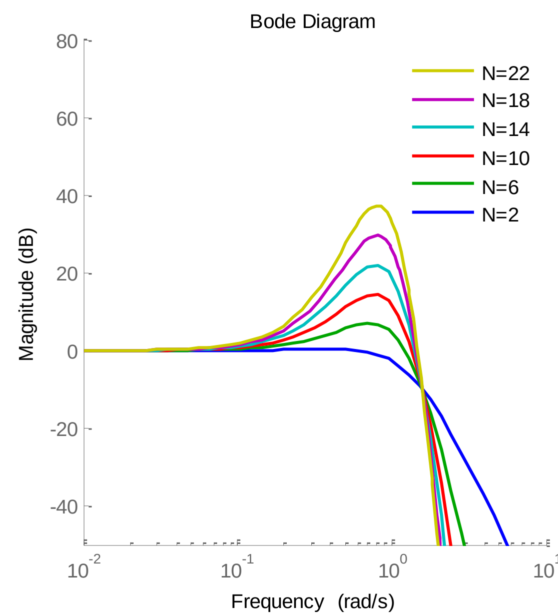
- The amplification factors will exponentially grow with increasing platoon size.
- These results are independent with controller gains, which means this is a fundamental drawback of PF topology when using identical linear controller

Appendix: Robust issue

1. Scaling Trend of $\mathcal{H}_\infty$ norms under PF topology-simulation results



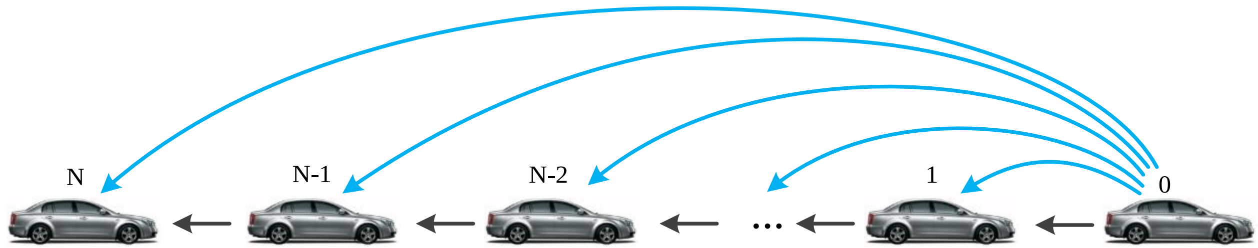


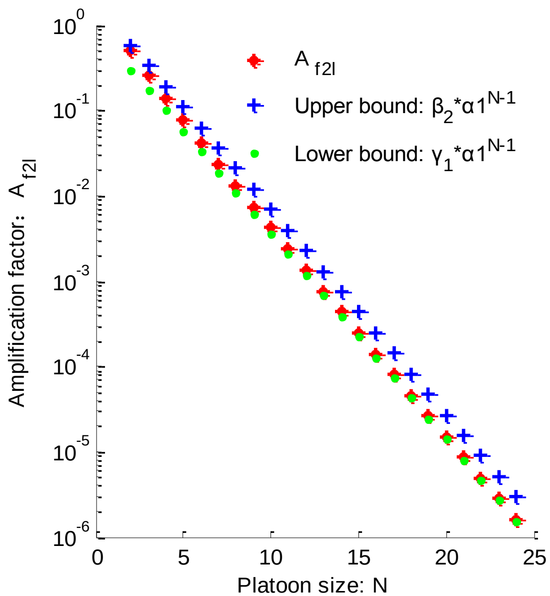


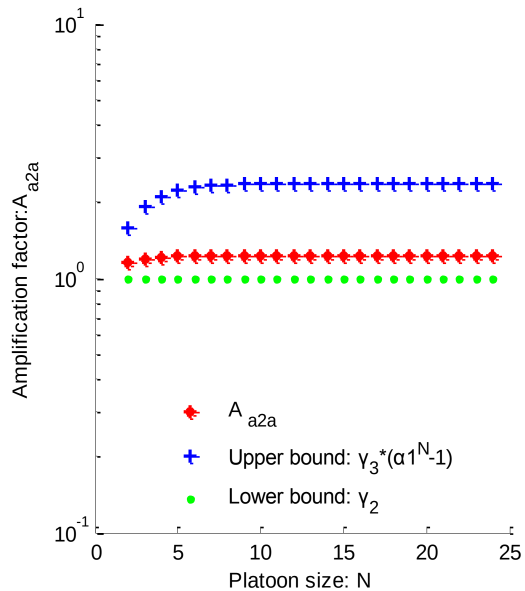
- ☐ The amplification factors AF_{f2l} and AF_{a2a} indeed exponentially grow with increasing platoon size.

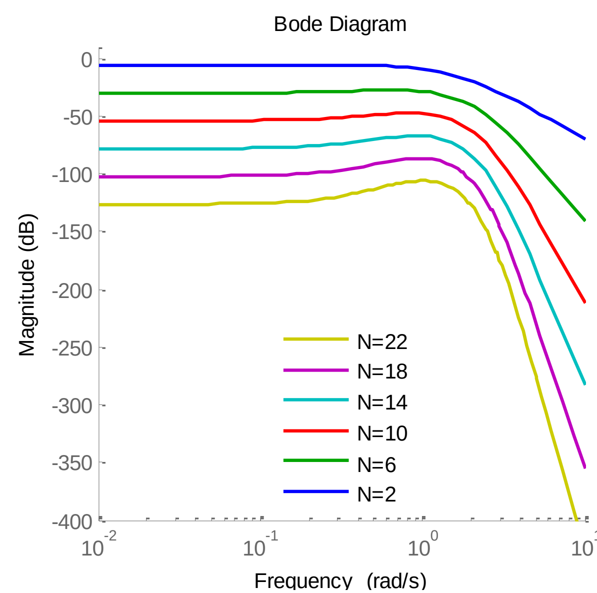
Appendix: Robust issue

2. Scaling Trend of $\mathcal{H}_\infty$ norms under PLF topology-simulation results









- If the leader broadcast its information to all the following vehicle, resulting in predecessor-leader following topology, then amplification factor will become better

Appendix: Robust issue

3. Scaling Trend of $\mathcal{H}_\infty$ norms under BD topology

Consider a homogeneous platoon under the BD topology with linear controllers given by

$$G(s) = \left[I_N \otimes (\tau s^3 + s^2) + (\mathcal{L} + \mathcal{P}) \otimes \left(k_p + k_v s + k_a s^2\right)\right]^{-1}$$

(4.1) the amplification factor AF_{a2a} at least increase with the platoon size as $O(N^2)$