Forecasting models for cost evolution of network components

and

Risk analysis based on uncertainties in demand forecasts and cost predictions

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Forecasting models for cost evolution of network components

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Agenda

- **Write and Crawford's learning curve model**
- **The extended learning curve model**
- **Discussion of different type of parameters in the models**
- **Examples**
- **Conclusion on cost prediction models**

Learning curve

T. P. Wright proposed the concept of learning curves:

$$T_n = n^{-\alpha} \cdot T_0$$

where $\mathbf{T_n}$ is the average production time for **n** units, and $\mathbf{T_0}$ is the time to complete the first unit.

J.R.Crawford applied the same formula, but interpreted $\mathbf{T_n}$ to be the completing time for the $\mathbf{n^{th}}$ unit in a series.

Let us assume that the component cost (price) P_n is proportional to the production time T_n.

Learning curve coefficient K

$$P_n = n^{-\alpha} P_0$$

P_n is the average cost for the n^{th} unit.

The learning curve coefficient is defined by:

$$P_{2n} = K \cdot P_n$$

Then

$$K = (2)^{-\alpha}$$

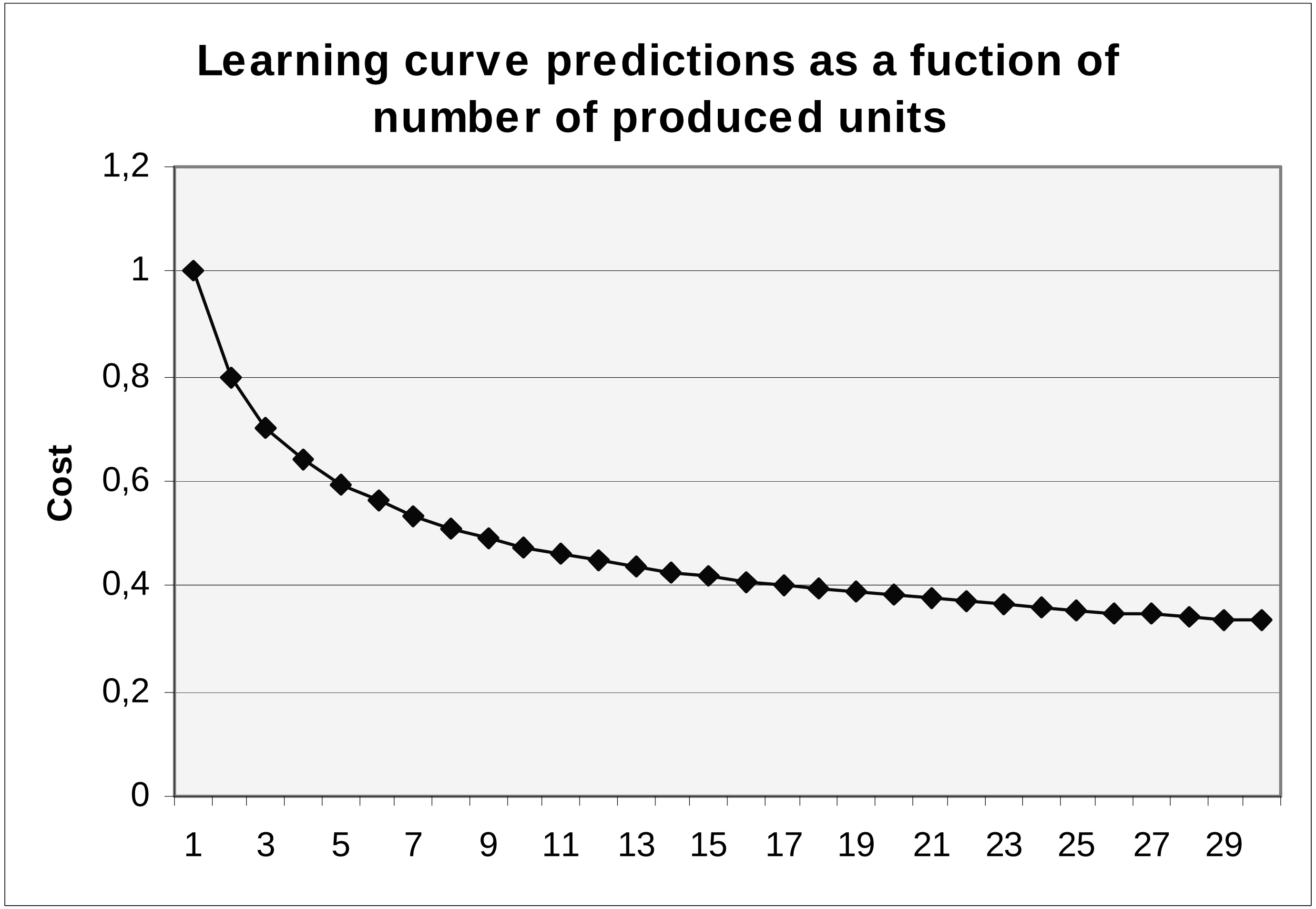
$$\alpha = -\log_2 K$$

Relevant K values of different network components

LearningCurveClass	K_Value
CivilWorks	100,00%
CopperCable	100,00%
Electronics	80,00%
SitesAndEnclosures	100,00%
FibreCable	90,00%
Installation (constant)	100,00%
AdvancedOpticalComponents	70,00%
Installation (decresing)	85,00%
OpticalComponents	80,00%

Learning curve prediction

K = 0,8 or α = 0,32



What we need:

Cost as a function of time

The answer

To combine the learning curves with volume forecasts of components.

$$P(t) = n(t)^{-\alpha} P(0) = n(t)^{-\log_2 K} P(0)$$

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To combine the learning curves with volume forecasts of components.

$$P(t) = n(t)^{-\alpha} P(0) = n(t)^{-\log_2 K} P(0)$$

(Olsen , Stordahl 1993) Extended learning curve model is given by inserting a Logistic model into the learning curve model

$$n(t) = M \cdot [1 + e^{(a+b \cdot t)}]^{-\gamma}$$

Parameters in the model

- **K or α:** Learning curve coefficient
- **P(0):** Production cost, unit no 1
- **M:** Saturation
- **a:** Parameter in the Logistic model
- **b:** Parameter in the Logistic model
- **γ:** Parameter in the Logistic model

Reformulation of the parameters

The normalized Logistic model is:

$$\mathbf{n_r\,(t) = n(t)/\,M}$$

The aggregated production volume the reference year, 0:

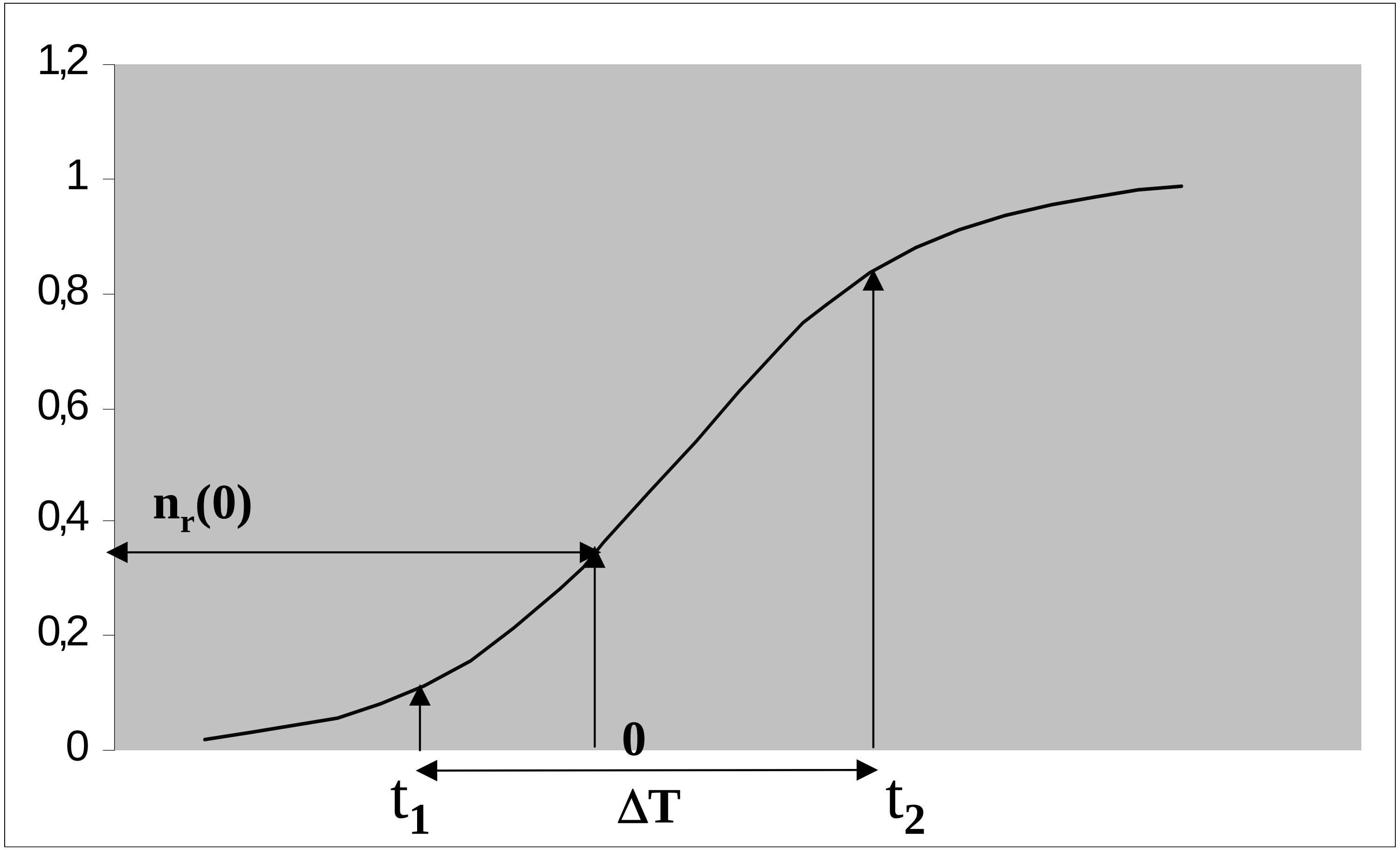
$$\mathbf{n_r\,(0)}$$

The growth period:

$$\mathbf{n_r\,(t_1) = 0{,}1} \qquad \mathbf{n_r\,(t_2) = 0{,}9}$$

$$\mathbf{\Delta T = t_1 - t_2}$$

Interpretation of the parameters



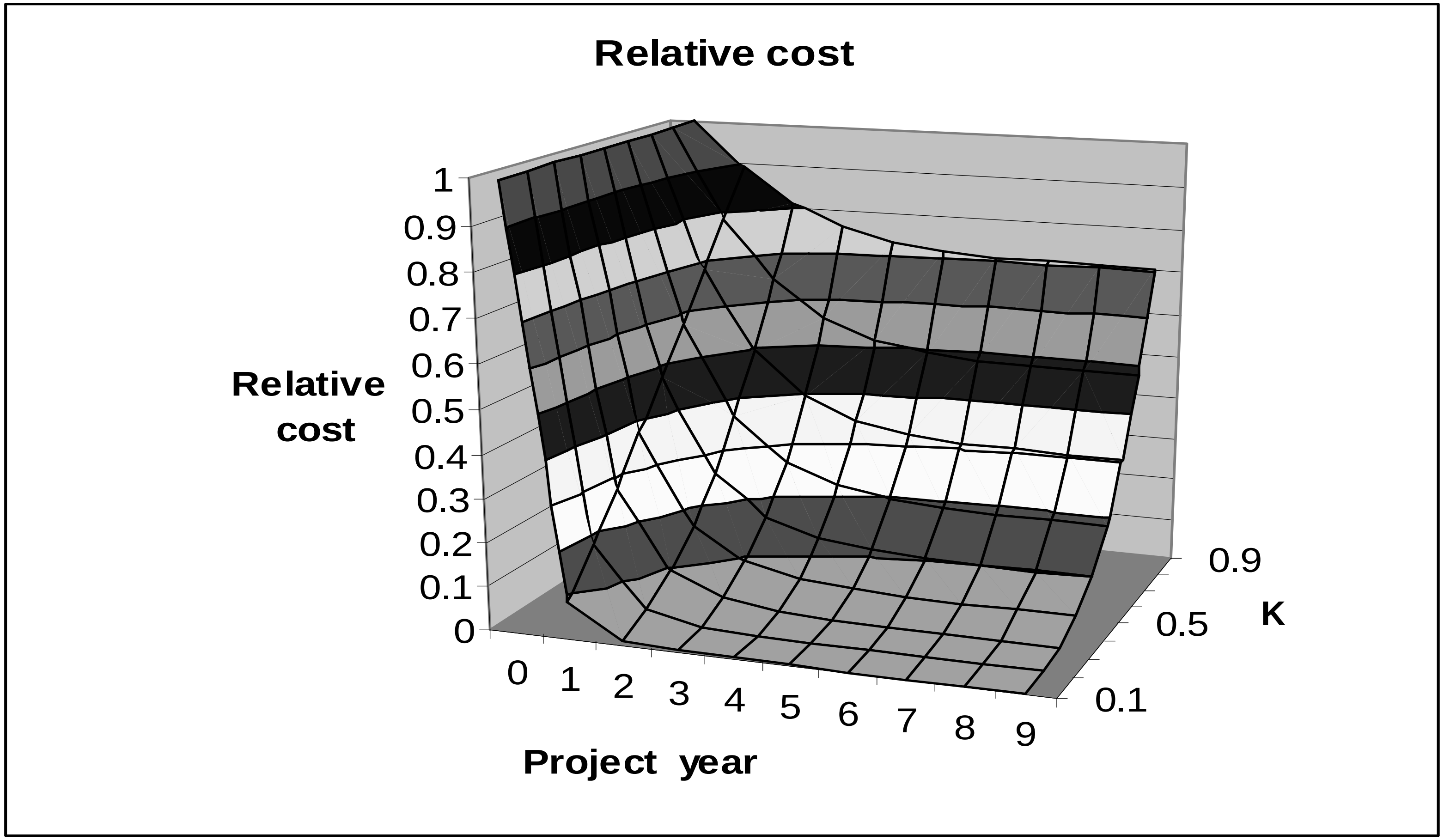
The extended learning curve ($\gamma = 1$)

$$P(t) = P(0) \cdot \left[n_r(0)^{-1} \cdot \left(1 + e^{\left\{ \ln\left[n_r(0)^{-1} - 1 \right] - \left[\frac{2 \cdot \ln 9}{\Delta T} \right] \cdot t \right\}} \right)^{-1} \right]^{\log_2 \cdot K}$$

- P(0): Production cost the reference year (0)
- $n_r(0)$: Relative accumulated production volume the reference year
- ΔT: Time for the accumulated volume to grow from 10% to 20%
- K: Learning curve coefficient

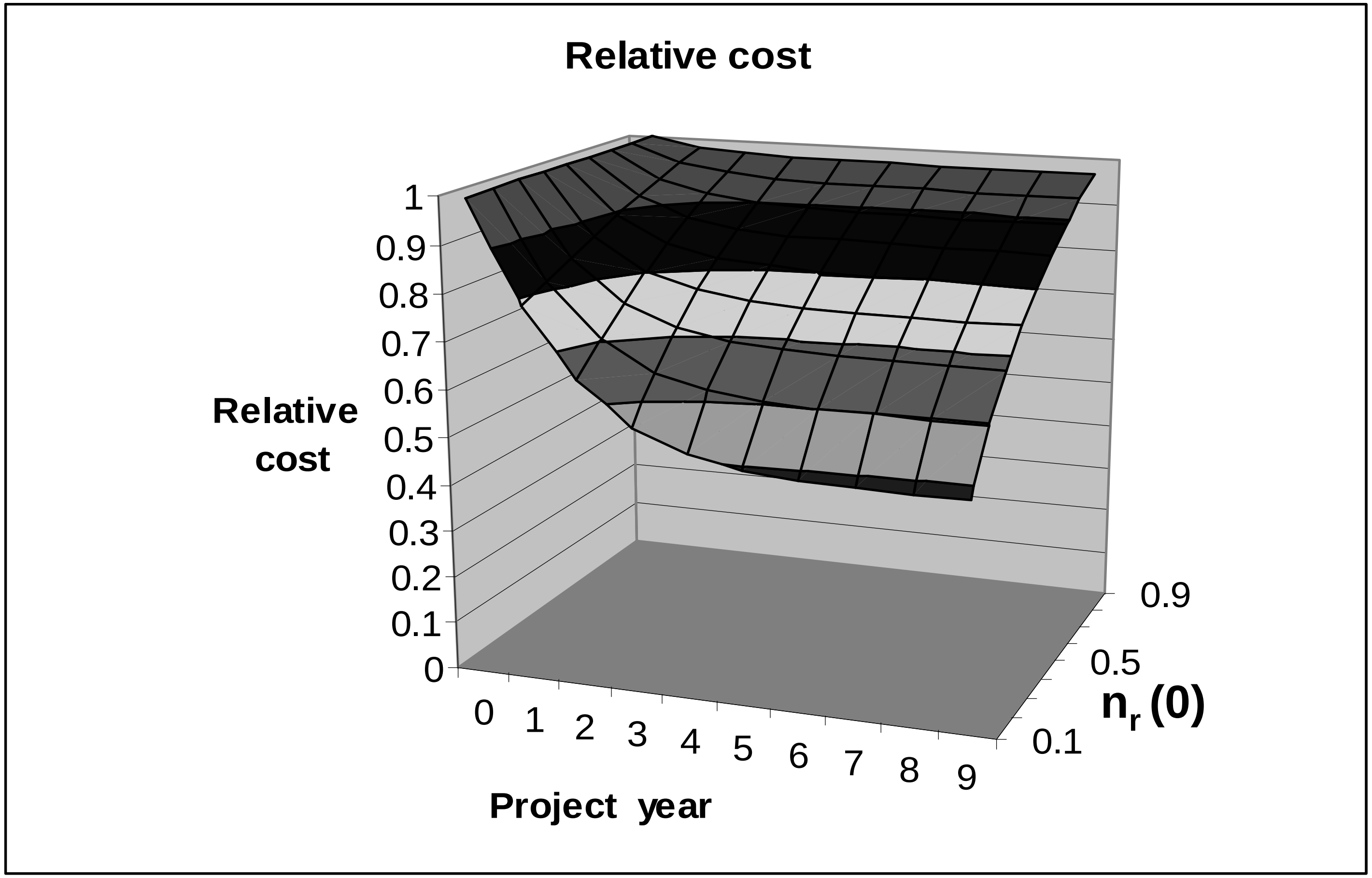
Cost as a function of K

n_r (0)=0.1, ΔT=5



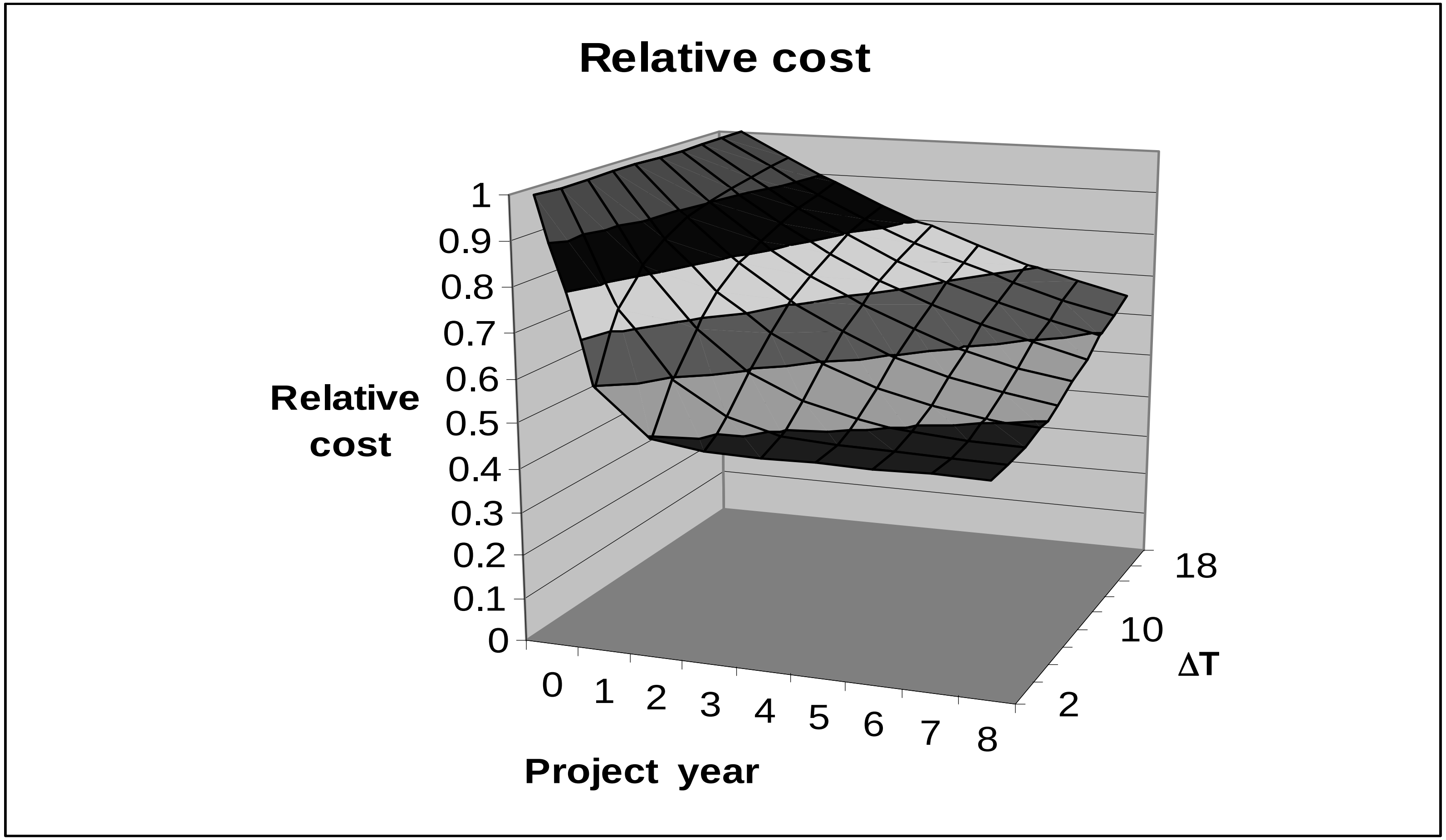
Cost as a function of $n_r(0)$

ΔT = 5, K = 0.8

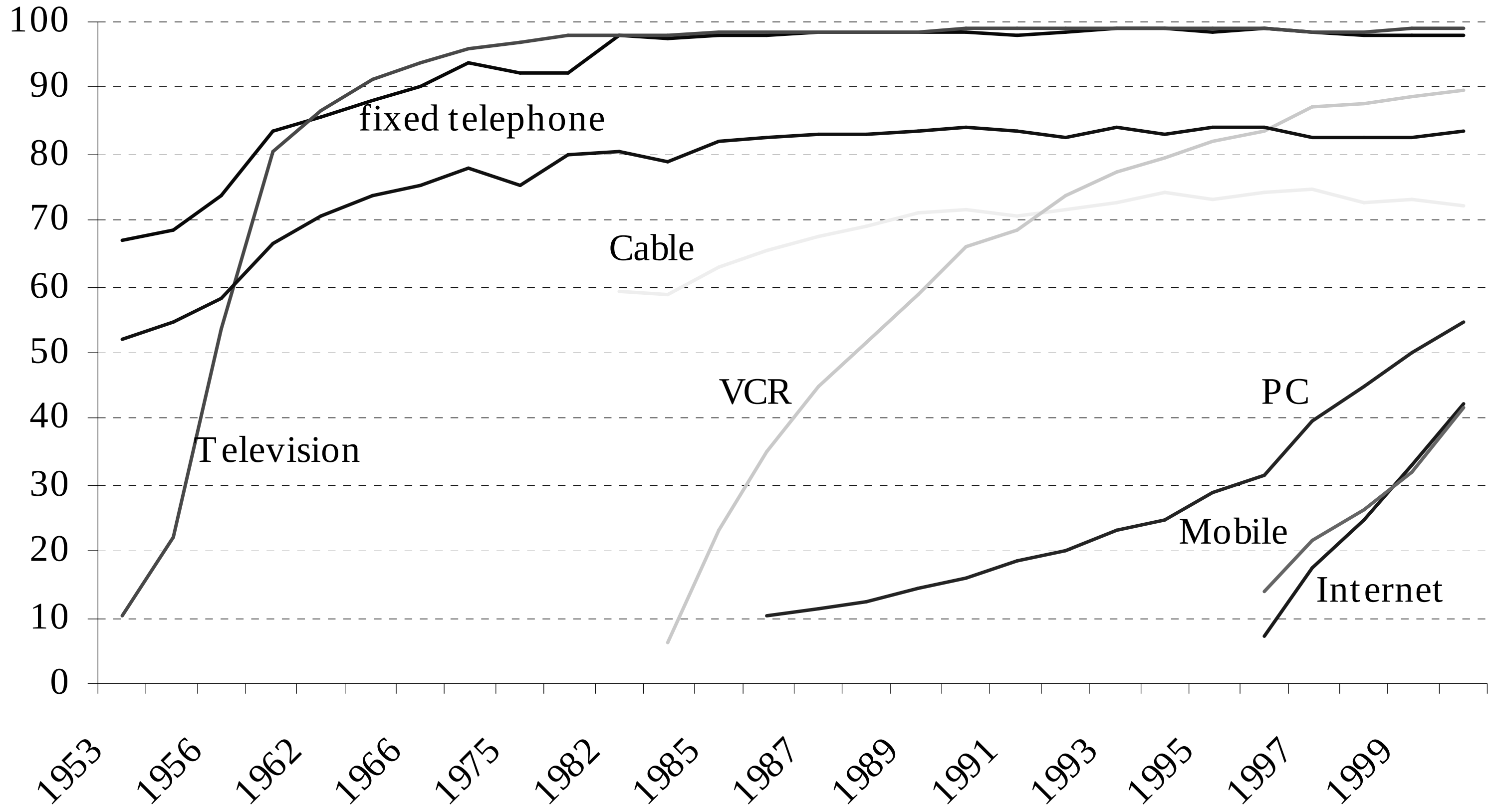


Cost as a function of ΔT

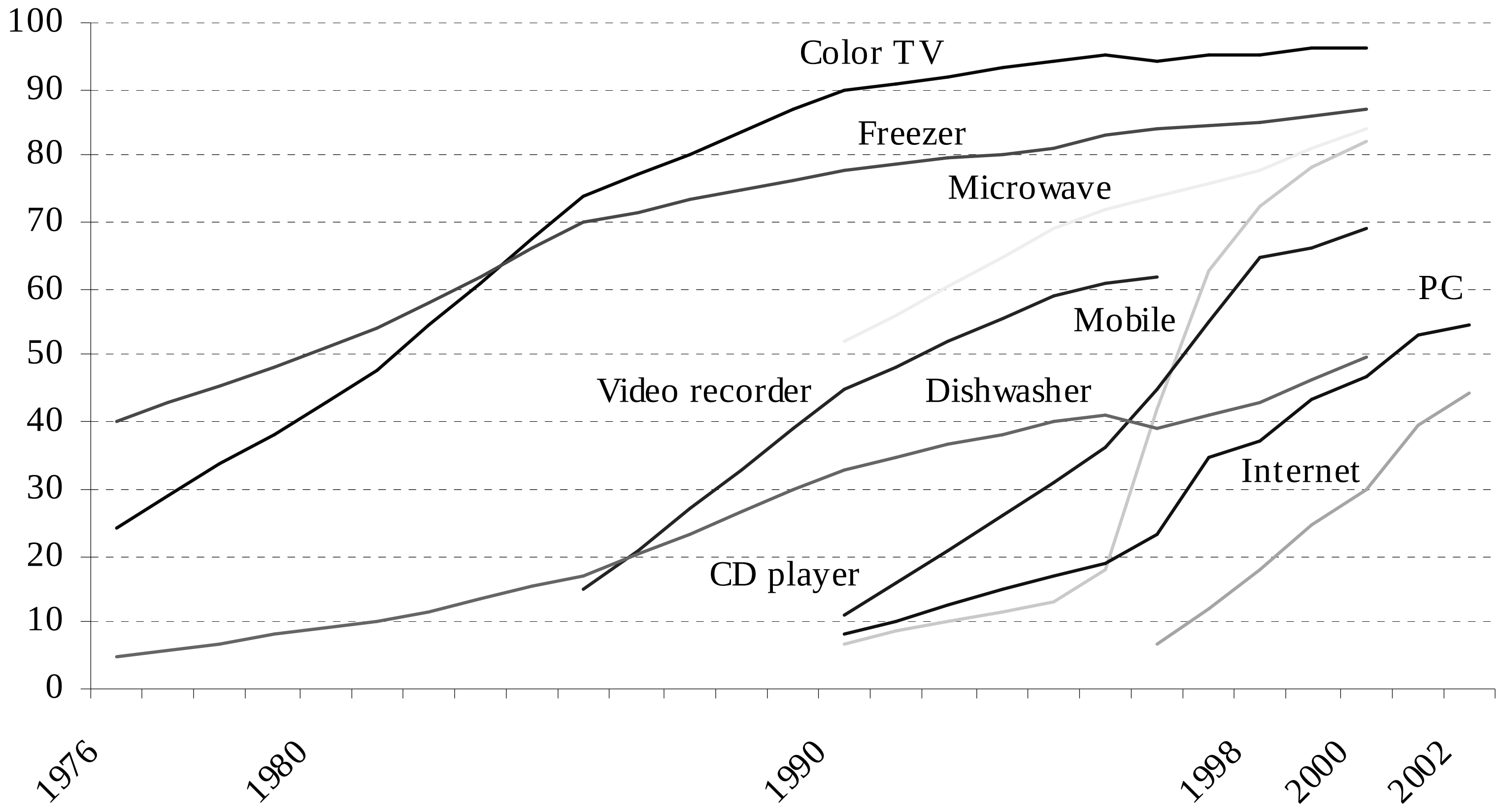
n_r (0) = 0.1, K = 0.8



Historical diffusions of selected goods in Canada.



Historical diffusions of selected goods in Finland.



Recommended values on the parameters when cost time series are not available

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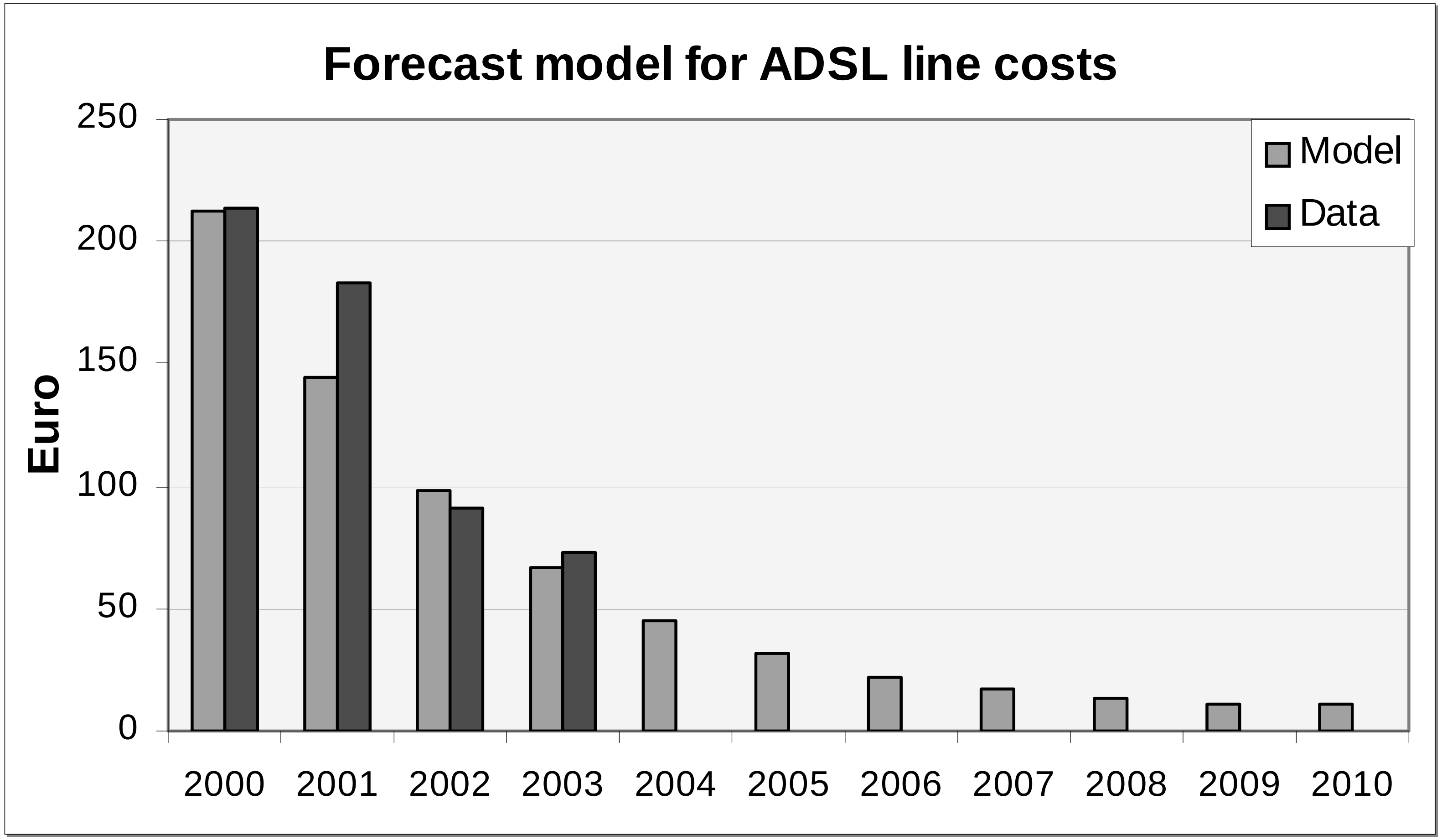
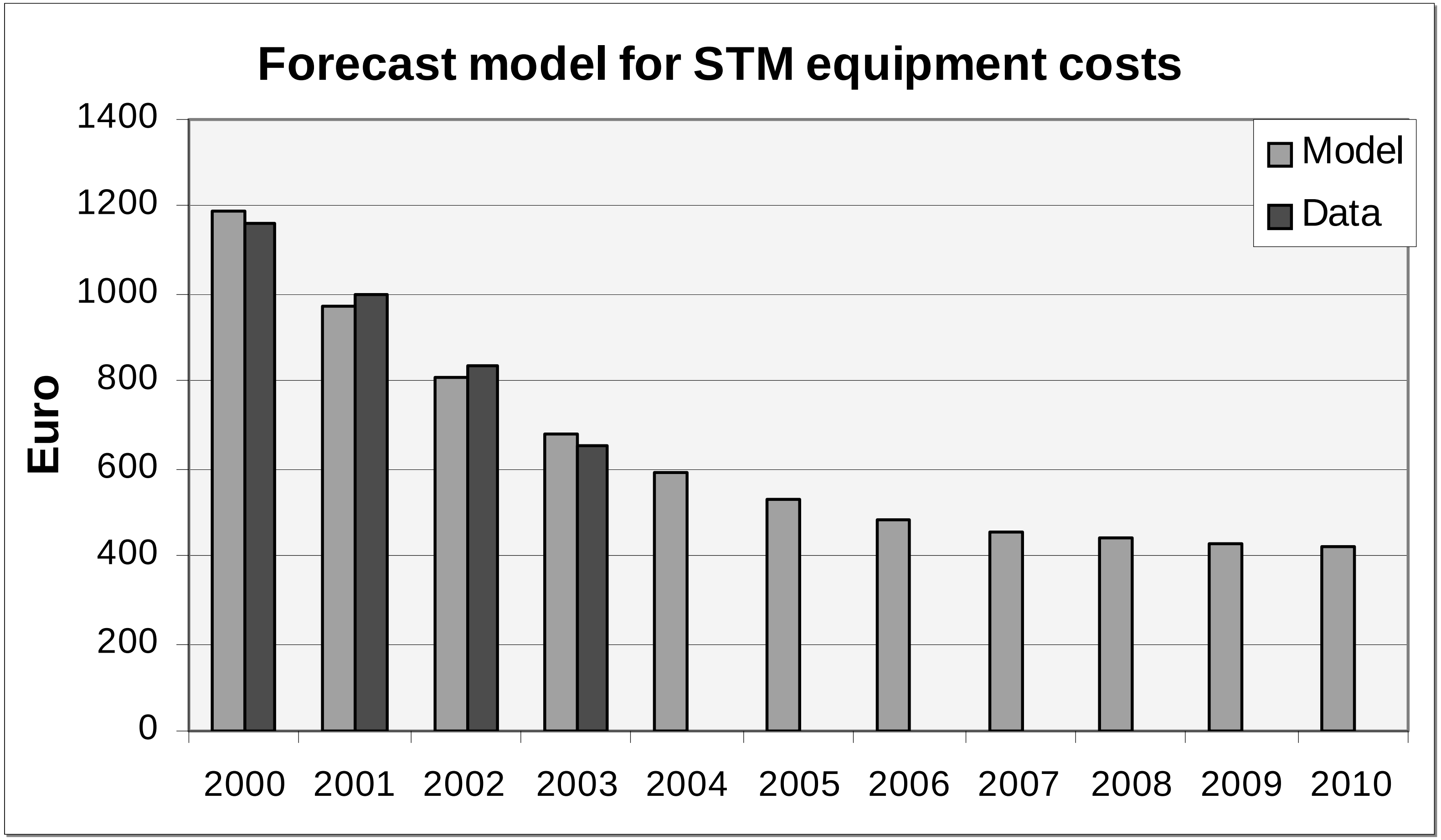
LearningCurveClass	K_Value
CivilWorks	100,00%
CopperCable	100,00%
Electronics	80,00%
SitesAndEnclosures	100,00%
FibreCable	90,00%
Installation (constant)	100,00%
AdvancedOpticalComponents	70,00%
Installation (decresing)	85,00%
OpticalComponents	80,00%

VolumeClass	$n_r(0)$	ΔT
Emerging_Fast	0,001	5,00
Emerging_Medium	0,001	10,00
Emerging_Slow	0,001	20,00
Emerging_VerySlow	0,001	40,00
Mature_Fast	0,1	5,00
Mature_Medium	0,1	10,00
Mature_Slow	0,1	20,00
Mature_VerySlow	0,1	40,00
New_Fast	0,01	5,00
New_Medium	0,01	10,00
New_Slow	0,01	20,00
New_VerySlow	0,01	40,00
Old_Fast	0,5	5,00
Old_Medium	0,5	10,00
Old_Slow	0,5	20,00
Old_VerySlow	0,5	40,00
Straight Line	0,1	1000,00

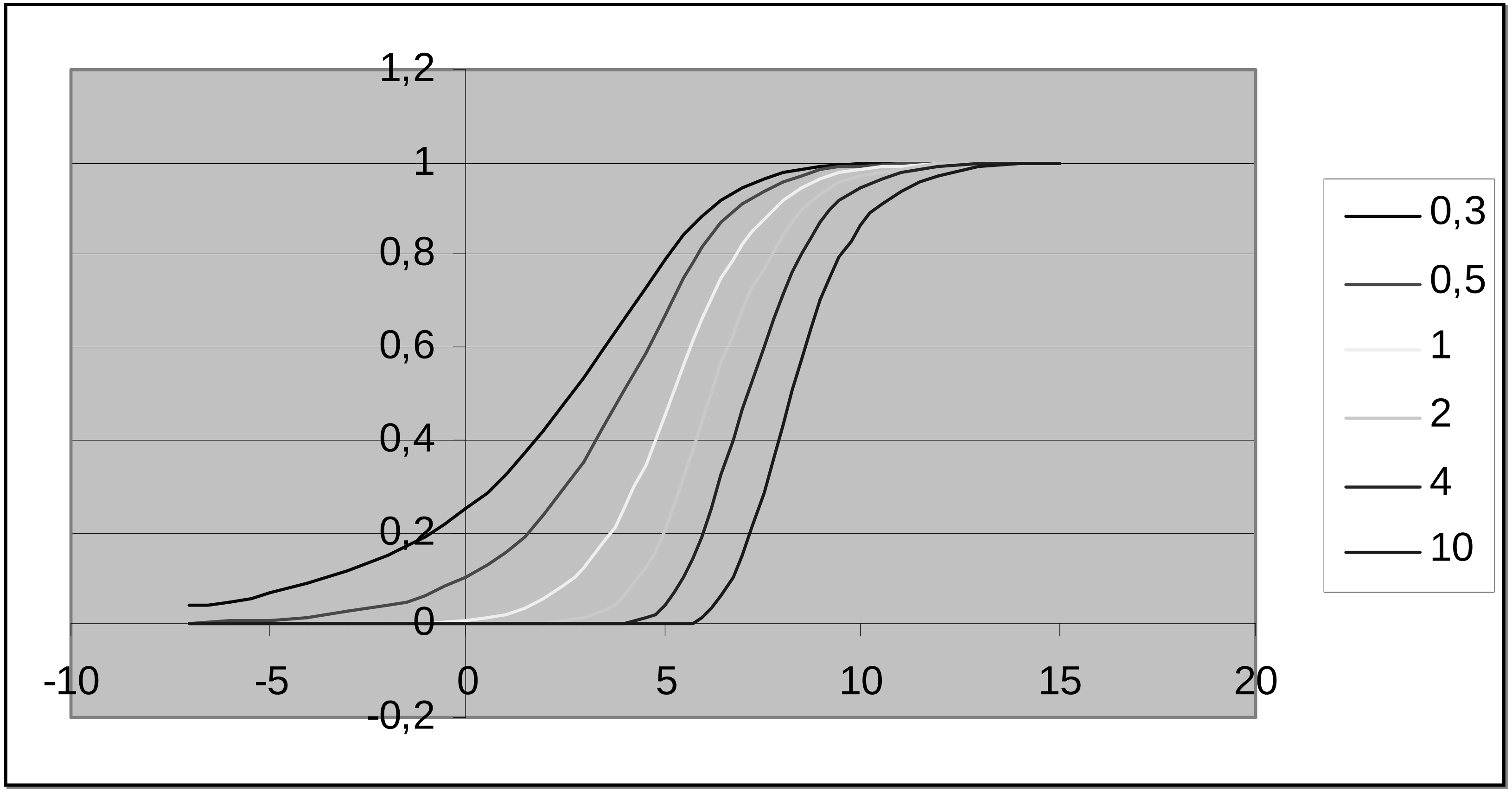
Forecasts for ADSL line costs. Estimation: P(2000)= 212 Euro, $\Delta T=8$, n(2000)=0,1%,K=0,736



Forecasts for STM equipment costs. Estimation: P(2000)=1188 Euro, ΔT=8, n(2000)=7,4%, K=0,756



Logistic model with different γ values



The extended learning curve (diff γ)

$$P(t) = P(0) \cdot \left[n_r(0)^{-1} \cdot \left(1 + e^{\left\{ \ln\left[n_r(0)^{\frac{-1}{y}} - 1 \right] + \left[\frac{\ln \delta}{\Delta T} \right] \cdot t \right\}} \right)^{-\gamma} \right]^{\log_2 \cdot K}$$

- P(0): Production cost the reference year (0)
- $n_r(0)$: Relative accumulated production volume the reference year

- ΔT: Time for the accumulated volume to grow from 10% to 20%
- K: Learning curve coefficient
- γ: Parameter asymmetry

Use of the the Extended learning curve model

- RACE 2087/TITAN 1992-1996
- AC 226/OPTIMUM1996-1998
- AC364/TERA1998-2000
- IST-2000-25172 TONIC2000-2002
- ECOSYS / CELTIC 2004-
- Many Eurescom projects
  - P306, P413, P614, P901 etc
- Within Telenor and other project partners organizations

Advantages with the extended learning curve model

- The model makes forecasts of component costs (predictions as a function of time
- The model has the possibility to include both a priori knowledge and statistical information at the same time
- The model can be used to forecast component costs evolution even if no cost observations are known
- The model can be used to forecast component costs based on estimation of the parameters when historical costs are available

Risk analysis based on uncertainties in demand forecasts and cost predictions

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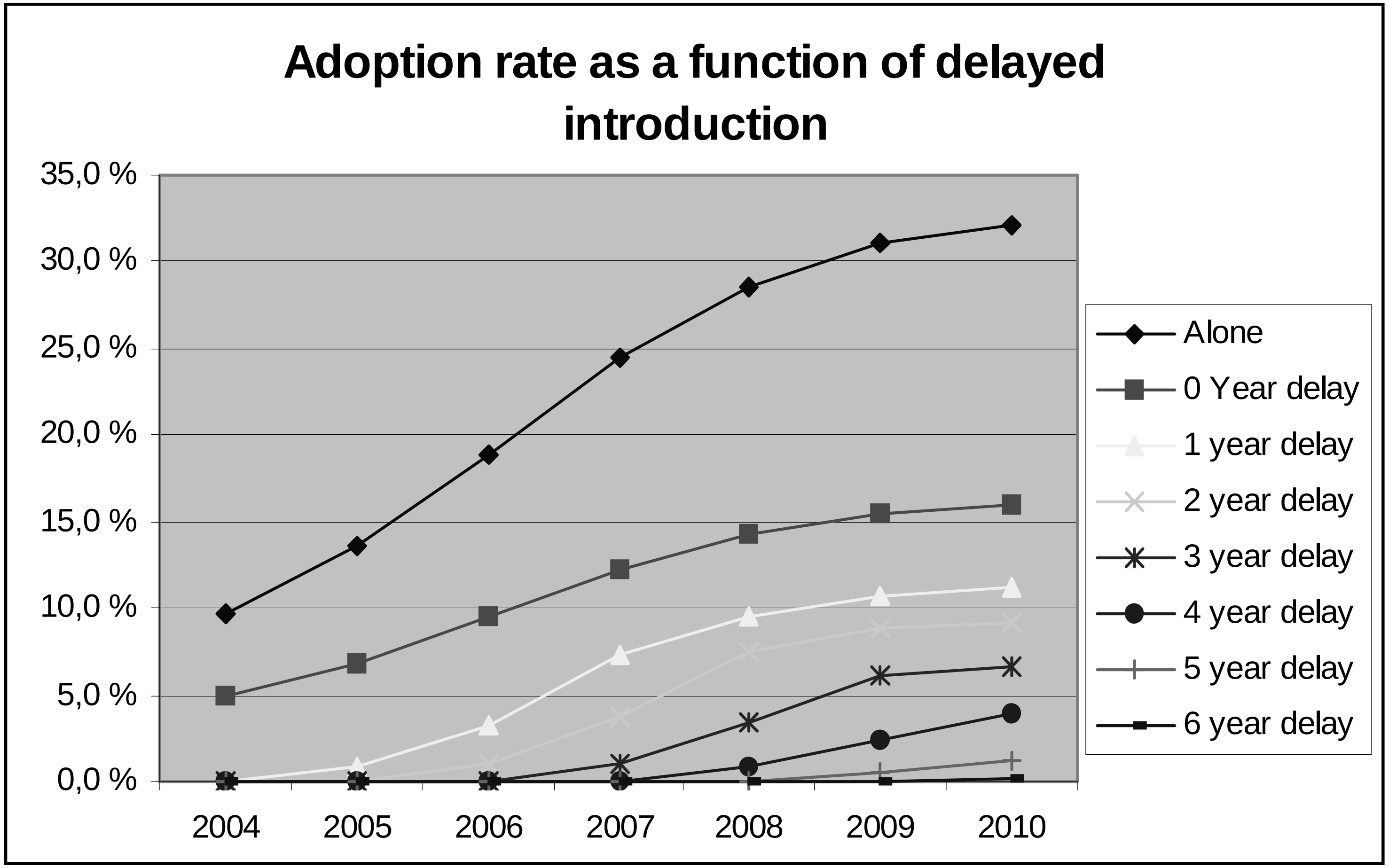
Agenda

- Business case: Roll out of ASDL2+/VDSL
- Adoption rate forecasts
- Evaluation of 6 different roll out scenarios
- Calculation of net present values for the roll out scenarios
- Framework for risk analysis
- Modelling dependency between variables in risk analysis
- Results from risk analysis
- Conclusions

Important factors for ADSL2+/VDSL roll out

- Broadband demand forecasts
- Substitution effects between broadband technologies
- Competition (Same technology and other technologies)
- Size of the access area
- Distribution of the copper line length
- Standardisation of network technology/components
- Component price and functionality
- Maintenance costs
- Expected ARPU (Average revenue per user)

Adoption rate forecasts for ADSL2+/VDSL



Market segmentation

Market segment	Exchange size N	Percent households
Area 1	15.000 < N	10 %
Area 2	10.000 < N =< 15.000	15 %
Area 3	5.000 < N =< 10.000	20 %
Area 4	2.000 < N =< 5.000	20 %
Area 5	N =< 2.000	35 %

Generic case study

Population and coverage

Population	**60 000 000**					
Persons per Household (HH)	**2,4**					
total number of HH	25 000 000					
	Area 1	Area 2	Area 3	Area 4	Area 5	
Distribution of HHs	**14 %**	**21 %**	**29 %**	**23 %**	**13 %**	100 %
Averagen number of HHs per CO	**12 000**	**8 000**	**2 600**	**1 400**	**400**	
Coverage level	**10 %**	**15 %**	**20 %**	**15 %**	**0 %**	***60 %***
HH (in %) within 2 km	**75 %**	**75 %**	**75 %**	**75 %**		
Coverage (HP)	2 500 000	3 750 000	5 000 000	3 750 000		15 000 000
Total number of HHs in covered echanges	3 333 333	5 000 000	6 666 667	5 000 000		
Number of upgraded exchanges	278	625	2 564	3 571		7 038
Number of HHs in areas without deployment	166 667	250 000	583 333	750 000	3 250 000	

The 6 scenarios studied

- Scenario 1 – Market equality, no overlap
- Scenario 2 – Market equality, 50% overlap
- Scenario 3 – Market equality, 75% overlap
- Scenario 4 – Incumbent 2 years delayed
- Scenario 5 – Incumbent 1 years delayed
- Scenario 6 – Incumbent offensive roll out

Scenario 1

”Market equality, no overlap”

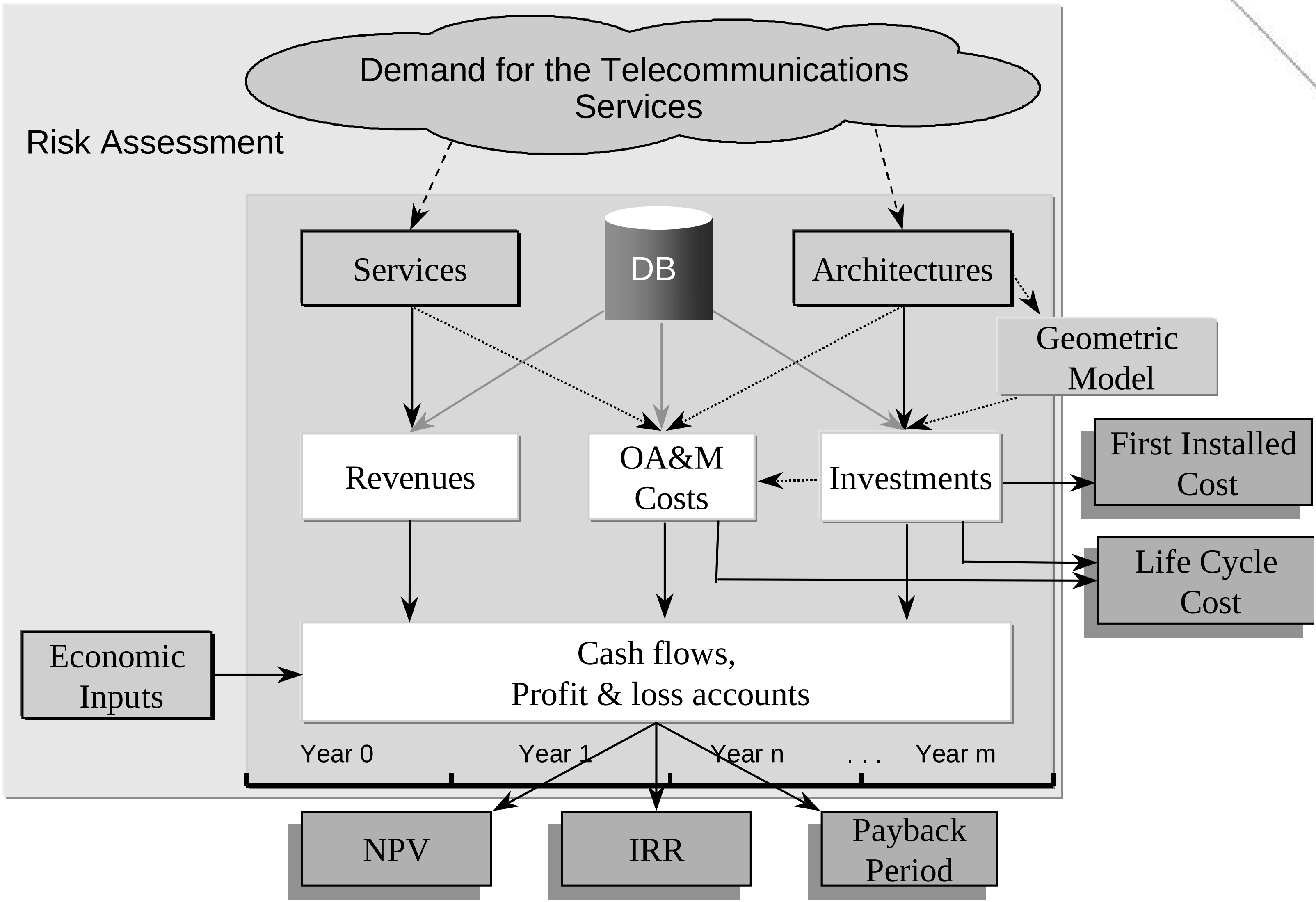
Year	Incumb.	Other	Overlap	Incumb.	Other	Incumb.	Other	Incumb.	Other	Incumb.	Other
				Area 1	Area 1	Area 2	Area 2	Area 3	Area 3	Area 4	Area 4
2004	2,5 %	2,5 %	0,0 %	**2,5 %**	**2,5 %**						
2005	7,5 %	7,5 %	0,0 %	**2,5 %**	**2,5 %**	**2,5 %**	**2,5 %**				
2006	12,5 %	12,5 %	0,0 %			**5,0 %**	**5,0 %**				
2007	17,5 %	17,5 %	0,0 %					**5,0 %**	**5,0 %**		
2008	22,5 %	22,5 %	0,0 %					**5,0 %**	**5,0 %**		
2009	27,5 %	27,5 %	0,0 %							**5,0 %**	**5,0 %**
2010	30,0 %	30,0 %	0,0 %							**2,5 %**	**2,5 %**
Sum				5,0 %	5,0 %	7,5 %	7,5 %	10,0 %	10,0 %	7,5 %	7,5 %

Scenario 4

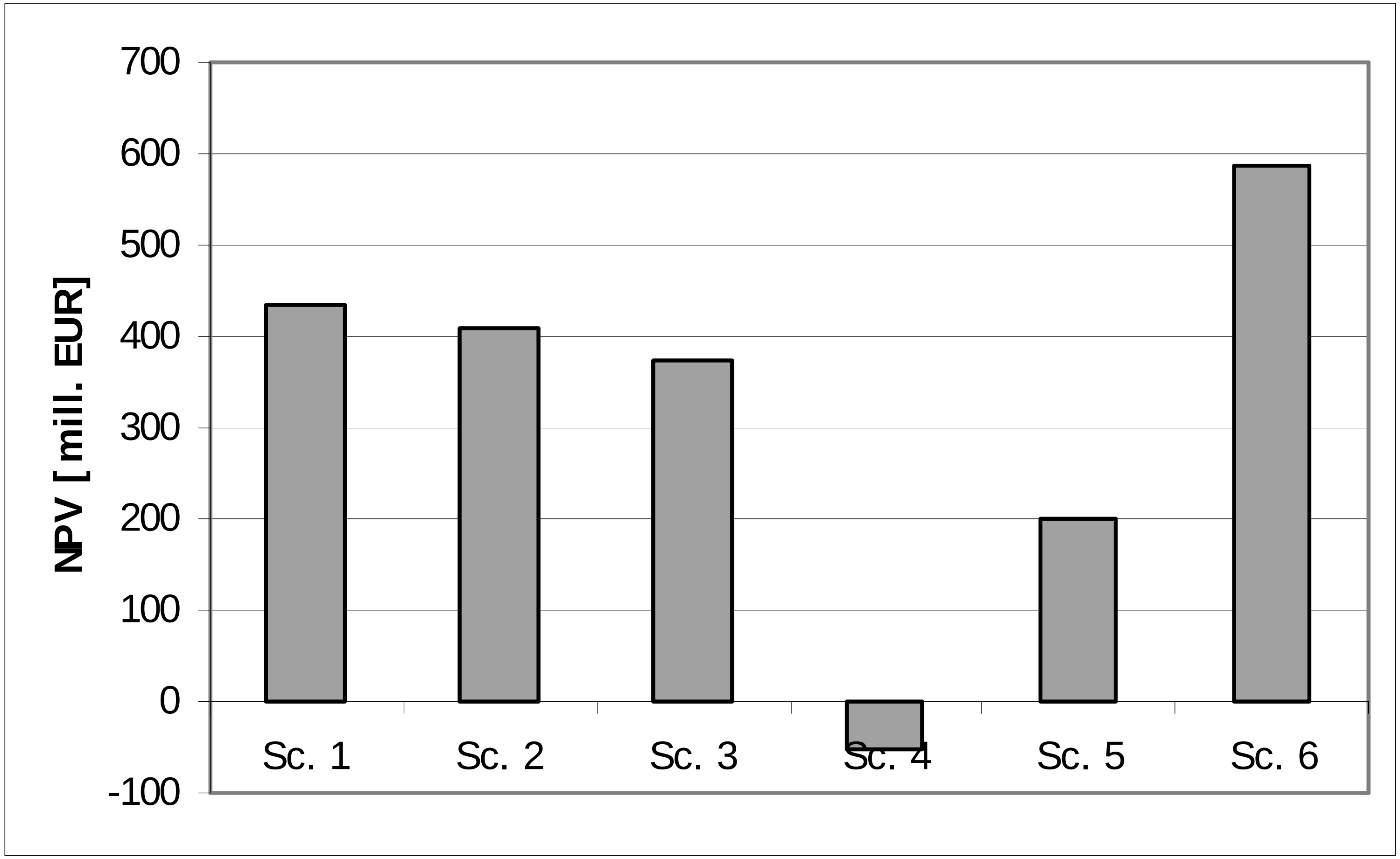
"Incumbent 2 years delayed"

Year	Incumb.	Other	Overlap	Incumb.	Other	Incumb.	Other	Incumb.	Other	Incumb.	Other
				Area 1	Area 1	Area 2	Area 2	Area 3	Area 3	Area 4	Area 4
2004	0,0 %	5,0 %	0,0 %		**5,0 %**						
2005	0,0 %	15,0 %	0,0 %		**5,0 %**		**5,0 %**				
2006	10,0 %	25,0 %	5,0 %	5,0 %		**5,0 %**	**5,0 %**		**5,0 %**		
2007	22,5 %	37,5 %	15,0 %	5,0 %			5,0 %	**7,5 %**	**7,5 %**		
2008	35,0 %	45,0 %	27,5 %			5,0 %		2,5 %	5,0 %	**5,0 %**	**2,5 %**
2009	45,0 %	50,0 %	37,5 %			5,0 %		2,5 %	2,5 %	**2,5 %**	**2,5 %**
2010	47,5 %	52,5 %	45,0 %					7,5 %			**2,5 %**
Sum				10,0 %	10,0 %	15,0 %	15,0 %	20,0 %	20,0 %	7,5 %	7,5 %

The business case approach:



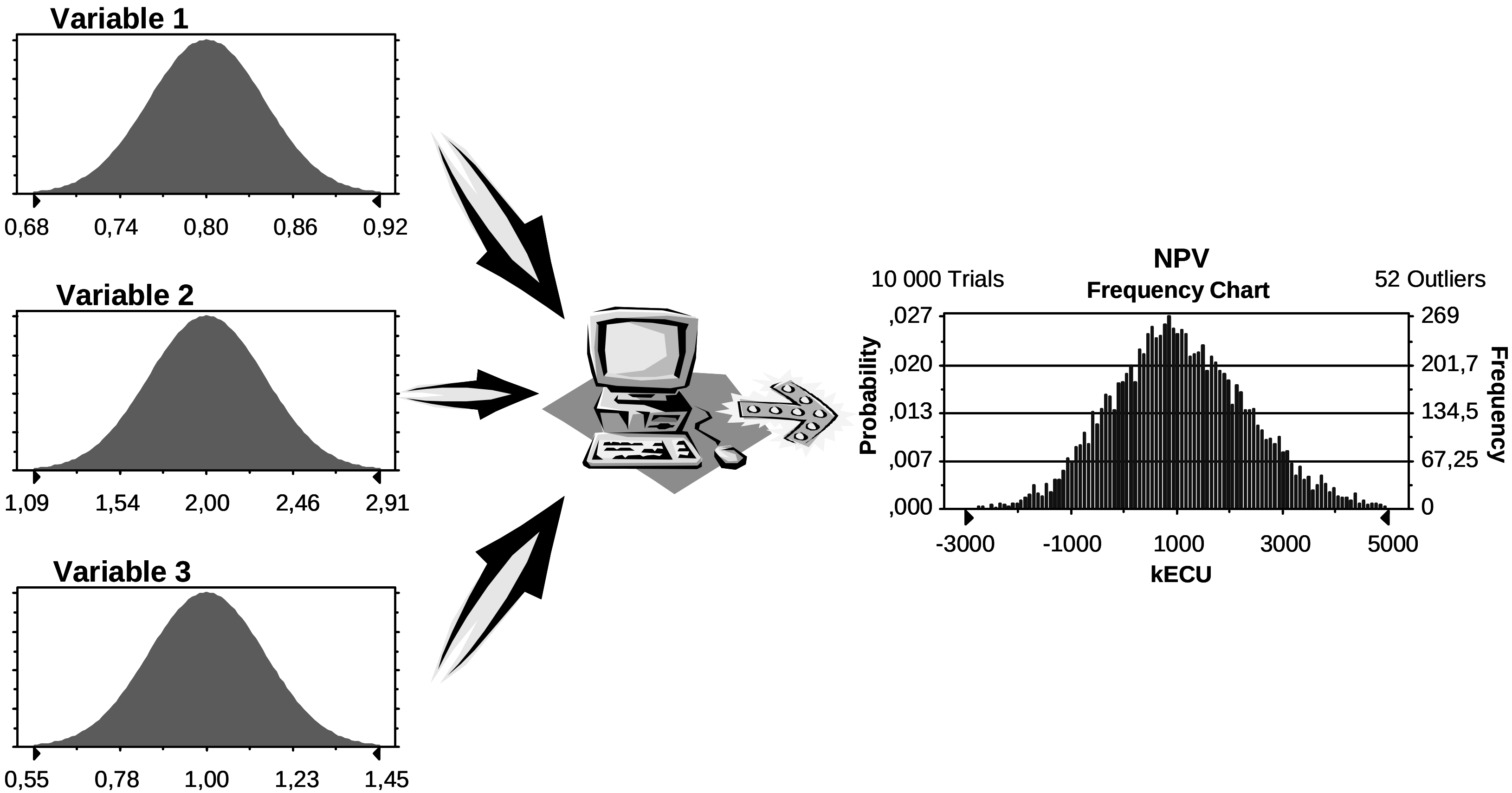
Net present values for the roll out scenarios



Conclusions

- A framework for analysing ADSL2+/VDSL rollout has been developed
- The first step is to enter the market with a cherry picking strategy
- Delay in roll out causes significant loss
- The best strategy is to enter the areas as the first operator starting with the largest areas
- But what about the uncertainty and the risks?

Risk analysis principles



Evaluation of the output distribution

Suppose Net Present Value is the output distribution

Alternative measures:

- Mean value
- Confidence interval
- 10% percentage
- 5% percentage
- Percentage of observations below NPV=0

Fitting of probability densities for the input variables

The probability densities have the ability not to give negative values.

The following input are convenient for defining the probability densities:

- Default value
- Minimum value
- 5% percentile
- 95% percentile
- Maximum value

Beta function and the fitting

$$p(y) = \frac{1}{\mathrm{B}(\alpha,\beta)(b-a)^{\alpha+\beta-1}}(y-a)^{\alpha-1}(b-y)^{\beta-1}$$

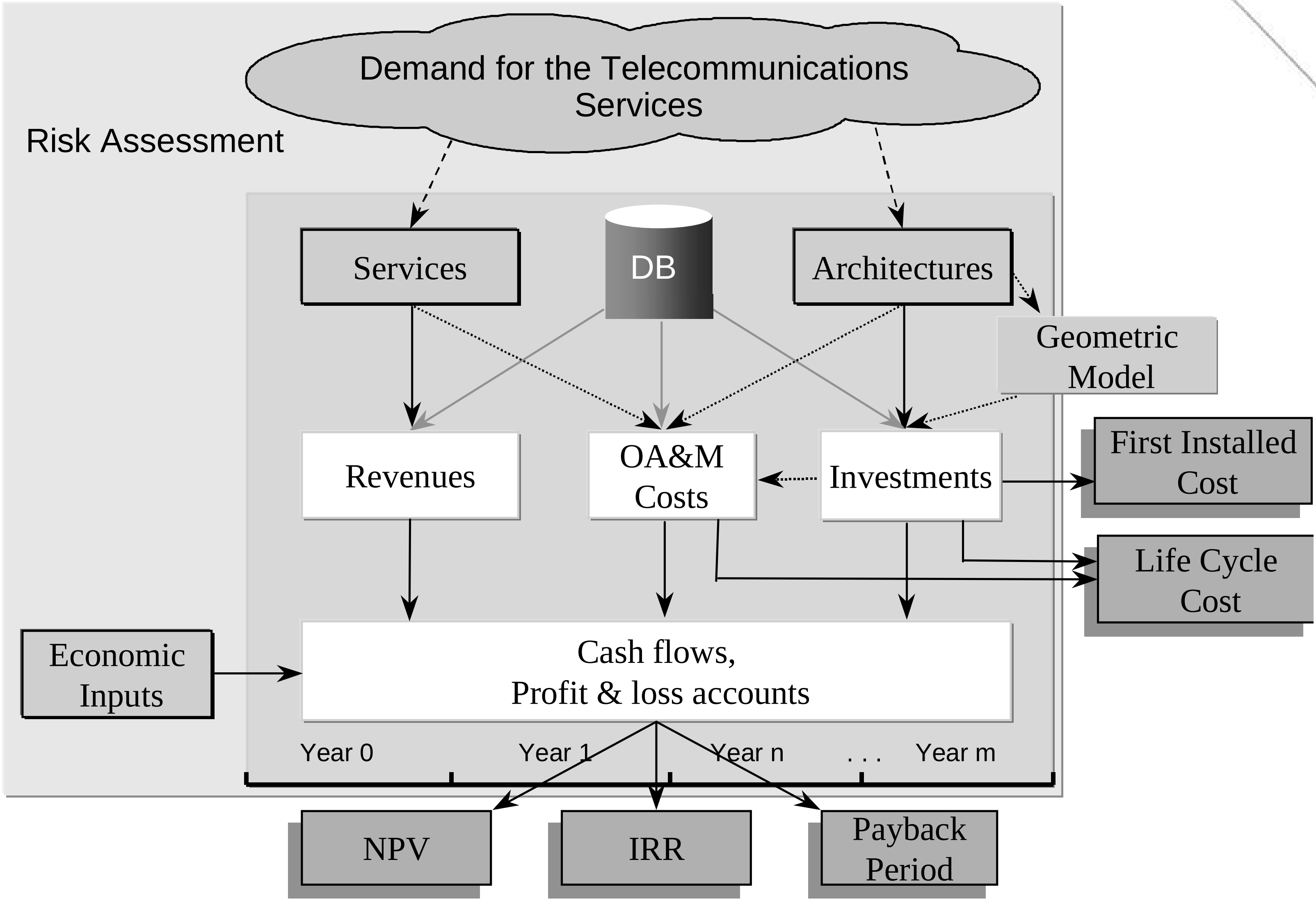
$$\mathrm{B}(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

- The parameters/variables a, b, α and β in the Beta function are found base on the shown input
- Solver is used in the calculations
- The distribution is multiplied with the default value to map the real distribution

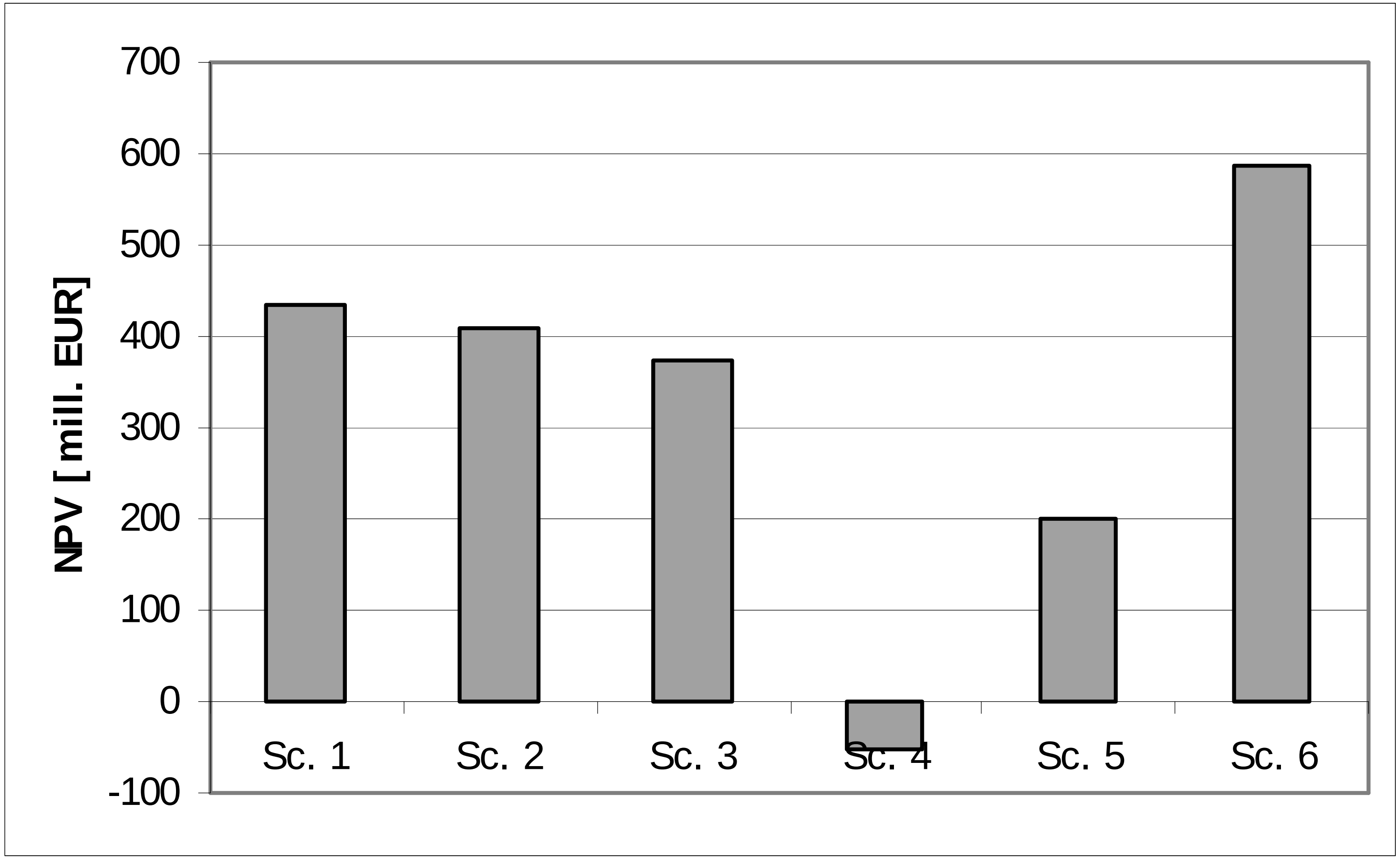
Definition of probability functions for critical variables

Variable Name	Minimum Value	5% percentile	Default Value	95% percentile	Maximum Value	α	β
Monthly ARPU	90	95	**100**	108	124.7	5.11	11.16
Line Card Price	1,200	1,400	**1,600**	1,800	2,000	4.94	4.94
Sales Costs	25%	27.5%	**30%**	32.5%	35%	4.94	4.94
Provisioning Costs	50	60	**65**	70	80	11.77	11.77
Equipment Price Reduction Rate	5%	8%	**10%**	12%	15%	8.02	8.02
Adoption Rate, final year	26%	29%	**32%**	37%	42%	4.02	6.04
Customer Installations Cost	100	110	**120**	130	140	4.95	4.95
Content Costs	50%	55%	**60%**	65%	70%	4.95	4.95
Smart Card Costs	20	25	**30**	35	40	4.94	4.94
Customer Operations & Maintenance	15	20	**25**	30	35	4.95	4.95

The business case approach:



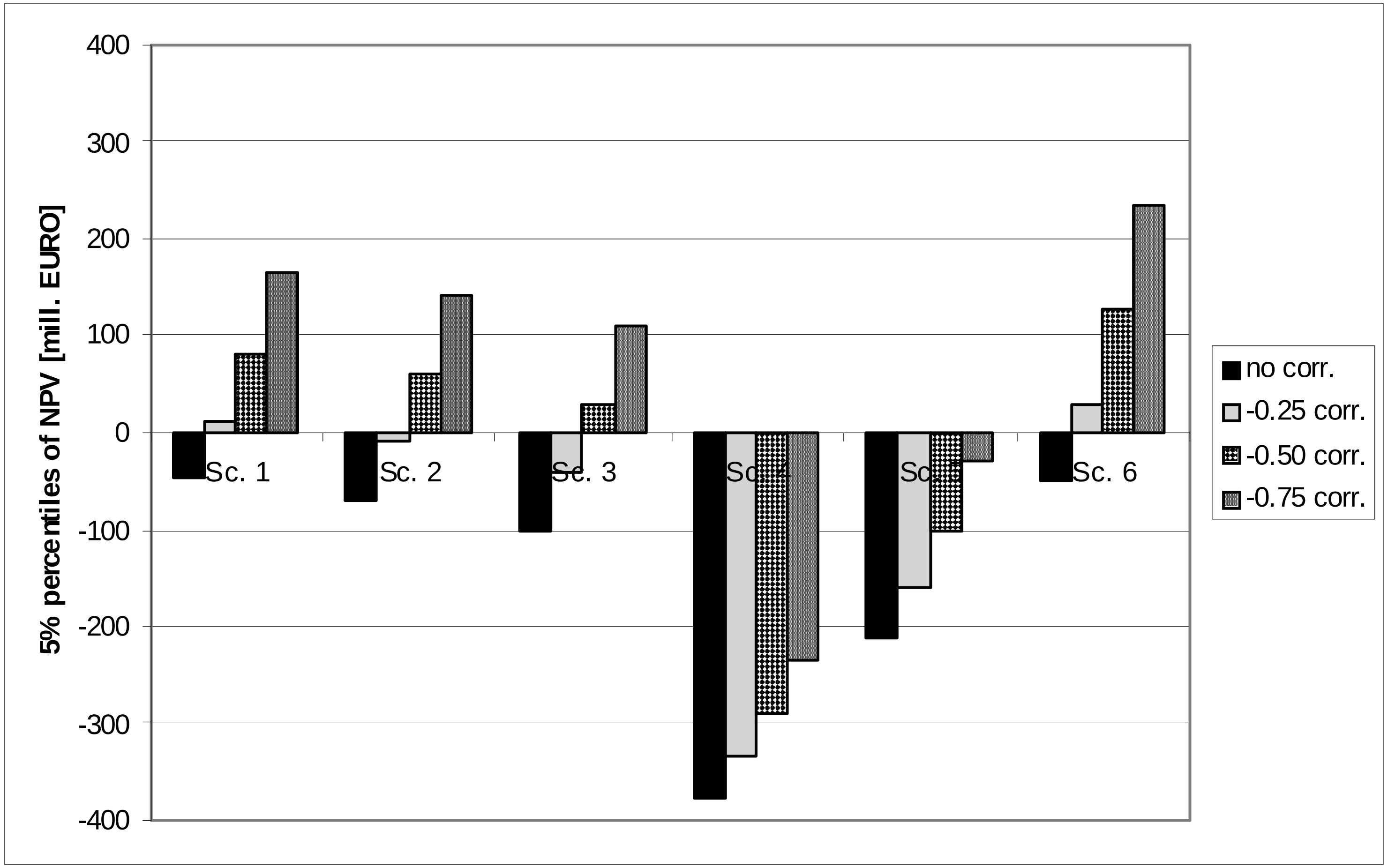
Net present values for the roll out scenarios



Net present value results from risk analysis. Different simulations

Scenario	10 variables simulated			Adoption rate and ARPU simulated			Only Adoption Rate simulated		
	5% perc.	95% perc.	σ	5% perc.	95% perc.	σ	5% perc.	95% perc.	σ
Sc. 1	-48.5	1 155.7	364.1	-5.1	1 114.5	339.5	58.6	928.4	263.4
Sc. 2	-70.6	1 138.4	365.7	-27.4	1 097.3	341.2	37.6	909.7	265.6
Sc. 3	-102.3	1 108.3	366.4	-58.9	1 066.1	341.9	4.3	882.5	266.6
Sc. 4	-376.4	438.5	264.8	-341.4	408.4	227.6	-287.4	261.6	166.7
Sc. 5	-212.0	830.8	315.9	-174.2	795.8	293.8	-112.9	624.9	224.1
Sc. 6	-50.6	1 561.9	487.7	6.0	1 507.2	455.9	90.6	1 256.9	356.1

5% percentile for NPV for the scenarios for different correlation between demand and ARPU



Conclusions

- The risk framework has been developed through the European programs RACE, ACTS and IST, by the projects RACE 2087/TITAN, AC 226/OPTIMUM, AC364/TERA and IST-2000-25172 TONIC.
- The presentation shows how risk analysis is applied on a specific business case for evaluation of the economic risks.
- The methodology for fitting the probability densities of critical variables in the business case is described
- The analysis also show the effect by modelling dependency between ARPU and demand in the risk simulations

Time for Questions & Answers

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