An Investment Criterion Incorporating Real Options

James Alleman, Hirofumi Suto &
Paul Rappoport

Experts Dialogue: Managing Risk in the Competitive Environment
International Telecommunication Union
Geneva, Switzerland
28-29 October 2004

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
5. Application

1. Objectives

- Develop an Investment Criterion Incorporating Real Options

  or

- A Simple Decision-Making Criterion Under Uncertainty

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
5. Application

Agenda

1. Objectives
2. Option Pricing
   - 2.1 Call Option
   - 2.2 Binomial Lattice Model
   - 2.3 Continuous Additive Model
3. Real Options
4. New Criterion
5. Application

Agenda

1. Objectives
2. Option Pricing
   - 2.1 Call Option
   - 2.2 Binomial Lattice Model
   - 2.3 Continuous Additive Model
3. Real Options
4. New Criterion
5. Application

2.1 Call Option

- The right to buy a stock, not obligation
  - At certain price = K: exercise price
  - At certain time = T: time to expiration

2.1 Call Option

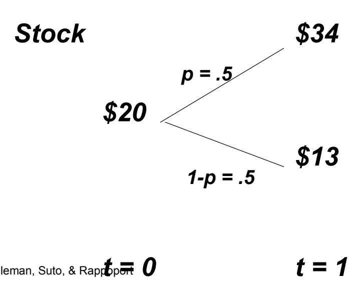
- Example: IBM Stock Call Option
  - Exercise Price K: $23
  - Time to Expire T: 1 year
- How much would you pay for this option if the stock is traded at $20 now?

Agenda

1. Objectives
2. Option Pricing
   - 2.1 Call Option
   - 2.2 Binomial Lattice Model
   - 2.3 Continuous Additive Model
3. Real Options
4. New Criterion
5. Application

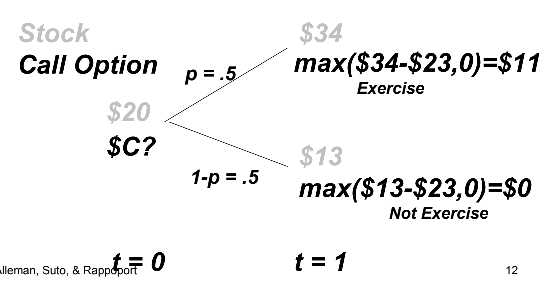
2.2 Binomial Lattice Model

- 2.2.1 Stock behavior



2.2 Binomial Lattice Model

- 2.2.2 Payoff of Call Option (*K= $23*)



2.2 Binomial Lattice Model

- 2.2.2 Payoff of Call Option (*K*= $23)

Call Option

$C? — p = .5 → $11

$C? — 1-p = .5 → $0

t = 0 t = 1

2.2 Binomial Lattice Model

- 2.2.3 One Price Principle

Payoffs are same

Prices should be same

One price principle,
No arbitrage principle

2.2 Binomial Lattice Model

- 2.2.4 Replicating Portfolio

$x of Stock: x ($20) → 1.7x ($34) / 0.65x ($13)

\+ *$b of Bond*: b → 1.08b / 1.08b

2.2 Binomial Lattice Model

- 2.2.4 Replicating Portfolio

$x of Stock: x → 1.7x / 0.65x

\+ *$b of Bond*: b → 1.08b / 1.08b

2.2 Binomial Lattice Model

- 2.2.4 Replicating Portfolio

$x of Stock: x → 1.7x / 0.65x

\+ *$b of Bond*: b → 1.08b / 1.08b

Replicating Portfolio: x+b → 1.7x+1.08b / 0.65x+1.08b

2.2 Binomial Lattice Model

- 2.2.4 Replicating Portfolio

$x of Stock: x → 1.7x / 0.65x

\+ *$b of Bond*: b → 1.08b / 1.08b

Replicating Portfolio: x+b → 1.7x+1.08b = 11 / 0.65x+1.08b = 0

2.2 Binomial Lattice Model

- **2.2.4 Replicating Portfolio**

1.7x+1.08b = 11
0.65x+1.08b = 0

2.2 Binomial Lattice Model

- **2.2.4 Replicating Portfolio**

1.7x+1.08b = 11
0.65x+1.08b = 0

$$x = \frac{11-0}{1.7-.65} = 10.5$$

$$b = \frac{1}{1.08} \times \frac{1.7 \times 0 - .65 \times 11}{1.7-.65} = -6.3$$

2.2 Binomial Lattice Model

- **2.2.4 Replicating Portfolio**

1.7x+1.08b = 11
0.65x+1.08b = 0

$$x = \frac{11-0}{1.7-.65} = 10.5$$

$$b = \frac{1}{1.08} \times \frac{1.7 \times 0 - .65 \times 11}{1.7-.65} = -6.3$$

Price of Portfolio= $x + b = 10.5 - 6.3 = 4.2$

2.2 Binomial Lattice Model

- **2.2.4 Replicating Portfolio**

1.7x+1.08b = 11
0.65x+1.08b = 0

$$x = \frac{11-0}{1.7-.65} = 10.5$$

$$b = \frac{1}{1.08} \times \frac{1.7 \times 0 - .65 \times 11}{1.7-.65} = -6.3$$

Price of Portfolio= $x + b = 10.5 - 6.3 = 4.2$

Price of Call Option, C = 4.2

2.2 Binomial Lattice Model

- **2.2.5 Risk-neutral Probability**

$$C = x + b = \frac{1.08-.65}{1.7-.65} \times \frac{11}{1.08} + \frac{1.7-1.08}{1.7-.65} \times \frac{0}{1.08}$$

2.2 Binomial Lattice Model

- **2.2.5 Risk-Neutral Probability**

$$C = x + b = \frac{1.08-.65}{1.7-.65} \times \frac{11}{1.08} + \frac{1.7-1.08}{1.7-.65} \times \frac{0}{1.08}$$

q = .4 *1-q = .6*

Risk-Neutral Probability

2.2 Binomial Lattice Model

- 2.2.6 General Option Pricing Formula

$$C = x + b = \frac{1.08-.65}{1.7-.65}\times\frac{11}{1.08}+\frac{1.7-1.08}{1.7-.65}\times\frac{0}{1.08}$$

$$C = q\times\frac{C_u}{R}+(1-q)\times\frac{C_d}{R}$$

2.2 Binomial Lattice Model

- 2.2.6 General Option Pricing Formula

$$C = x + b = \frac{1.08-.65}{1.7-.65}\times\frac{11}{1.08}+\frac{1.7-1.08}{1.7-.65}\times\frac{0}{1.08}$$

$$C = q\times\frac{C_u}{R}+(1-q)\times\frac{C_d}{R}$$

P

2.2 Binomial Lattice Model

- 2.2.6 General Option Pricing Formula

$$C = \hat{E}[P]$$

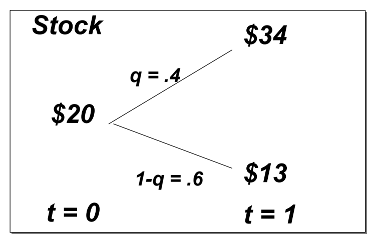
$\hat{E}[\bullet]$ *:Expectation with q*

P = PV[payoff] $\left(\frac{C_u}{R} \text{ or } \frac{C_d}{R}\right)$

= Expectation of P with Risk-neutral Probability

2.2 Binomial Lattice Model

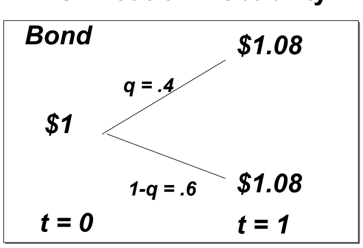
- 2.2.7 Risk-neutral Probability



$$20 = .4\times\frac{34}{1.08}+.6\times\frac{13}{1.08}$$

2.2 Binomial Lattice Model

- 2.2.7 Risk-neutral Probability



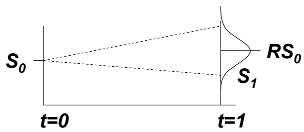
$$1 = .4\times\frac{1.08}{1.08}+.6\times\frac{1.08}{1.08}$$

Agenda

1. Objectives
2. Option Pricing
   - 2.1 Call Option
   - 2.2 Binomial Lattice Model
   - 2.3 Continuous Additive Model
3. Real Options
4. New Criterion
5. Application

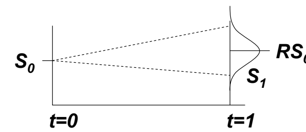
2.3 Continuous Additive Model

– 2.3.1 Build Model that satisfies risk-neutral



2.3 Continuous Additive Model

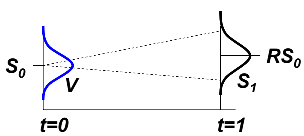
– 2.3.1 Build Model that satisfies risk-neutral



$$E\left[\frac{S_1}{R}\right] = \frac{1}{R} RS_0 = S_0$$

2.3 Continuous Additive Model

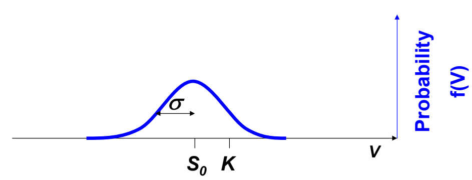
– 2.3.1 Build Model that satisfies risk-neutral



$$\frac{S_1}{R} \equiv V$$

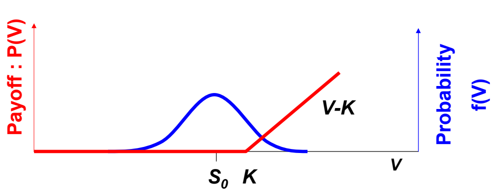
2.3 Continuous Additive Model

– 2.3.2 Payoff of Call Option



2.3 Continuous Additive Model

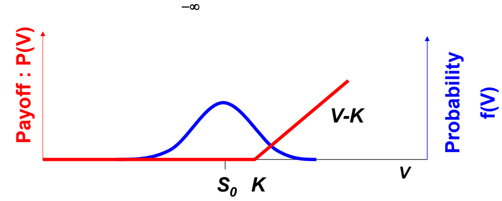
– 2.3.2 Payoff of Call Option



2.3 Continuous Additive Model

– 2.3.3 Call Option Price

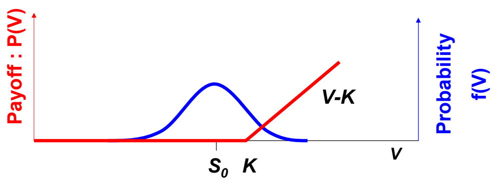
$$C = \hat{E}[P(V)] = \int_{-\infty}^{+\infty} P(V) f(V) dV$$



2.3 Continuous Additive Model

– 2.3.3 Call Option Price

$$C = \hat{E}[P(V)] = \int_{-\infty}^{+\infty} P(V) f(V) dV = \int_{K}^{+\infty} (V - K) f(V) dV$$



Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
5. Application

Agenda

1. Objectives
2. Option Pricing
3. Real Options
   - 3.1 What's Real Options
   - 3.2 Defer Option
   - 3.3 An Example Project
   - 3.4 ExNPV
4. New Criterion
5. Application

3.1 What 's Real Options?

– Value Operation Flexibility

– Applying Option Pricing Theory

– Types of Real Options
- Defer
- Expand
- Switch
- Abandon etc.

Agenda

1. Objectives
2. Option Pricing
3. Real Options
   - 3.1 What's Real Options
   - 3.2 Defer Option
   - 3.3 An Example Project
   - 3.4 ExNPV
4. New Criterion
5. Application to DSL

3.2 Defer Option

– Real Option:
Right to wait to invest
until the market is good

– Call Option Analogy:
Right to buy the stock if the stock price becomes high

3.2 Defer Option

Defer Option		Call Option
Present value of a project's future cash flow	S	Stock price
Investment to acquire the project assets	K	Exercise price
Length of time the decision may be deferred	T	Time to expiration
Time value of money	r_f	Risk-free rate of return
Riskiness of the project assets	σ^2	Variance of returns on stock

Agenda

3.3 Example: A Project

Defer Option	Variable	
Present value of operating future cash flow	S	\$100 million
Investment to Equipment at time T=1	K_T	\$110 million
Length of time the decision may be deferred	T	1 year
Risk-free rate	r_f	6%
Riskiness of the project	σ	\$30 million

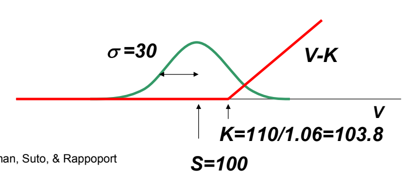
3.3 Example: A Project

- NPV
  - PV[Cash out] = PV[K_T]
    = 110/1.06
    = 103.8
  - PV[Cash in] = S
    = E[PV[S_1]]
    = 100
  - NPV = PV[cash in] – PV[cash out]
    = 100 – 103.8
    = - 3.8

3.3 Example: A Project

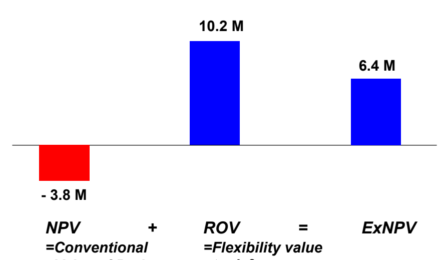
- ROV (Real Option Value)

$$C = \int_K^{+\infty} (V - K) f(V) dV = 10.2$$



3.4 ExNPV

- Expanded NPV



Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. **New Criterion**
5. Application

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. **New Criterion**
   - **4.1 Basic Ideas**
   - **4.2 Case: NPV < 0**
   - **4.3 What do d and D* mean?**
5. Application

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
   - **4.1 Basic Ideas**
   - 4.2 Case: NPV < 0
   - 4.3 What do d and D* mean?
5. Application

4.1 Basic Ideas

- **When is ExNPV = 0, when NPV < 0 ?**

4.1 Basic Ideas

When is ExNPV = 0, when NPV < 0 ?

ExNPV > 0	***ExNPV < 0***
Wait and watch the market!	***Do not invest***

ExNPV = 0

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
   - 4.1 Basic Ideas
   - **4.2 Case: NPV < 0**
   - 4.3 What do d and D* mean?
5. Application to DSL

4.2 Case: NPV < 0

If NPV < 0, when is ExNPV = 0? i.e.

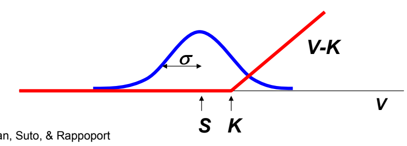
- ***ExNPV = ROV + NPV = 0***

4.2 Case: NPV < 0

When is ExNPV = 0, give NPV < 0?

- ***ExNPV = ROV + NPV = 0***

$$\int_{K}^{+\infty}(V-K)f(V)dV+(S-K)=0$$



4.2 Case: NPV < 0

- **To solve the equation**

$$v=\frac{V-S}{\sigma}$$ ***: Normal distribution (0,1): $f_N(v)$***

4.2 Case: NPV < 0

- **To solve the equation**

$$v=\frac{V-S}{\sigma}$$ ***: Normal distribution (0,1): $f_N(v)$***

$$D\equiv\frac{|S-K|}{\sigma}=\frac{|NPV|}{riskiness}$$

4.2 Case: NPV < 0

- ***Solve the equation***

$$\int_{K}^{+\infty}(V-K)f(V)dV+(S-K)=0$$

$$\downarrow$$

$$\int_{D}^{+\infty}(v-D)f_N(v)dv-D=0$$

4.2 Case: NPV < 0

- ***Solve the equation***

$$\int_{K}^{+\infty}(V-K)f(V)dV+(S-K)=0$$

$$\downarrow$$

$$\int_{D}^{+\infty}(v-D)f_N(v)dv-D=0$$

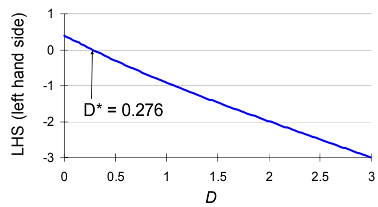
$$\downarrow$$

$$\underline{\underline{\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}D^2\right)-D\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-D}\exp\left(-\frac{1}{2}v^2\right)dv-D}}=0$$

LHS (left hand side)

4.2 Case: NPV < 0

- ***Solve the equation***



4.2 Case: NPV < 0

Criterion for Case:

ExNPV > 0	***ExNPV < 0***

4.2 Case: NPV < 0

Criterion for Case:

D < D*	***D > D****
Wait and watch the market!	

where $D \equiv \frac{|S-K|}{\sigma} = \frac{|NPV|}{riskiness}$

4.2 Case: NPV < 0

Criterion for Case:

D < D*	***D > D****
Wait and watch the market!	***Do not invest***

where $D \equiv \frac{|S-K|}{\sigma} = \frac{|NPV|}{riskiness}$

Agenda

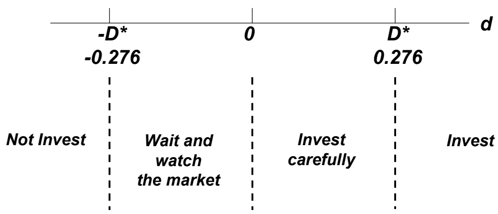
1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
   - 4.1 Basic Ideas
   - 4.2 Case: NPV < 0
   - 4.3 What do d and D* mean?
5. Application to DSL

Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
   - 4.1 Basic Ideas
   - **4.2 Case: NPV < 0**
   - 4.3 What do d and D* mean?
5. Application to DSL

4.2 & 4.3 Combined Criterion

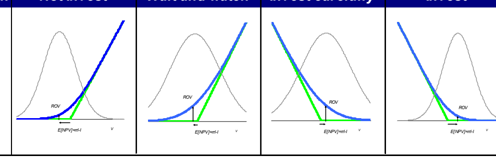
when $d \equiv \dfrac{S-K}{\sigma}$



Combined Criterion

- **Summary of Criterion**

NPV	NPV < 0		NPV > 0	
ROV	|NPV| > ROV	|NPV| < ROV	|NPV| < ROV	|NPV| > ROV
d	*d* < - D*	- D* < *d* < 0	0 < *d* < D*	D* < *d*
Decision	*Not Invest*	*Wait and watch*	*Invest carefully*	*Invest*



4.4 Meaning of d and D*?

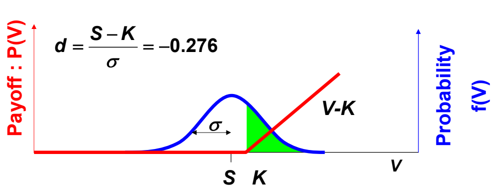
- **d = NPV/riskiness**
  - **Uncertainty adjusted NPV**
  - **Risk normalized NPV**

4.4 Meaning of d and D*?

- **d = NPV/riskiness**
  - **Uncertainty adjusted NPV**
  - **Risk normalized NPV**
- **d = D***
  - **The point of ExNPV = 0**
  - **Break-even point of NPV plus ROV**

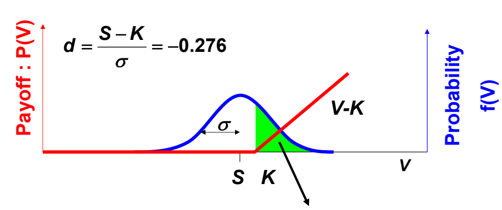
4.4 Meaning of d and D*?

If d = - D*, what is the probability the project payoff > 0 ?



4.4 Meaning of d and D*?

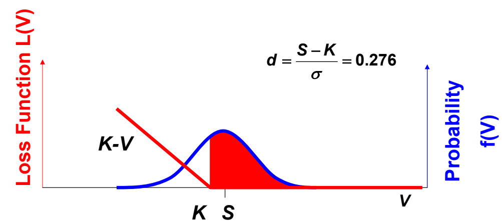
If d = - D*, what is the probability the project payoff > 0 ?



Probability = 39%

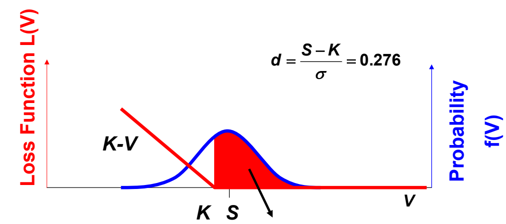
4.4 Meaning of d and D*?

If d = D*, what is the probability the project payoff > 0 ?



4.4 Meaning of d and D*?

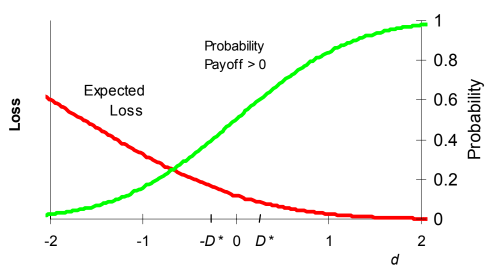
If d = D*, what is the probability the project payoff > 0 ?



Probability = 61%

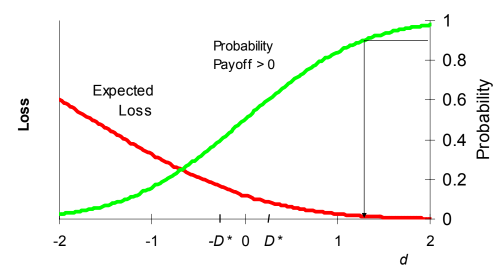
4.4 Meaning of d and D*?

Tradeoffs of Losses



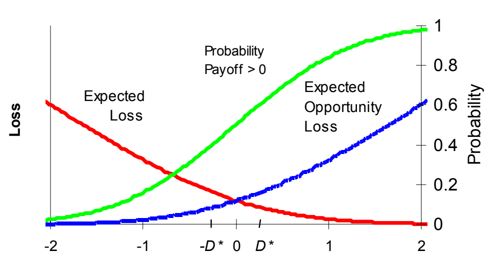
4.4 Meaning of d and D*?

Tradeoffs of Losses



4.4 Meaning of d and D*?

Tradeoffs of Losses



Agenda

1. Objectives
2. Option Pricing
3. Real Options
4. New Criterion
5. Application

5.1 Simple Case

Six Independent DSL Projects

Variable	A	B	C	D	E	F
S	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
K_T	$90.00	$90.00	$110.00	$110.00	$110.00	$110.00
T	0.0	2.0	0.0	0.5	1.0	2.0
σ	30%	30%	30%	20%	30%	40%
r_f	6%	6%	6%	6%	6%	6%

S : Current value of future CF
K_T : Investment at time T
T : Time to expiration
σ : Volatility
r_f : Risk-free rate of return

5.1 Simple Case

Six Independent DSL Projects

Variable	A	B	C	D	E	F
S	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
K_T	$90.00	$90.00	$110.00	$110.00	$110.00	$110.00
T	0.0	2.0	0.0	0.5	1.0	2.0
σ	30%	30%	30%	20%	30%	40%
r_f	6%	6%	6%	6%	6%	6%

S : Current value of future CF
K_T : Investment at time T
T : Time to expiration
σ : Volatility
r_f : Risk-free rate of return

$$Riskiness = S\sigma\sqrt{T}$$

$$PV(K_T) = \frac{K_T}{(1+r_f)^T}$$

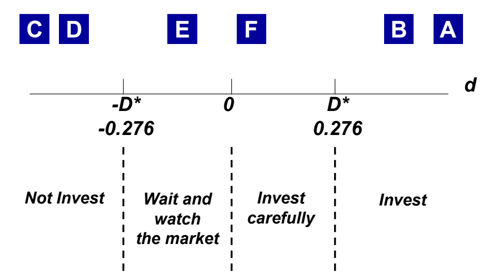
$$d = \frac{NPV}{Riskiness} = \frac{S - PV(K_T)}{S\sigma\sqrt{T}}$$

5.1 Simple Case

Six Independent DSL Projects

Variable	A	B	C	D	E	F
S	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
K_T	$90.00	$90.00	$110.00	$110.00	$110.00	$110.00
T	0.0	2.0	0.0	0.5	1.0	2.0
σ	30%	30%	30%	20%	30%	40%
rf	6%	6%	6%	6%	6%	6%
Riskiness	$0.00	$42.43	$0.00	$14.14	$30.00	$56.57
NPV	$10.00	$19.90	-$10.00	-$6.84	-$3.77	$2.10
d	+infinite	0.469	-infinite	-0.484	-0.126	0.037
Exercise decision	invest	invest	do not invest	do not invest	wait & watch	invest carefully

5.1 Simple Case



Conclusion

- **Decision index d = NPV/Riskiness gives uncertainty adjusted NPV**
- **d = D* = 0.276 gives the break-even point of NPV plus ROV**
- **Make a decision by observing d**

An Investment Criterion Incorporating Real Options

James Alleman, Hirofumi Suto &
Paul Rappoport

Experts Dialogue: Managing Risk in the Competitive Environment
International Telecommunication Union
Geneva, Switzerland
28-29 October 2004