Quantitative tools for Market Analysis

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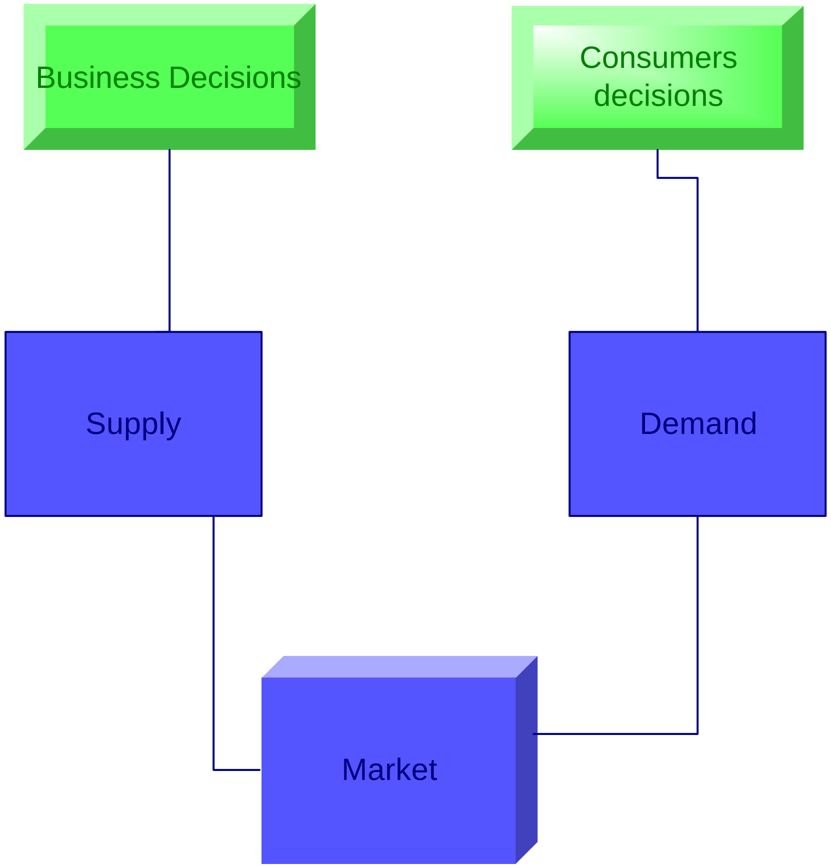
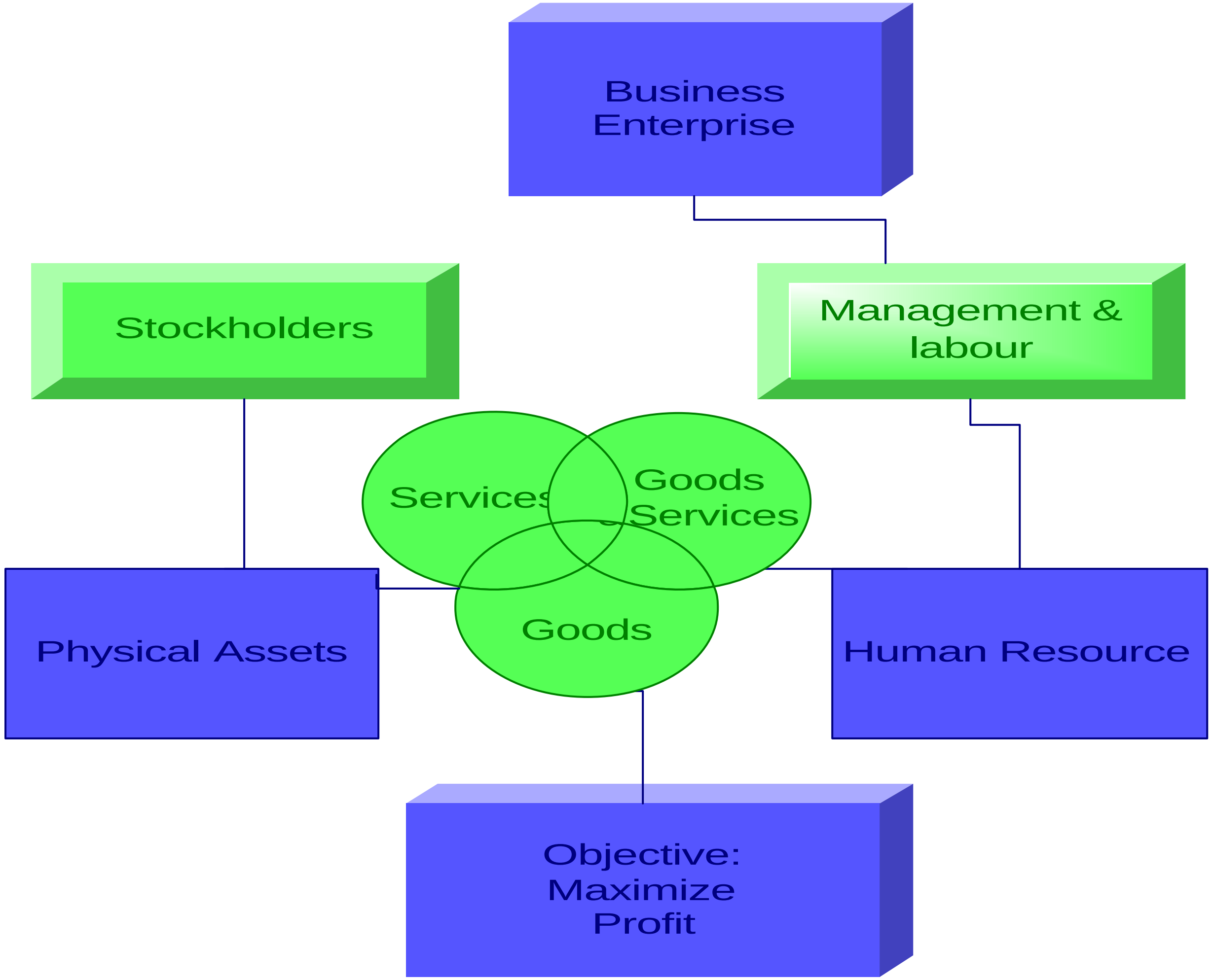
Fundamentals of Markets
Diagram 1



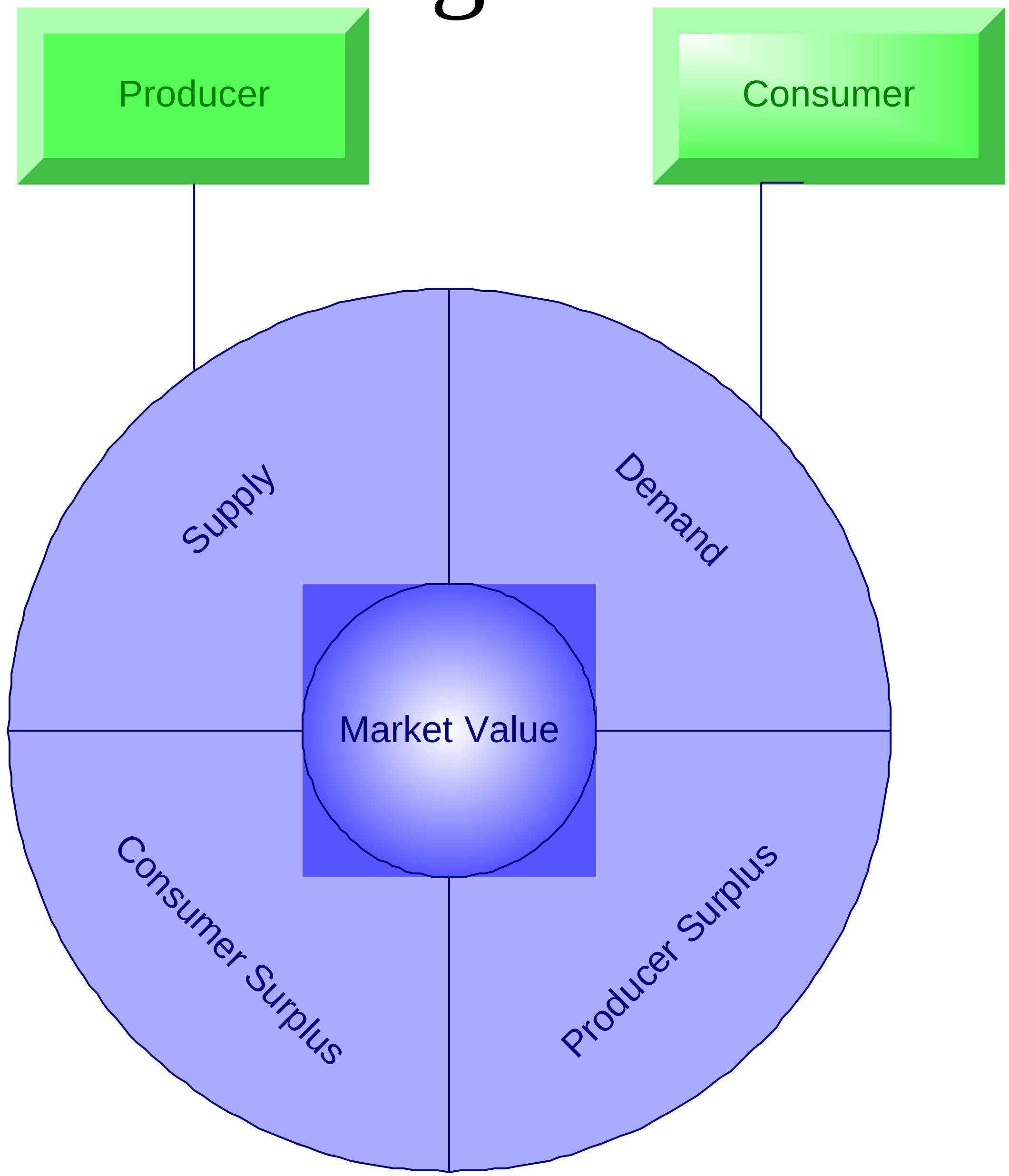
Why Analytical Tools?

- Analysis of Producers/Business Behaviour.
- Analysis of Consumers Behaviour.
- Analysis of Environment.
- Why Need for Behavioral Analysis?
- Premise: Past behaviour influences present attitude and provides indicators of future behaviour.

Supply Side Functionality Diagram 2



Markets Redefine Objectives
Diagram 3



Key Market Functionality

- **Determines/Assigns value to goods & services.**
- **Determines/assigns allocation of resources for production and distribution of goods & services;**
- **Determines social welfare.**
- **Analysis of value is critical to market analysis.**

Investment

- **Investment decision function of expected return. I = ƒE(V)**
- **Investment, expected to mature in year n is normally made in year$_1$.**
- **So investment decision in year$_1$ is a function of E(V^n) discounted to its value in year$_1$ (present value (PV))**

1.0. $PV = p/(1+i) + p/(1+i)^2 \cdots\cdots p/(1+i)^n$

- **Where p = expected profit (return) and**
- **i = discount rate and n time period usually in years.**

Investment Decision (?)

1.1. $?_t = {}_t S_1^n \frac{R_t - C_t}{(1 + i)^t}$

- ❖R = Total revenue
- ❖C= Total cost
- ❖Total revenue and total cost expansion:
- ❖$R_t = S_t * P_t$
- ❖$C_t = U_t * Ç_t$

Revenue/Cost Permutations

$$1.2.\ ?R = M_r = ?S/?P = \frac{(S_2 - S_1)/S_1}{(P_2 - P_1)/P_1} = \frac{d\,S}{d\,P}$$

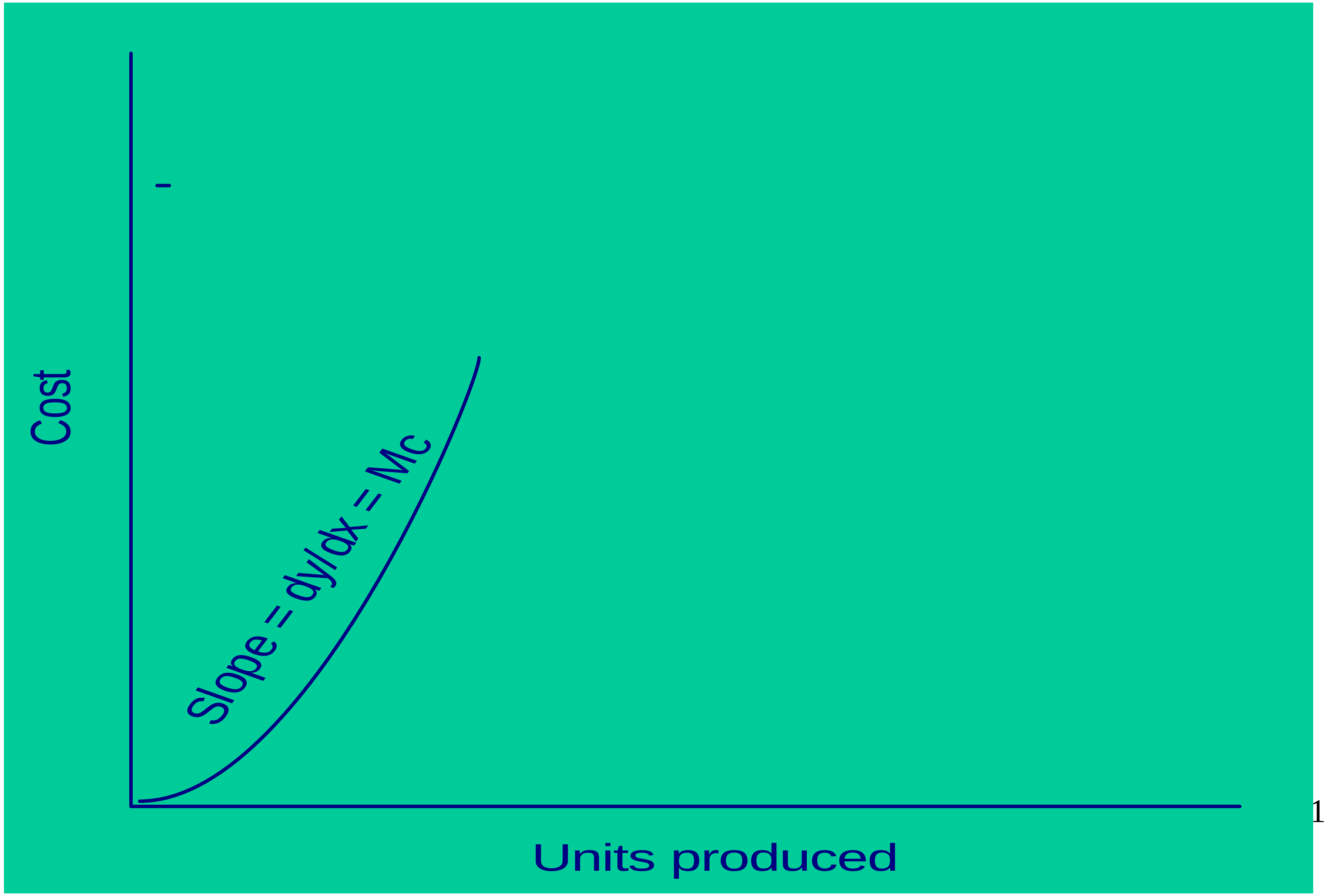
$$1.3.\ ?C = Mc = ?U/?\text{Ç} = \frac{(U_2 - U_1)/U_1}{(\text{Ç}_2 - \text{Ç}_1)/\ \text{Ç}_1} = \frac{d\,U}{d\,\text{Ç}}$$

- M_r = Marginal revenue
- M_c = Marginal cost
- d = Partial derivative

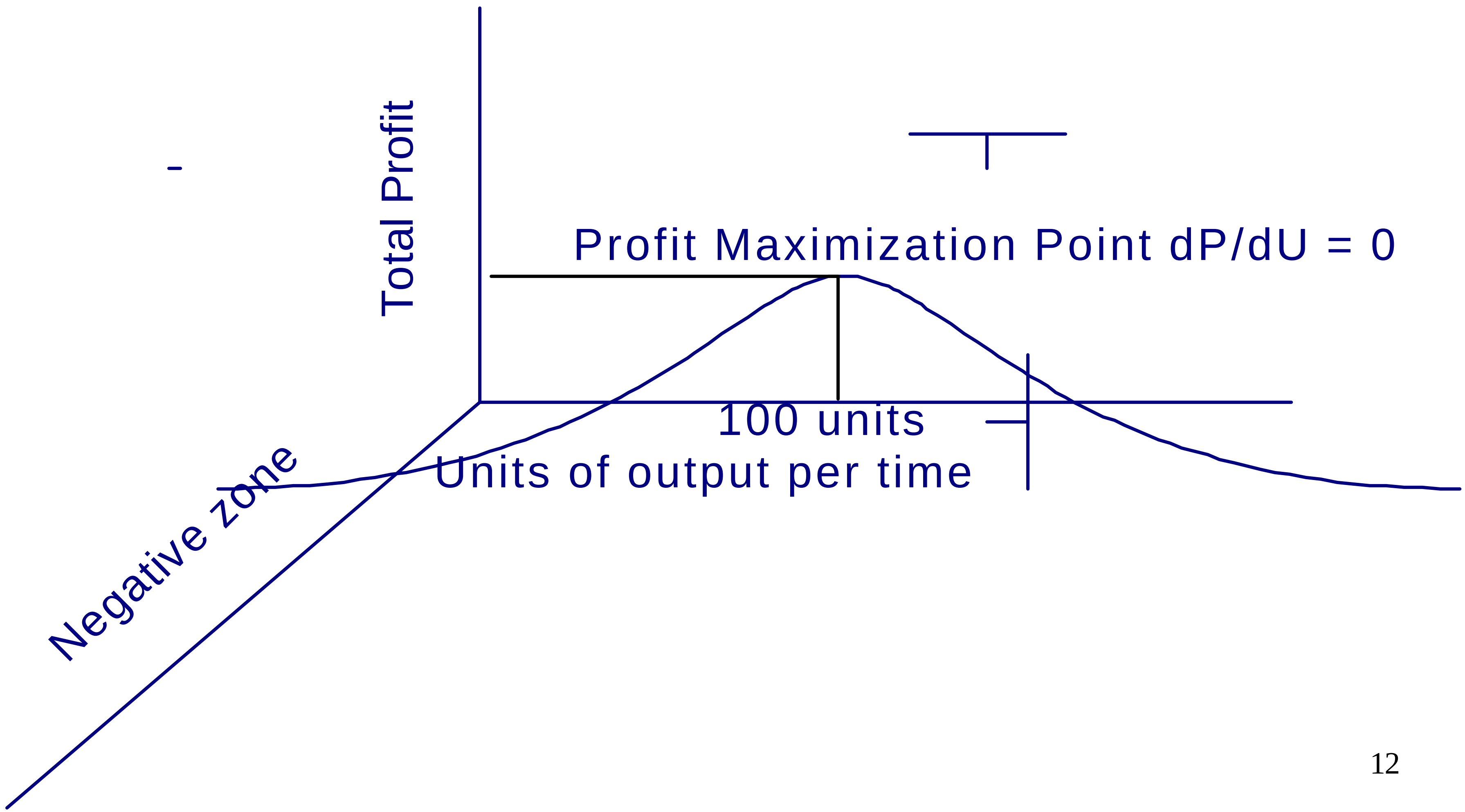
Variables

- **S = Quantity of goods/services sold**
- **P = price, good/service**
- **U = units of goods/services produced**
- **Ç = unit cost of goods/services**

Function Slope = dy/dx
Diagram 4.



Maximising Profit Diagram Diagram 5.



Profit Function

- **Let profit function (P) be expressed as:**

1.4. $P = -\$10\,000 + \$\,400U - 2U^2$

- **When output (U) is zero the firm incurs a loss of**
- **$ 10, 000**
- **As output increases profit increases.**
- **Profit is maximised/minimised at the point:**
- **$dP/dU = 400 - 4U = 0$**
- **$4U = 400$**
- **$U = 100$.**
- **Beyond 100 units profit begins to decline.**

- To ascertain whether profit is maximised or minimised, take the second derivative:
- $\underline{d^2P}$
- d^2U
- Point of maximization is where:
- $\underline{d^2P\ \ }$ is negative and minimization where
- d^2U it is positive
- So where:
- $\underline{dP\ }$ = 400 – 4U = 400 – 4(100) = 0
- dU
- $\underline{d^2P =}$ -4 confirms that at output 400 profit is
- d^2U maximised.

Quantitative Analysis of Discount Rate

- ❑**Interest is the return on investment.**
- ❑**Correlation between level of return and risk**
- ❑**Positive correlation Risk free investment and low return.**

1.5. ? = *f*(µ)

Where:

- ❖**? = risk; and**
- ❖**µ = uncertainty.**

Estimating Uncertainty

- No precise estimator.
- Probability: likelihood of occurrence of an event is normally used as an indicator of uncertainty.
- Assume there is a 70% chance that per minute tariff will fall with the introduction of a new mobile operator then:
- Likelihood of price fall = 0.7
- Likelihood of no price fall = 0.3
- The sum of probabilities of an event should = 1

Application

- **Assume company is considering the options of investing $ 1 million in either:**

(i) Expansion 2.5G network

(ii) Introduction 3G network

Assume that both projects are dependent on the level of domestic economic activity as given at table x.

Table x

State of Economy	Probability	Project 2.5G	Project 3G
Recession	0.2	$ 400	$ 0
Normal	0.6	$ 500	$ 500
Boom	0.2	$ 600	$ 1,000

Expected Profit

- **Expected profit (Ê(?)) is the weighted average of profits (?) in various states of the economy.**

1.6. $\hat{E}(?) = {}_i S_1^n ?_i P_i$

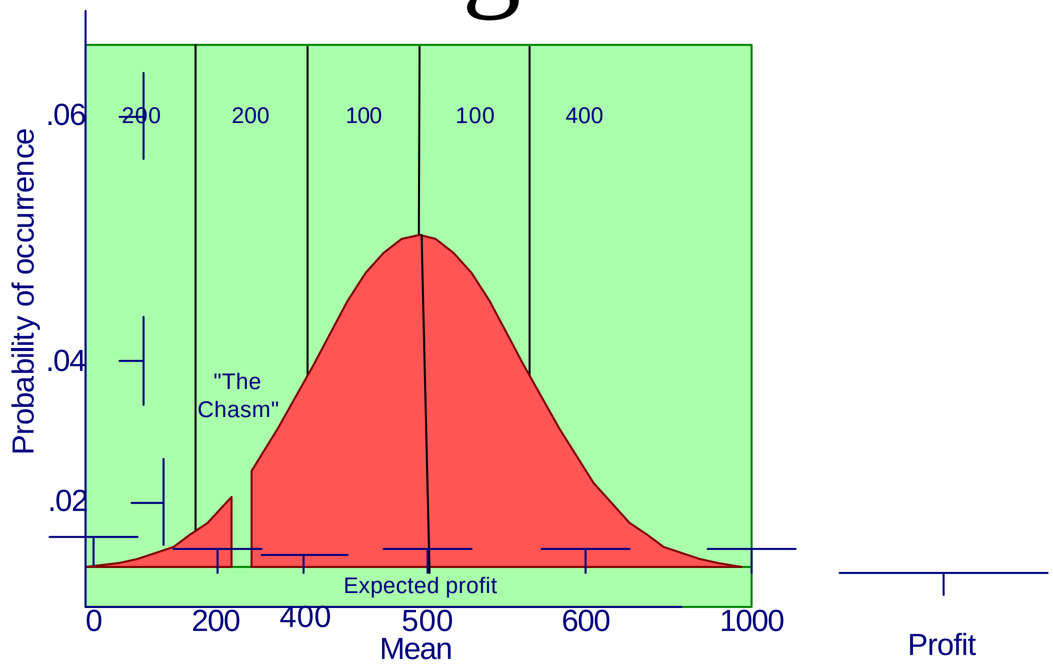
- **Where:**

- **P indicates probability and i = 1-n**
- **E.g. the expected profit for the projects at Table X are:**

1.7. $\hat{E}(?_a) = 400(0.2) + 500(0.6) + 600(0.2) = 500$

1.8. $\hat{E}(?_b) = 500(0.6) + 1{,}000(0.2) = 500$

Range of outcome
Diagram 6



Observations

- ❖Inverse relationship between probability of distribution and risk ;
- ❖Risk is higher the further the dispersion from the expected value
- ❖Risk is lower when variation of probable outcome is closer to the expected value
- ❖Therefore dispersion around expected value determines degree of risk.
- ❖Range for Project A = 400-600
- ❖Range for Project B = 0-1000

Estimating Dispersion, (Standard Deviation)

1. **Calculate expected value (mean of distribution):**

Ê? = S ?$_{i}$ P$_{i}$

2. Calculate deviation from Ê? :

= ?$_{i}$ - Ê?

3.Calculate the variance of probability of distribution:

s^{2} = S (?$_{i}$ - ?)^{2}P$_{i}$

- ❖ **Standard Deviation s = (s^{2})^{½}**

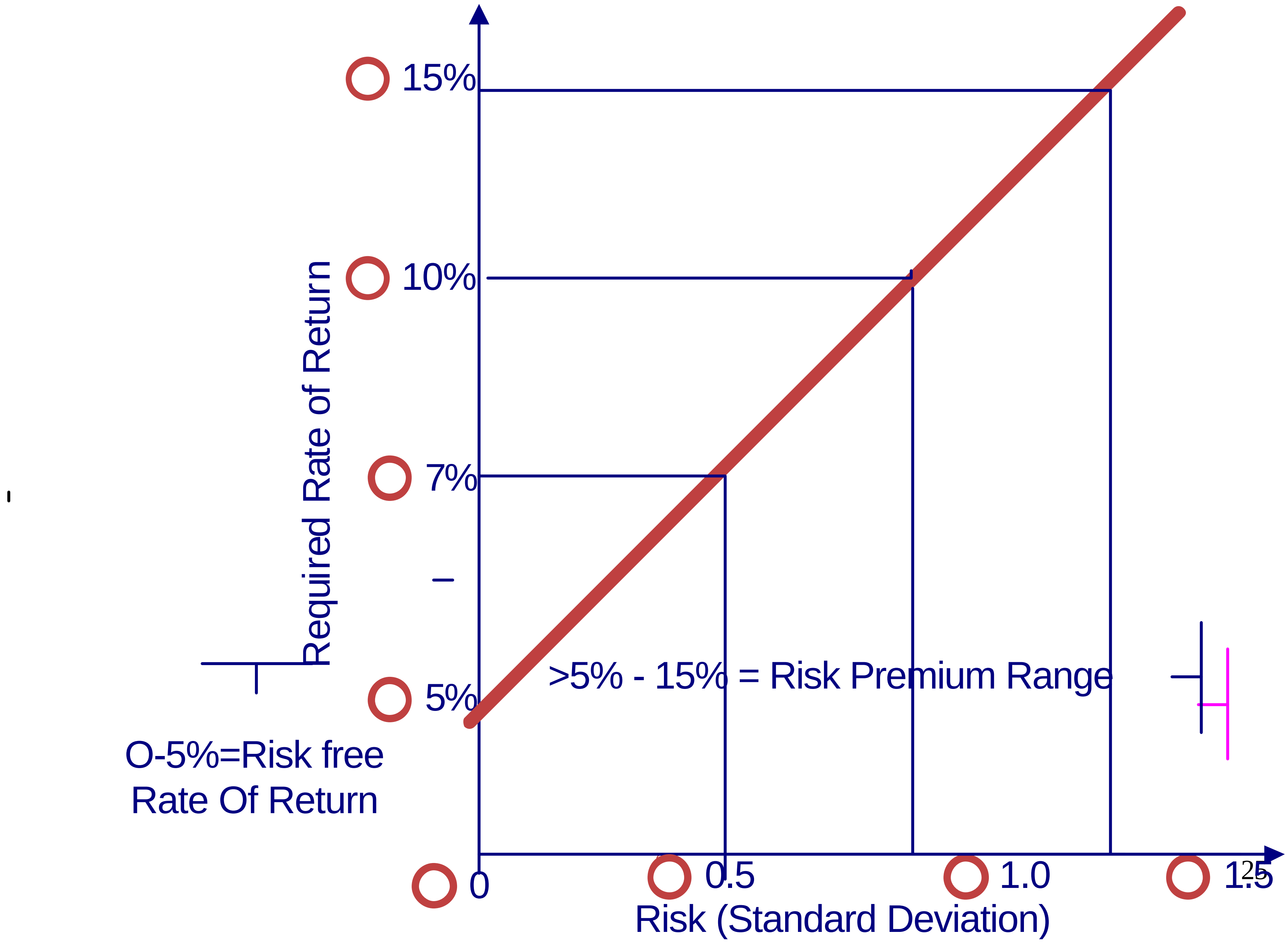
Deviation (? $_i$ - Ê?)	Variance (? $_i$ - ?)^2	Variance * Probability (? $_i$ - ?)^{2}P$_i$	Coefficient of Variation = s / Ê?
400 – 500 = - 100	10,000	10,000 * 0.2	= 2,000
500 – 500 = 0	0	0* 0.6	= 0
600 – 500 = 100	10,000	10,000* 0.2	= 2000
s ^{2}	= 4,000	s	= 63.25

Adjusting Market Value for Risk

- **Rewriting Equation (1.0):**

1.9. PV = $_{t}S^{n}_{1}\ p/(1+i)^{t}$,

- **Risk adjustment:trade off investors are likely to make based on the degree of uncertainty.**
- **Risk premium is the difference between the required rate of return and risk free rate of return on an investment.**
- **Risk adjustment is effected through changes in the discount rate i**


15%
10%
7%
5%
0
Required Rate of Return
>5% - 15% = Risk Premium Range
O-5%=Risk free
Rate Of Return
0.5
1.0
1.5
Risk (Standard Deviation)

- The Adjusted Present Value to account for risk (RVP) is :

2.0. RPV = $_{t}S_{1}^{8}$ p/(1+ r)^{t}

Working Example:

- Project A : Roll out GSM network in region Y
- Project B : Roll out GSM network in region z
- Investment on both projects = $100,000
- Expected annual return over 8 yrs

  = $ 20,000 (Project A) & $23,000 (Project B)
- s for Project A = 1.0
- s for Project B = 1.5
- Adjusted Costs of capital (r) are 10 % project A & 15% Project B.

Risk Adjusted Value (RPV)

2.2. **RPV$_{Project\ A}$** $= \frac{20,000}{(1.10)^8} - 100,000$

$= 20,000 * \frac{1}{(1.10)^8} - 100,000$

= 20,000 x 5.335 – 100,000

= 6,700

Risk Adjusted Value (RPV)

2.3. **$RPV_{Project\ B}$ = $\frac{23,000}{(1.15)^8}$ - 100,000**

- **= 23,000* $\frac{1}{(1.15)^8}$ - 100,000**

= 23,000 x 4.487 -100,000

= 3,200

Other Risk Assessment Methodologies

- **Coefficient of Variation = s / Ê?**
- **Beta method: measures risk for which investors require compensation.**

Quantitative Analysis, Demand

- **Application of Quantitative methods to study, simulate and forecast consumers response to change in factors that are likely to influence their purchase of a good/service.**
- **A Demand Function may be expressed as:**
- **$(Q_t) = f\ {}_t\Sigma_1^n(x_{1t}, x_{2t}\ , x_{3t}, \ldots\ldots x_{nt})$.**
- **The fundamental problem is how to measure the relationship between Q_t & ΣX_{it}.**

Demand & Profit

2.4. $?p_t = f(?R_t – ?C_t) = f[?(Q_t* P_t) – ?C_t]$

- **Assuming:**

  $?Q_t = f\ S?(P_t, Y_t, A_t, p_t, T_t \ldots.u)$

Where:

P = Price of Q

Y= Disposable Income

A = Direct & indirect advertisement

p= price of related goods/services

T= taste

u= other factors.

Sensitivity

- **Problem:**
- **To estimate the sensitivity of interrelationship between:**

1. **(P_t, Y_t, A_t, p_t, T_t ….u_t) & Q_t**
2. **(P_t, Y_t, A_t, p_t, T_t ….u_t) & revenue (R) and in turn**
3. **(P_t, Y_t, A_t, p_t, T_t ….u_t) & the level of profit (p).**

- **Sensitivity is normally involves measurement elasticity (?) the multiplier that prescribes the degree of responsiveness of one variable to a change in another variable, ceteris paribus.**

Methods of Measurement of Elasticity

- ? = dQ * X Point elasticity
- dX Q

- Or
- É = dQ * X! Arc elasticity
- dX Q!
- ! indicates the mean

Measuring Demand Sensitivity

- **Assume that demand (annual) for a service (a) is expressed in terms of the linear function:**

2.5. $Q_a = {}_{a1}P + {}_{a2}Y + {}_{a3}Pop + {}_{a4}Ic + {}_{a5}A$

- **Where**
- **P = Price of a**
- **Y= Income**
- **Pop = Population**
- **Ic = Index of credit availability**
- **A = Advertising expenditure**
- $_{a1} - {}_{a5}$ **are coefficients of the independent variables.**

- Assume that the respective coefficients were measured as:
- 3000 for price
- 1,000 for income
- 0.05 for population
- 1, 500,000 for Index of credit availability
- 0.05 for Advertisement.

Then:

2.6. $Q_a = - 3000P + 1{,}000Y + 0.05Pop + 1{,}5000{,}000Ic + 0.05A$

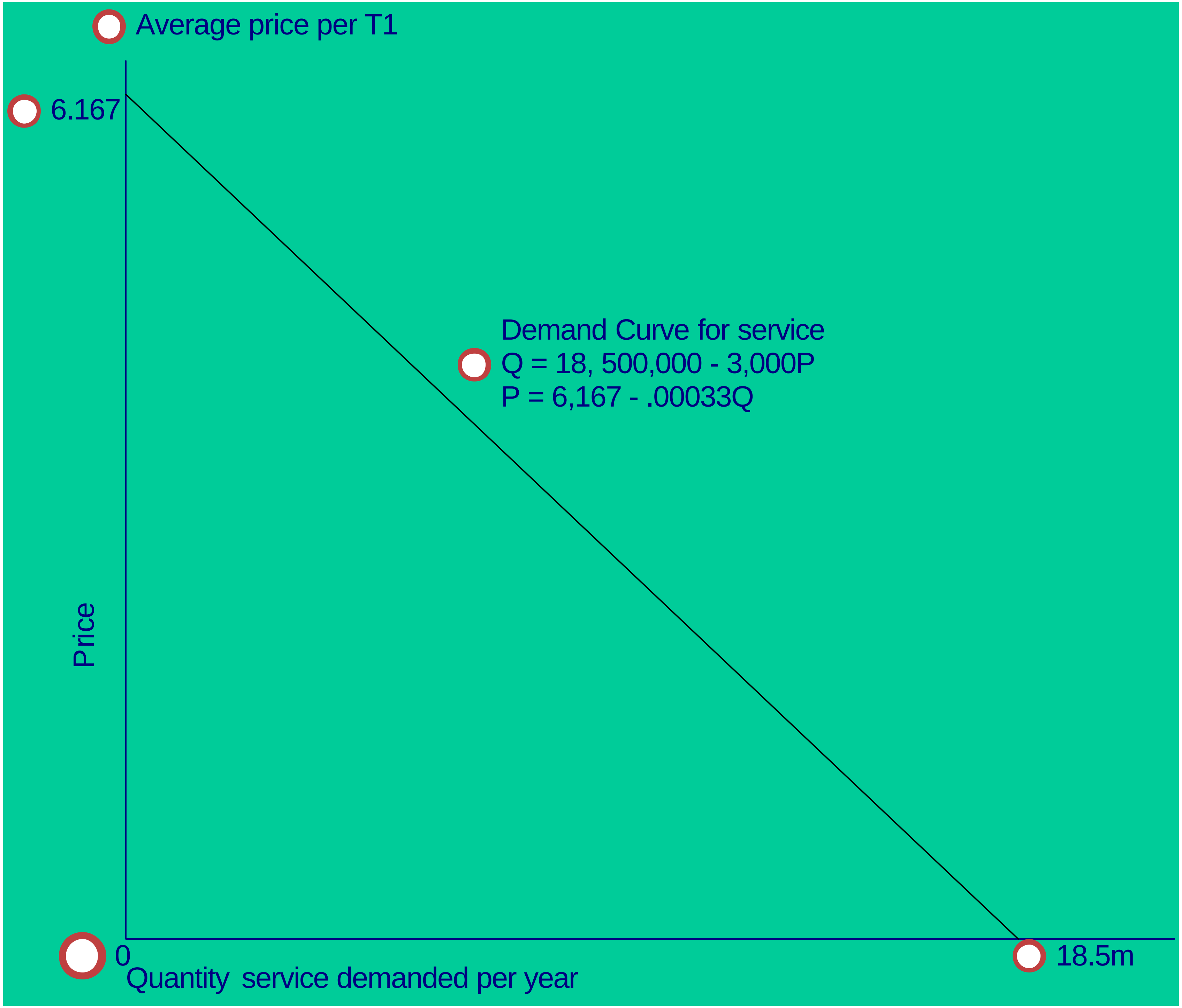
- **Assuming that:**
  - **Y = 2,000**
  - **Pop= 200, 000,000**
  - **Ic = 1.0**
  - **A = 100,000,000**
- **In terms of Price:**

2.7. Q_a = - 3000P + 1,000(2,000) +

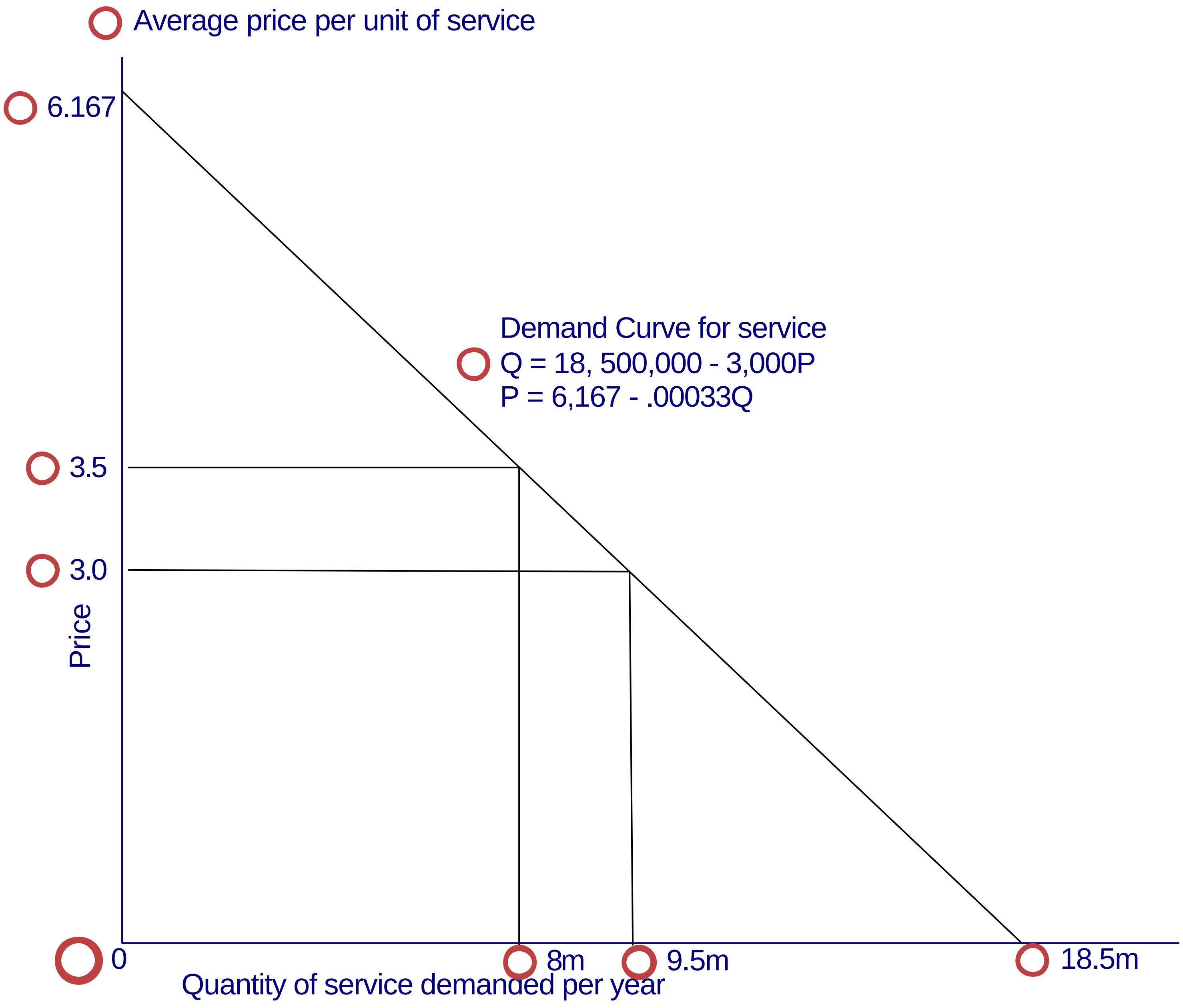
0.05(200,000,000) + 1,5000,000(1)

+ 0.05(100,000,000)

= 18,500,000 – 3000P



Average price per T1
6.167
Demand Curve for service
Q = 18, 500,000 - 3,000P
P = 6,167 - .00033Q
Price
0
Quantity service demanded per year
18.5m


Average price per unit of service
6.167
Demand Curve for service
Q = 18, 500,000 - 3,000P
P = 6,167 - .00033Q
3.5
3.0
Price
0
8m
9.5m
18.5m
Quantity of service demanded per year

Point Elasticity

(2.7. Q_a = - 3000P + 1,000Y + 0.05Pop
+ 1,5000,000Ic + 0.05A)

Assume a proposed price increase from 3,000 to 3,500.

2.8. $\frac{dQ_a}{dP}$ **.= - 3,000 a constant**

At:

P = 3,000 and Q = 9,500,000

2.9. ? = - 3000(3000/9,500,000)
= - 0.95.

❑**At:**

P = 3,500 and Q = 8,000,000

3.0. ? = -3,000(3,500/8,000,000)

= - 1.3

❑**So elasticity can change along a demand Curve and could very between:**

- **Inelastic**
- **Unitary**
- **Elastic**

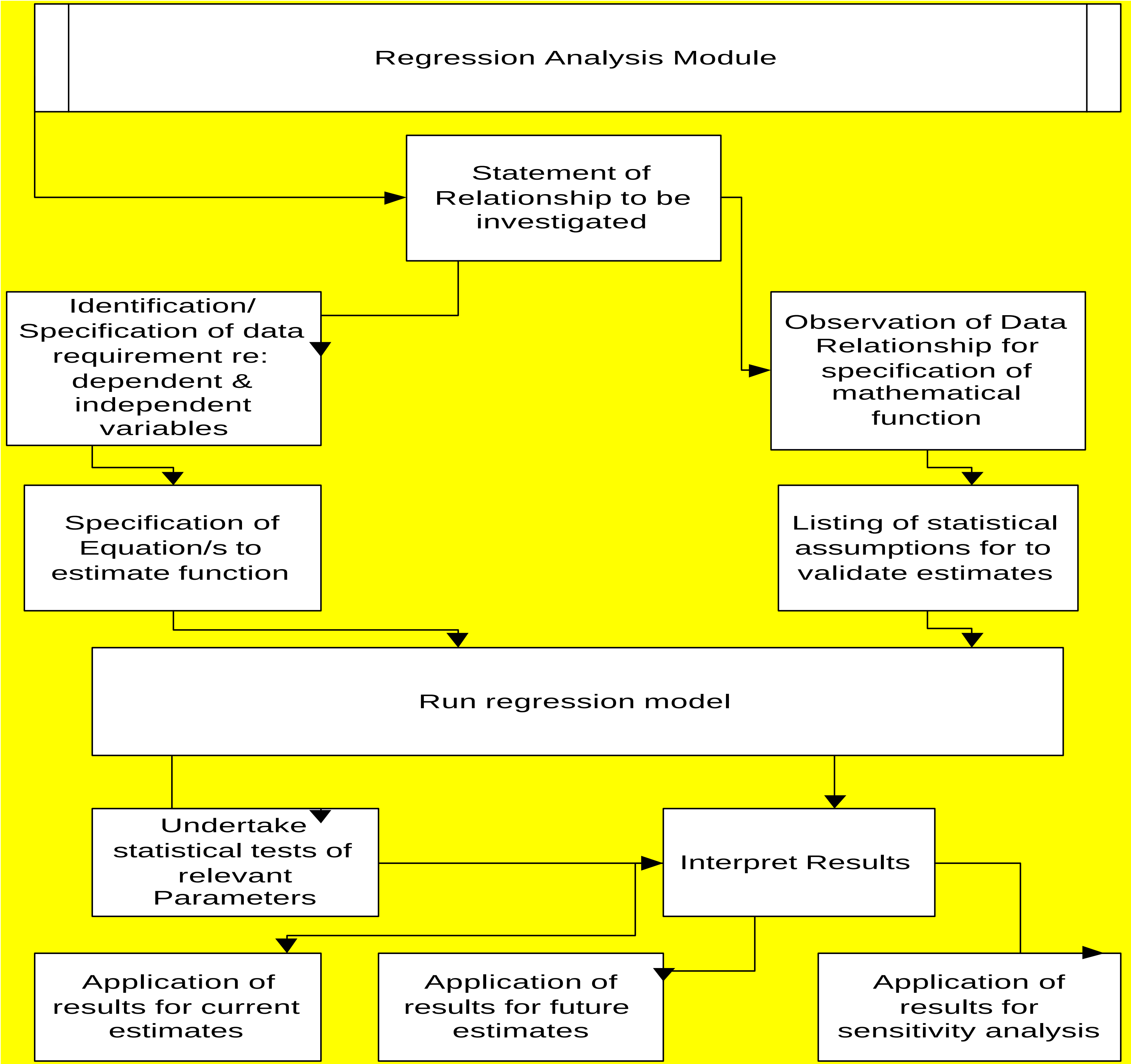
Arc Elasticity

- É = -3,000(3,000 + 3,500)/2
- (8,000,000 + 9,500,000)/2
- = 1.114
- **GENERALLY, NOT MANY ENTERPRISES HAVE DATA TO CALCULATE POINT ELASTICITIES.**

Real World Calculation of Decision Parameters

***Regression Analysis*:**

- ❑ **Basically, a combination of mathematical and statistical methods to estimate values of parameters.**
- ❑ **Widely used for:**
  - o **Simulation exercises;**
  - o **Forecasting/predicting socio-economic/financial/market behaviour.**


Regression Analysis Module
Statement of Relationship to be investigated
Identification/ Specification of data requirement re: dependent & independent variables
Observation of Data Relationship for specification of mathematical function
Specification of Equation/s to estimate function
Listing of statistical assumptions for to validate estimates
Run regression model
Undertake statistical tests of relevant Parameters
Interpret Results
Application of results for current estimates
Application of results for future estimates
Application of results for sensitivity analysis

Identification/Specification of Issue

<u>Statement of issue:</u>

- **E.g. Service provider applies for an increase in monthly residential telephone access rates in rural area X.**
- **One of the factors the Regulator and operator should seek to establish for decision making is the sensitivity of households in area X to changes in telephone access rates.**

Identification of Demand Function

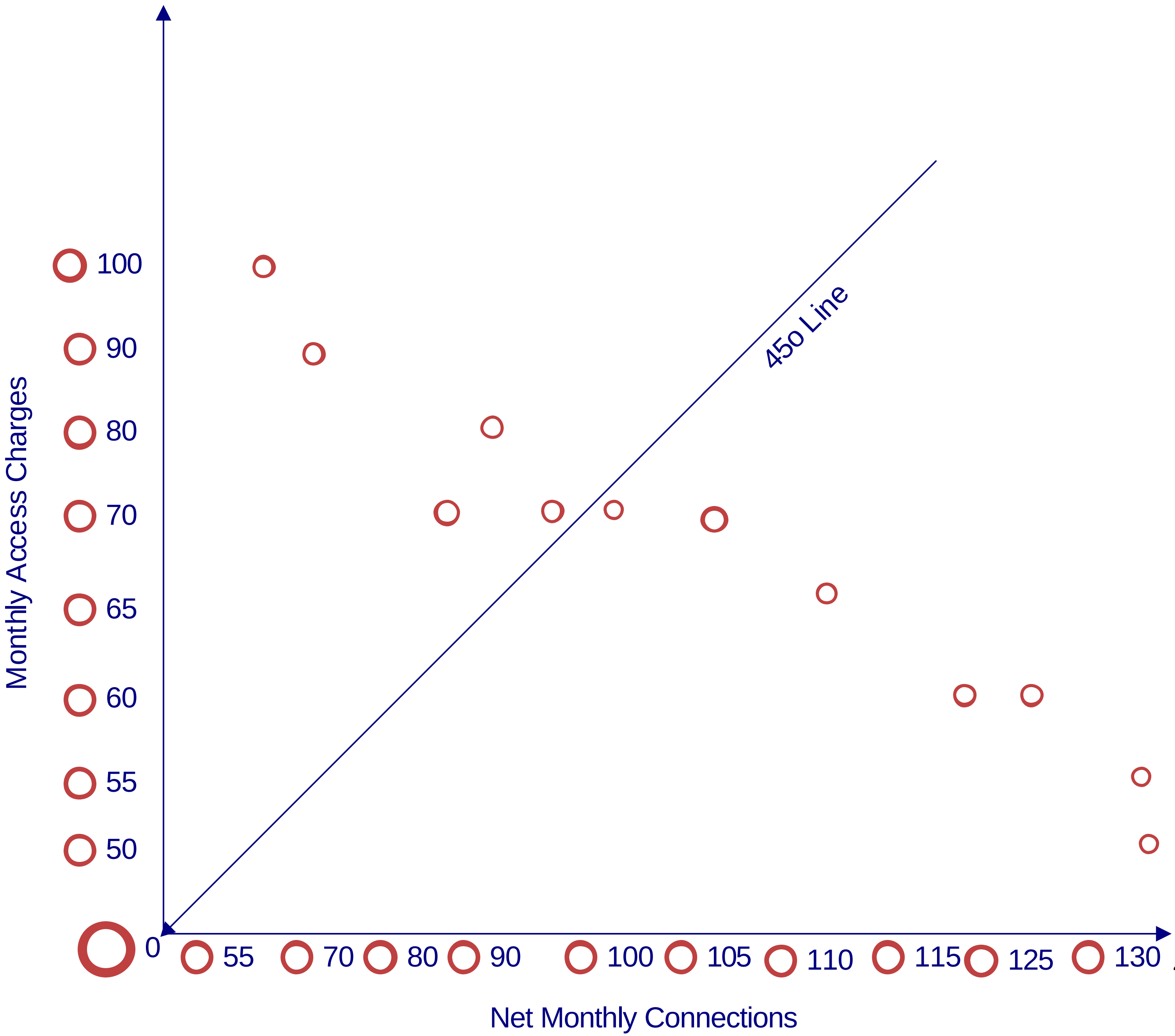
Identification of critical factors that impact net monthly residential telephone connections **(Q_t)**

Given the data at the following table it seems Reasonable to assume that (Q_t) is affected by:

- **Monthly access rate for residential telephone, P_t;**
- **Monthly Disposable income per household, Y_t in area X.**

3.1. $Q_t = f(P_t, Y_t)$

Q	P	Y
55	100	5.50
70	90	6.30
90	80	7.20
100	70	7.00
90	70	6.30
105	70	7.35
80	70	5.60
110	65	7.15
125	60	7.50
115	60	6.90
130	55	7.15
130	50	6.50



Monthly Access Charges
100
90
80
70
65
60
55
50
0
55
70
80
90
100
105
110
115
125
130
450 Line
Net Monthly Connections

Specification of Regression Equation

3.1. $Q_i = ß_1 - ß_2 P_i + ß_3 Y_i + u_i$

- U is the stochastic term involving the following assumptions:
  - o u_i is normally distributed
  - o $E(u_i) = 0$
  - o $E(u_i^2) = s^2$
  - o $E(u_i u_j) = 0$
  - o The explanatory variables are non-stochastic
- Note $ß_1$ is the mean of Q when each of the explanatory variables is equal to zero.

Least Sq Estimates of Regression Coefficient

3.2. $S = {}_{i}S_{1}^{13}(Q_i - ß_1 - ß_2 P_i \; - ß_3 Y_i)^2$

- *Differentiating S with respect to* $ß_1$*,* $ß_2$ *and* $ß_3$*:*

3.3. $\frac{dS}{d\,ß_1} = -2\;{}_{i}S\,(Q_i - ß_1 - ß_2 P_i \; - ß_3 Y_i)$

3.4. $\frac{dS}{d\,ß_2} = -2\;{}_{i}S\; P_i(Q_i - ß_1 - ß_2 P_i \; - ß_3 Y_i)$

3.5. $\frac{dS}{d\,ß_2} = -2\;{}_{i}S\; Y_i(Q_i - ß_1 - ß_2 P_i \; - ß_3 Y_i)$

- **Minimizing and rearranging terms:**

3.6. $S\ Q_i = ßn_1 + ß_2\ S\ P_i + ß_3\ S\ Y_i$

3.7. $S\ P_i\ Q_i = ß_1 S\ P_i + ß_2\ S\ P^2_i + ß_3\ S\ P_i\ Y_i$

3.8. $S\ Y_i Q_i = ß_1 S\ Y_i + ß_2\ S\ P_i\ Y_i + ß_3\ S\ Y^2_i$

- **So :**

3.9. $ß_1 = S\ Q_i/n - ß_2\ S\ P_i/n - ß_3\ S\ Y_i/n$

- **Substituting $ß_1$ into the normal equations and converting them into matrix form.**

- **Note most regression package would do these iterations.**

$$\beta_2 = \frac{\begin{vmatrix} a_{q2} & a_{23} \\ a_{q3} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix}} = \frac{a_{q2}a_{23} - a_{q3}a_{23}}{a_{22}a_{23} - a^2_{23}}$$

$$\beta_3 = \frac{\begin{vmatrix} a_{22} & a_{q2} \\ a_{23} & a_{q3} \end{vmatrix}}{\begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix}} = \frac{a_{22}a_{q3}a_{23}a_{q2}}{a_{22}a_{23} - a^2_{23}}$$

$$\frac{-3550*4.86-(-54)*125.25}{2250*4.86-(-54)^2} = \frac{10,631.5}{8019} = -1.326$$

$$\frac{2250*125.5-(-54)*-(350)}{8019} = \frac{90,112.5}{8019} = 11.237$$

4.0. $ß_1$= 100-(-1.326)*70 – 11.237*6.7 = 117.532

- ***Other important calculations from the regression equation are:***

S Q_i/n = 100	**d Q_i * S P_i/n d P_i S Q_i/n**	**= 1.326* 70/100 = 0.93**
S P_i/n = 70	**?$_Y$ = 11.237* 6.7/100**	**= 0.75**
S Y_i/n = 6.7	**$R^{2 = 0.981}$**	**F= 41.5**

4.1. Q_i = 117.532 – 1.326P_i + 11.237Y_i + u_i

Statistical Tests

equation	Estimated t values	T values for 13-9 Degrees of Freedom	Confidence Interval 95% two tail test	Test for auto correlation
!ß - ß* / $s_{ß}$	$ß_1$ = 2. 44	2.262	Significant	Test for multi-collinearity
	$ß_2$ = 2.30		Significant	
	$ß_2$ = 2.41		Significant	

Annex 1
Basics for Optimization Techniques

1. $Y = aX^b$

$dy/dx = baX^{b-1}$

2. $Y = U*V$

- $dy/dx = V\ du/Dx + U\ dv/dx$
- E.g. $Y = 3x^2(3-x)$
- $dy/dx = (3-x)(6x) + 3x^2(-1)$

- $Y = U/V$
- $\frac{dy}{dx} = \frac{V \frac{du}{dx} - U \frac{dv}{dx}}{V^2}$
- $Y = \frac{2x - 3}{6x^2}$
- $\frac{dy}{dx} = \frac{6x^2(2) - (2x - 3)12x}{36x^4}$
- $= \frac{3 - x}{3x^3}$

- $Y = 2U – U^2$
- Where $U = 2x^3$
- dy/dx = dy/du* du/dx =
- dy/du $= 2 – 2(2x^3)$
- du/dx $= 6x^2$
- $= (2-4x^3)6x^2$
- $= 12x^2 – 24x^5$

- Second order derivatives:
- if $du/dx = 2cQ – 3dQ^2$
- Then:
- $du^2/dx^2 = 2c – 6dQ$

- ✓ $Y = ae^{bx}$ then $dy/dx = bae^{bx}$

- ✓ Y = a log bx then dy/dx = a/x

- ✓ $Y = a^x$ then $dy/dx = a^x \log x$
- ✓ Y = a sin bX then dy/dx = ab cos bX
- ✓ Y = a cos bX then dy/dx = -ab sin bX