REPORT ITU-R SA.2098

Mathematical gain models of large-aperture space research service earth station antennas for compatibility analysis involving a large number of distributed interference sources

(2007)

1 Introduction

Compatibility studies between space research service (SRS) earth stations and high-density fixed systems are being conducted in the 31.8-32.3 GHz and 37-38 GHz bands.

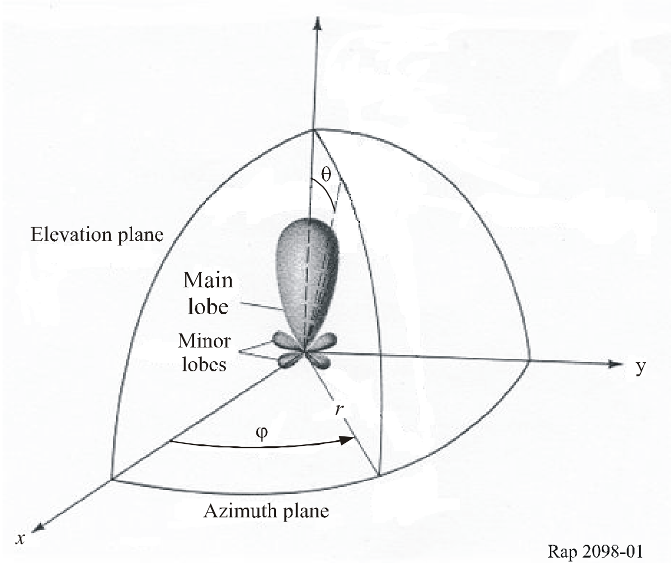
One of the key parameters that needs to be defined to determine the level of interference that may occur at SRS earth stations is the antenna pattern to be used in the calculations. A peak envelope radiation pattern for fixed wireless systems is provided in Recommendation ITU-R F.699 and a radiation pattern representing average side-lobe levels for line-of-sight point-to-point radio-relay systems is provided in Recommendation ITU-R F.1245. The Recommendation ITU-R F.699 pattern, when applied only to polar angles larger than 1°, is the same as the pattern in Recommendation ITU-R SA.509. Peak envelope radiation patterns for Earth stations operating in the fixed-satellite service (FSS) are given in Recommendations ITU-R S.580 and ITU-R S.465 and a radiation pattern representing average side-lobe levels of fixed-satellite service earth stations is provided in Recommendation ITU-R S.1428. This Report compares the performance of these patterns and introduces a new model.

Actual and realistic patterns involve many factors, too complicated and diverse to be exactly accounted for in a simple theoretical computation. For example, the position of nulls and peaks in the side-lobe regions vary as a function of antenna gravitational loading, winds, etc., and are best represented by an envelope. Over the years many pattern models have been suggested for large reflector antennas, (see e.g. Recommendation ITU-R F.1245-1 – Mathematical model of average and related radiation patterns for line-of-sight point-to-point radio-relay system antennas for use in certain coordination studies and interference assessment in the frequency range from 1 GHz to about 70 GHz. Recommendation ITU-R SA.509-2 – Space research earth station and radio astronomy reference antenna radiation pattern for use in interference calculations, including coordination procedures. Recommendation ITU-R SA.1345 – Methods for predicting radiation patterns of large antennas used for space research and radio astronomy and [Jamnejad, 2003].

A simple but effective method of characterizing an actual antenna pattern is to use a model which is based on many theoretical and experimental results and provide an upper- and/or lower-bound or envelope for the antenna which can be easily applied to many situations. Ideally, as discussed in Recommendation ITU-R F.1245, following the definition of directivity of an antenna, the gain model G given in dB should obey the equation for the average gain ratio, g_a:

$$g_a = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} g(\theta, \varphi) \sin(\theta) d\theta d\varphi = 1$$

in which θ is the polar angle from boresight and φ is the azimuth angle, as shown in the following figure:



For a circularly symmetric pattern, the equation reduces to:

$$g_a = \frac{1}{2}\int_0^{\pi} g(\theta)\sin(\theta)d\theta = 1$$

Typically gain models are given in dB as parameter G, which is related to the gain ratio g by:

$$G(\theta, \varphi) = 10\log(g(\theta, \varphi))$$

or

$$g(\theta, \varphi) = 10^{\frac{G(\theta, \varphi)}{10}}$$

In the models which are usually proposed in the literature, since an upper limit envelope or some other approximation is used instead of the actual pattern, the average gain values, as calculated from the integrals above, are much larger than unity (or larger than 0 in dB). However they can be used as a validity check for evaluating the general accuracy of the model compared to an actual antenna pattern. Typically a value of less than or around 2 (less than 3 dB) would provide a reasonable approximation.

Here, we evaluate the left-hand side involving the integral numerically for a number of circularly symmetric gain models and provide plots for variation of its value as a function of antenna parameters, such as frequency and aperture diameter.

2 Gain models

In all the models given below, gain values are specified in dB, angles are specified in degrees, and:

D: diameter of the main aperture of the antenna (m)

$$\lambda = \frac{c}{f} = \frac{0.3}{f_{GHz}}, \text{ wavelength (m)}$$

We are only considering large apertures with $D/\lambda > 100$ in this paper.

a) The Recommendation ITU-R F.699-7 model

Recommendation ITU-R F.699-7 proposes the following radiation pattern (maximum envelope) for the frequency range of 1-70 GHz:

$$G(\theta) = G_{max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda} \theta \right)^2 \qquad \text{for } 0 < \theta < \theta_m$$

$$G(\theta) = G_1 \qquad \text{for } \theta_m \leq \theta < \theta_r$$

$$G(\theta) = 32 - 25 \log_{10} (\theta) \qquad \text{for } \theta_r \leq \theta < 48°$$

$$G(\theta) = -10 \qquad \text{for } 48° \leq \theta \leq 180°$$

Note that if we take θ_r to be 1°, then the Recommendation ITU-R F.699-7 gain model defined from 1° to 180° is the same as the model in Recommendation ITU-R SA.509.

With:

$$G_{max} = 10 \log \left(\eta \left(\frac{\pi D}{\lambda} \right)^2 \right)$$

$$G_1 = 2 + 15 \log_{10} \left(\frac{D}{\lambda} \right)$$

$$\theta_m = 20 \sqrt{G_{max} - G_1} \left(\frac{D}{\lambda} \right)^{-1} \qquad \text{degrees}$$

$$\theta_r = 15.85 \left(\frac{D}{\lambda} \right)^{-0.6} \qquad \text{for } D/\lambda > 100$$

b) The Recommendation ITU-R RA.1631 model

France has proposed to use the model given in Recommendation ITU-R RA.1631. It is not a peak envelope but an average pattern defined by:

$$G(\theta) = G_{max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda} \theta \right)^2 \qquad \text{for } 0° < \theta < \theta_m$$

$$G(\theta) = G_1 \qquad \text{for } \theta_m \leq \theta < \theta_r$$

$$G(\theta) = 29 - 25 \log_{10} (\theta) \qquad \text{for } \theta_r \leq \theta < 10°$$

$$G(\theta) = 34 - 30 \log_{10}(\theta) \qquad \text{for } 10^\circ \le \theta < 34.1^\circ$$

$$G(\theta) = -12 \qquad \text{for } 34.1^\circ \le \theta < 80^\circ$$

$$G(\theta) = -7 \qquad \text{for } 80^\circ \le \theta < 120^\circ$$

$$G(\theta) = -12 \qquad \text{for } 120^\circ \le \theta \le 180^\circ$$

with:

$$G_{max} = 20 \log\left(\frac{\pi D}{\lambda}\right)$$

$$G_1 = -1 + 15 \log_{10}\left(\frac{D}{\lambda}\right)$$

$$\theta_m = 20\sqrt{G_{max} - G_1}\left(\frac{D}{\lambda}\right)^{-1} \qquad \text{degrees}$$

$$\theta_r = 15.85\left(\frac{D}{\lambda}\right)^{-0.6} \quad \text{degrees,} \qquad \text{or } \theta_r = 10^{1.28k^2 - 0.08k}\left(\frac{D}{\lambda}\right)^{-0.6k} \quad \text{degrees}$$

$$\theta_r = 15.85\left(\frac{D}{\lambda}\right)^{-0.6} \quad \text{degrees, for } k = 1$$

c) **The Recommendation ITU-R F.1245-1 model**

Recommendation ITU-R F.1245-1 proposes the following average radiation pattern for the frequency range of 1-40 GHz and provisionally for the range of 40-70 GHz:

$$G(\theta) = G_{max} - 2.5 \times 10^{-3}\left(\frac{D}{\lambda}\theta\right)^2 \qquad \text{for } 0^\circ < \theta < \theta_m$$

$$G(\theta) = G_1 \qquad \text{for } \theta_m \le \theta < \theta_r$$

$$G(\theta) = 29 - 25 \log_{10}(\theta) \qquad \text{for } \theta_r \le \theta < 48^\circ$$

$$G(\theta) = -13 \qquad \text{for } 48^\circ \le \theta \le 180^\circ$$

with:

G_{max}: peak gain

$$G_1 = 2 + 15 \log_{10}\left(\frac{D}{\lambda}\right)$$

$$\theta_m = 20\sqrt{G_{max} - G_1}\left(\frac{D}{\lambda}\right)^{-1}$$

$$\theta_r = 12.02\left(\frac{D}{\lambda}\right)^{-0.6}$$

d) **The Jp model (peak envelope)**

This is a new model providing peak envelope for all frequency ranges of interest. It is similar to Recommendation ITU-R F.699 with some modifications. The modifications involve the following areas:

i) The main beamwidth of the pattern can vary somewhat, based on various parameters of the antenna such as aperture illumination, blockage, surface errors, etc. The one-sided half-power beamwidth is defined as $\theta_{hp} = 0.5C_{hp}/(D/\lambda)$, in which the constant C_{hp} has an approximate value between 65 and 71. For a more accurate modelling, this parameter can be varied according to the type and quality of the antenna used. Here, a value of $C_{hp} = 69$ is selected for compatibility with Recommendation ITU-R F.699.

ii) The flat shoulder area of the pattern is set to a more realistic value. This value is normally not dependent on the antenna dimensions or wavelength, but is a rather complicated function of aperture illumination and blockage. A value of 17 dB is used which can be adjusted if necessary.

iii) The pattern efficiency is taken into account in determining the peak as well as the slope of the model pattern. This is in contrast to other models which consider a fixed slope. The pattern efficiency is a combination of aperture illumination efficiency, blockage efficiency, spillover efficiency, and efficiency due to surface errors. We separate the efficiency into a surface tolerance component which is directly dependent on the frequency and lump all the others into a separate component which is more or less independent of the frequency.

iv) A 5 dB raised platform area in the flat far-side lobe region of the pattern, in the 80° to 120° range, is introduced to account for possible main reflector spillover effects whose exact height and location varies with F/D (focal length to diameter ratio) and other design parameters of the reflector antenna. In rare occasions this platform is subject to the provisions of Note 2 given below.

Thus, the model pattern in given by:

$$G(\theta) = G_0 - 3\left(\frac{\theta}{\theta_{hp}}\right)^2 \qquad \text{for } 0° \leq \theta \leq \theta_1$$

$$G(\theta) = G_0 - G_1 \qquad \text{for } \theta_1 < \theta \leq \theta_2$$

$$G(\theta) = G_0 - G_1 - G_2 \log_{10}\left(\frac{\theta}{\theta_2}\right) \qquad \text{for } \theta_2 < \theta \leq \theta_3$$

$$G(\theta) = G_3 \qquad \text{for } \theta_3 < \theta \leq 80°$$

$$G(\theta) = G_3 + 5 \qquad \text{for } 80° < \theta \leq 120°$$

$$G(\theta) = G_3 \qquad \text{for } 120° < \theta \leq 180°$$

in which:

$$G_0 = 10 \cdot \log_{10}\left[\eta_a\left(\frac{\pi D}{\lambda}\right)^2\right] - 4.343\left(\frac{4\pi h_{rms}}{\lambda}\right)^2$$

$$G_1 = 17$$

$$G_2 = 27 + 10\left[\log_{10}(\eta_a) - \log_{10}\left(60\frac{h_{rms}}{\lambda}\right)\right]$$

$$G_3 = -10$$

$$\theta_{hp} = 0.5\frac{C_{hp}}{(D/\lambda)} \quad (65 \le C_{hp} \le 70, \text{ nominal value} = 69)$$

$$\theta_1 = \theta_{hp}\sqrt{\frac{G_1}{3}}$$

$$\theta_2 = \theta_{hp}10^{\frac{G_1}{G_2}}\sqrt{\frac{G_2}{36}}$$

$$\theta_3 = \theta_2 10^{\frac{G_0 - G_1 - G_3}{G_2}}$$

The η_a value refers to the pattern-related (aperture illumination, spillover, blockage, etc.) antenna efficiency excluding that associated with surface tolerance. Note that in this model the gain at boresight decreases with η_a, but the gain slope, G_2, in angle range between θ_2 and θ_3 increases with η_a. This reflects the physical reality that a decrease in peak gain has to be accompanied with increases in the side-lobe regions. This feature is not incorporated in other models.

NOTE 1 – The gain loss due to the surface tolerance is separately included as a function of h_{rms}, the surface tolerance, which also affects the slope of the pattern model. The valid range of surface tolerance for use in the above formulae is:

$$\frac{1}{60} \le \frac{h_{rms}}{\lambda} \le \frac{1}{15}$$

Any value of h_{rms}/λ above 1/15 must be replaced by 1/15; any value below 1/60 must be replaced by 1/60.

Thus one can use the value 1/60 for a very good antenna, 1/30 for a moderately good antenna and 1/15 for a poor antenna.

NOTE 2 – In rare cases, for large surface errors, θ_3 might exceed 80°, and an overlap of the sloped side-lobe region with the flat bump at 80°-120° region occurs. In such cases, the maximum value of the two at each angle must be selected.

e) **The Ja model (average)**

In addition to the peak envelope, an average envelope for the gain at any given angle in the side-lobe region can be defined, which is meaningful in the following sense. Let us assume a number of antennas and a fixed given angle from boresight. Since the antennas are not identical, they might have their side lobes shifted such that for one antenna the peak of a lobe falls at the given direction while a different antenna might have a null in that direction and yet a third antenna might have a value between the peak and the null, etc. So, one presumably can use an average value for the gain at the given direction which is an average of all these values from null to peak. It turns out that for a given lobe with very sharp null, this average is close to 3 dB below the peak of the lobe (usually less if the nulls are not sharp and are to some extent filled in). Now, if one assumes that the peak envelope model touches all the peak points of the lobes, then an average envelope is parallel to this envelope but below it by about 3 dB.

Accordingly, a model for an "average" envelope is obtained by a simple modification of the above model by increasing the G_1 value by 3 dB, reducing the G_3 value by 3 dB, and modifying the θ_2 value accordingly. Caution should be used in the application of this model to particular situations of interest. It is given as:

$$G(\theta) = G_0 - 3\left(\frac{\theta}{\theta_{hp}}\right)^2 \qquad \text{for } 0° \leq \theta \leq \theta_1$$

$$G(\theta) = G_0 - G_1 \qquad \text{for } \theta_1 < \theta \leq \theta_2$$

$$G(\theta) = G_0 - G_1 - G_2 \log_{10}\left(\frac{\theta}{\theta_2}\right) \qquad \text{for } \theta_2 < \theta \leq \theta_3$$

$$G(\theta) = G_3 \qquad \text{for } \theta_3 < \theta \leq 80°$$

$$G(\theta) = G_3 + 5 \qquad \text{for } 80° < \theta \leq 120°$$

$$G(\theta) = G_3 \qquad \text{for } 120° < \theta \leq 180°$$

in which:

$$G_0 = 10 \log_{10}\left[\eta_a \left(\frac{\pi D}{\lambda}\right)^2\right] - 4.343\left(\frac{4\pi h_{rms}}{\lambda}\right)^2$$

$$G_1 = 20$$

$$G_2 = 27 + 10\left[\log_{10}(\eta_a) - \log_{10}\left(60\frac{h_{rms}}{\lambda}\right)\right]$$

$$G_3 = -13$$

$$\theta_{hp} = 0.5\frac{C_{hp}}{(D/\lambda)} \quad (65 \leq C_{hp} \leq 70,\ \text{nominal value} = 69)$$

$$\theta_1 = \theta_{hp}\sqrt{\frac{G_1}{3}}$$

$$\theta_2 = \theta_{hp} 10^{\frac{G_1 - 3}{G_2}}\sqrt{\frac{G_2}{36}}$$

$$\theta_3 = \theta_2 10^{\frac{G_0 - G_1 - G_3}{G_2}}$$

All the notes for the Jp model apply equally to the Ja model.

3 Numerical analysis and results

In order to calculate and plot and compare various gain models and their "averaged" gain, a few MATLAB programs have been written. These programs are very easy to use and provide a simple way to add new models for analysis and plotting. The following results have been obtained by these programs.

Each plot in Figs. 1-6 (a, b, c) shows several patterns for comparison. These include Recommendation ITU-R F.699-7 peak envelope (which for angles above 1° is the same as the model given in Recommendation ITU-R SA.509), the average envelope model in Recommendation ITU-R F.1245-1, the average envelope model in Recommendation ITU-R RA.1631, and finally a newly proposed peak envelope model "Jp" derived from the model contained in Recommendation ITU-R F.699.

Comparison at D/λ = 1 000 (e.g. 34 m antenna operating near 8.4 GHz)

Figures 1-3 are plotted for a 1 000-wavelength diameter antenna, corresponding to an aperture of 34 metre diameter operating near the 8.4 GHz band extensively used in deep space research. Patterns according to model Jp and Ja are given for the "poor", "average" and "good" quality antennas, corresponding to the root-mean-square (rms) surface tolerance of 1/15, 1/30, and 1/60 of wavelength, respectively. Cases a), b), and c) refer to linear, expanded linear and log representation of angle variable on the horizontal axis.

Comparison at D/λ = 4 000 (e.g. 34 m antenna operating near the 32 and 37 GHz bands)

Figures 4-6 are the corresponding cases for a 4 000 wavelength diameter antenna, corresponding to antennas with 34 m diameter operating near the 32 and 37 GHz bands where sharing between deep space research and HDFS is at issue.

Performance depending on surface tolerance

As can be seen for the proposed models Jp and Ja, the gain performances of the main beam and side-lobe regions change with the variation in surface tolerance. In models Jp and Ja, an initial aperture efficiency value of $\eta_a = 0.8$ is assumed, not including the surface tolerance effects. This is a typical value for the combination of aperture and spillover efficiency for a nominal 10-11 dB edge taper. This initial value is multiplied by a surface tolerance factor to arrive at the net aperture efficiency for the antenna. The surface tolerance factor are built in the formulas, and for the "poor", "average" and "good" cases are 0.5, 0.9 and 1.0 respectively. The net aperture efficiency for the three cases is therefore 0.4, 0.7 and 0.8 respectively. Note that this aperture efficiency is to be multiplied by other loss factors, such as the loss in the feed horn, to arrive at the overall efficiency of the antenna. In the case of other models the surface tolerance is not explicitly considered. An aperture efficiency of 0.7 is assumed for these models in all cases.

Among the large aperture antennas used in for deep space research in the NASA Deep Space Network, for example, surface tolerance of the 34 m antennas can be characterized as "good" at 8.4 GHz and 2.3 GHz, "average" to "good" at the 32 GHz, and potentially "average" at the 37 GHz when implemented. The surface tolerance of the 70 m antennas can be characterized as "good" at 8.4 GHz and 2.3 GHz, and potentially "poor" at 32 GHz if implemented.

Gain averaged over all angles

Figures 7-9 (a, b) show a comparison of the gain averaged over all angles according to equations given above for the various models discussed, using the "poor", "average", and "good" quality antenna models for the Jp and Ja cases. Cases a) and b) provide the averaged gain in dB and linear scale, respectively. As can be seen the Jp and Ja models are consistent across the range of D/λ (antenna diameter to wavelength) ratio. At higher D/λ ratio near 4 000, the "average" case shows a lower average gain and the "good" cases a much lower average gain, than other models.

Figure 10 (a, b) shows a similar set of plots for the case of the 34 m antenna (with 0.25 mm rms surface error) over the range of frequencies from 1 to 40 GHz. The figures show that model Jp (peak) provides a better average gain ratio than Recommendation ITU-R F.699, and similarly, the model Ja (average) provides a better average gain ratio than the average gain models Recommendations ITU-R F.1245 and ITU-R RA.1631, at all frequencies.

Figure 11 (a, b) shows the corresponding plots for the 70 m antenna (with 0.60 mm rms surface error) over the range of frequencies from 1 to 35 GHz. As can be seen, for the 70 m antenna of the deep space network at 32 GHz, the average gain ratio is somewhat higher than the other models due to the behaviour of this antenna at very high frequencies and the attempt by this model to provide a good fit to the higher pattern side lobes, which the other models do not. Should it become desirable to use this antenna at such high frequencies, the surface tolerance has to be improved.

Comparison with theoretical radiation patterns at various surface error correlation lengths

Figure 12 (a, b, c) includes theoretical patterns of a circular aperture calculated using a Lambda function (normalized Bessel function) approximation for the pattern, which also includes the effect of surface tolerance using Ruze's formulas [Ruze, 1966]. Several theoretical patterns corresponding to various "correlation lengths" for the surface errors are included. These figures, calculated for D/λ at 4 000, demonstrate that the Jp and Ja models provide a better envelope than the Recommendation ITU-R F.699-7 model. In the calculated theoretical patterns, a blockage ratio of 0.1, an aperture field illumination of the form $E = (1 - c) + c(1 - r^2)^n$ with an edge illumination of $ET = 10$ dB, $c = 1 - 10^{(-ET/20)}$, and a slope factor $n = 1$ are used.

Figure 12a compares the gain patterns for a "poor" antenna with rms surface error equal to (1/15) λ. Model Jp is a close upper envelope of all theoretical curves corresponding to various correlation length assumptions. The peak envelop model Recommendation ITU-R F.699-7, however, is exceeded significantly by the theoretical gain curves for several cases of correlation lengths at offset angles greater than 1°. It is also exceeded in some cases at offset angles between 0.05° and 0.1°. The "average gain" models, Recommendations ITU-R F.1245 and ITU-R RA.1631, are exceeded by the theoretical even more, as expected.

Figure 12b shows the comparison for an antenna with "average" surface tolerance. It is seen that the differences are getting smaller and both model Jp and Recommendation ITU-R F.699-7 are valid upper envelopes except for small violations in worst case. Model Recommendation ITU-R F.699 starts to exceed model Jp at angles between 0.1° and 10°.

In Fig. 12c with "good" antenna surface tolerance, it is seen that model Jp remains a valid upper envelope to the theoretical gain curves in all except the worst cases of surface error correlation length. The Recommendation ITU-R F.699 model exceeds the Jp model by about 5 dB between 0.1° and 30°. Even the average gain models, Recommendations ITU-R F.1245 and ITU-R RA.1631, exceed Ja by about 2 dB.

4 An average gain model

Average gain model based on Ja

In angular regions between 0.1° and 50° the radiation pattern of an antenna oscillates as indicated in the theoretical patterns discussed above. A peak envelope in this region would overestimate the antenna gain at many angles. In a compatibility study involving many interfering sources distributed over all angles in this region it is desirable to reduce the gain of the envelope to minimize bias in the estimation of aggregate interference. As discussed in § 2, case e), the average in this region can be considered approximately 3 dB lower than the peak levels of the oscillation.

Since the radiation pattern of the particular earth station antenna is not known, and since the peak-envelop model such as Jp is nearly the minimum envelope of the class of antennas under consideration, it is reasonable to reduce the gain levels of model Jp by 3 dB in the region specified and use the result as "average gain pattern" in this region. This "average gain pattern" would still be higher at certain angles than the average gains of the actual radiation pattern, if it were known.

This “average gain pattern” derived from the envelope model Jp is included in the comparisons shown in Figs. 1 through 12.

Deviation of gain from the average pattern

The 3 dB reduced between 0.1° and 50°, can be taken as a measure of the uncertainty level, or tolerance, of the antenna gain pattern used in a Monte Carlo statistical simulation. Lacking data on the distribution of this class of antennas, we suggest that variation from the average gain be Gaussian, with a 3-σ value equal to 3 dB.

5 Conclusion

A mathematical gain model, Jp, is proposed representing an envelope of the class of large aperture antennas currently in use at the SRS earth stations. It takes into account the effect of surface tolerance on gain distribution in the main beam and at side-lobes. It includes aperture efficiency in a way affecting both the peak and the side-lobe regions. It is demonstrated to have properties superior to the existing models in many respects.

An “average gain” model, Ja, is also proposed to allow more accurate estimation of aggregate interference of a large number of distributed interference sources by statistical (Monte Carlo) simulation. A simple model describing the uncertain deviation from the average gain pattern is also provided for use in the simulation.

For a typical 34 m or 70 m antenna in deep space research operating at 2.3 GHz or 8.4 GHz band, the model Jp (peak) provides a closer envelope than the envelope model Recommendation ITU-R F.699-7 and a better average gain ratio, and the model Ja (average) provides a closer approximation and a better average gain ratio than the average gain models Recommendations ITU-R F.1245 and ITU-R RA.1631. For a 34 m antenna operating at 32 GHz or 37 GHz band the above is still true.

In view of all the variations and uncertainties of radiation patterns among antennas, and the fact that the proposed model Jp is a closer upper envelope based on physical principles, it should be used in all compatibility and sharing studies using a single deterministic antenna gain pattern. The average gain model, Ja, should only be used when there are a large number of distributed interference sources appearing on a wide range of angles off bore-sight.

6 References

JAMNEJAD, V. [March 8-13, 2003] Simple gain probability functions for large reflector antennas of JPL/NASA. IEEE Aerospace Conference, Big Sky, Montana.

RUZE, J. [April 1966] Antenna tolerance theory-A review. *Proc. IEEE*, Vol. 54, p. 633-640.

FIGURE 1a

**A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "poor" antenna with surface error h_{rms} = (1/15) λ
(Linear angle axis)**

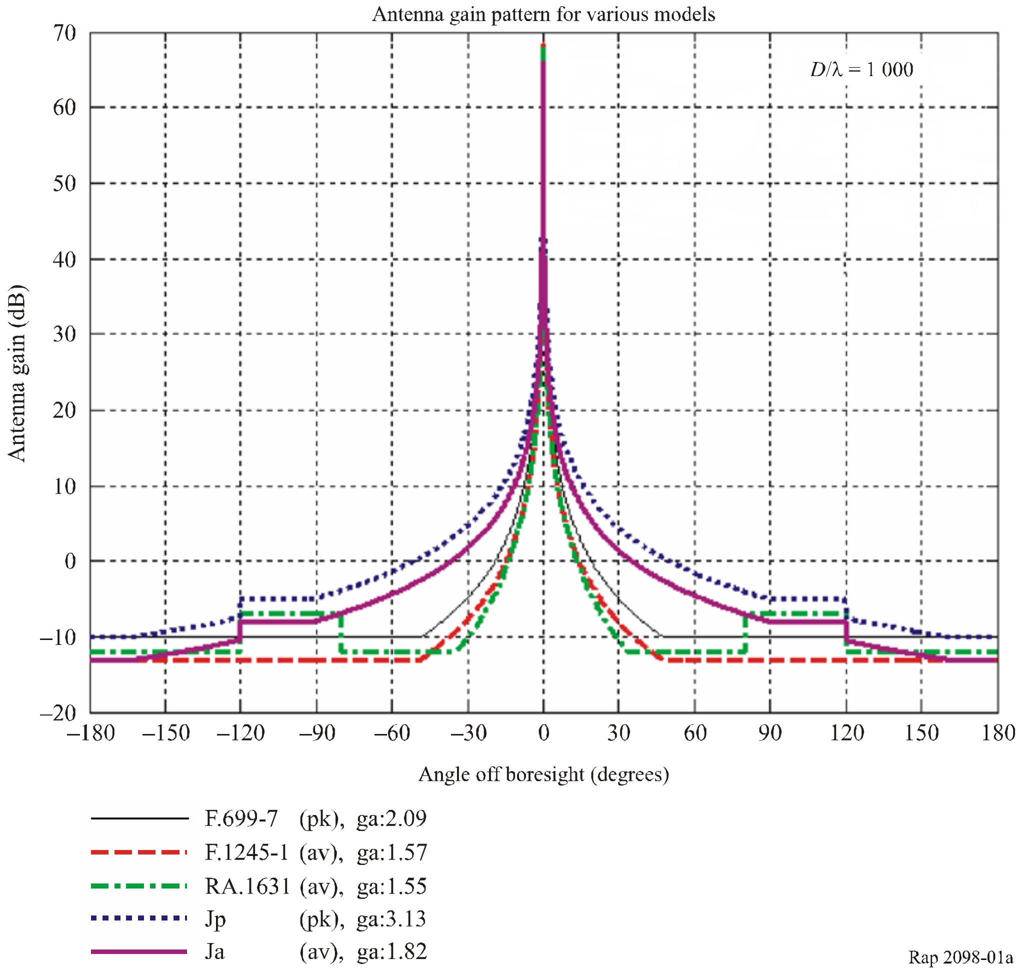


FIGURE 1b

A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "poor" antenna with surface error h_{rms} = (1/15) λ
(Expanded linear axis)

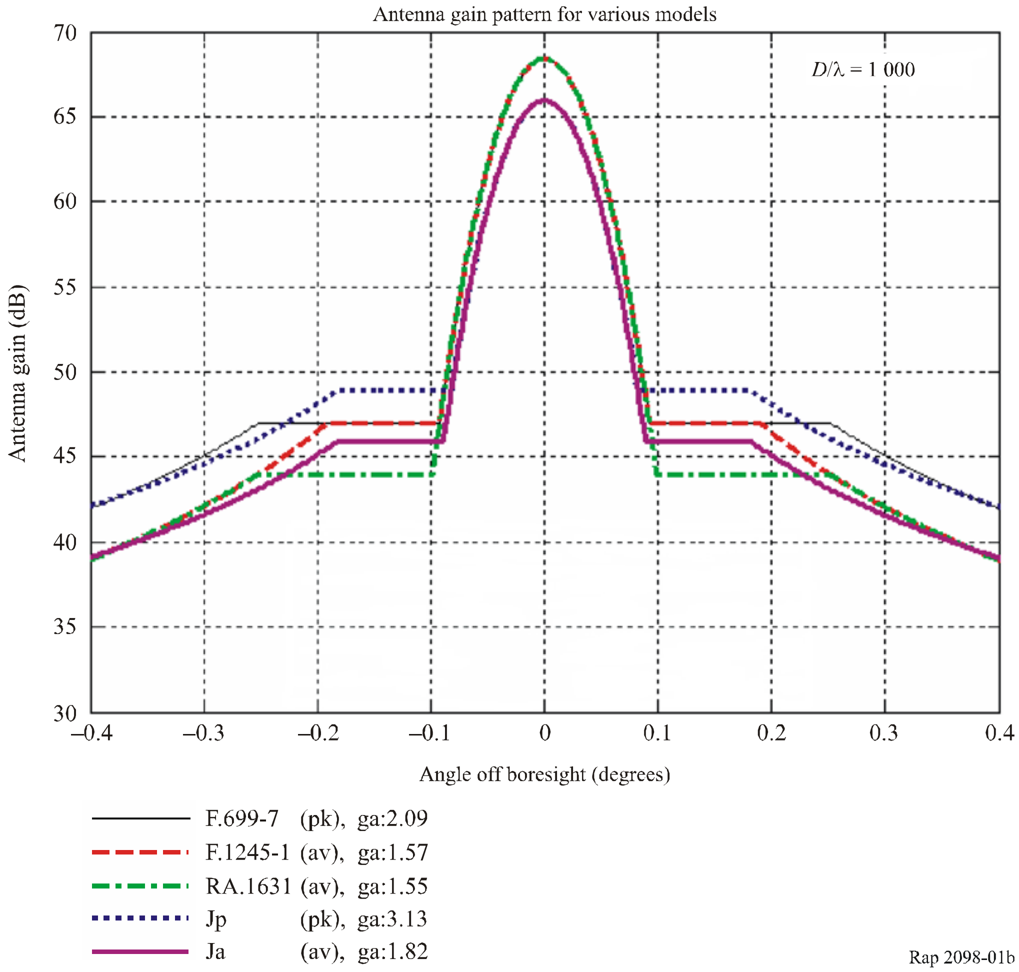


FIGURE 1c

A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "poor" antenna with surface error h_{rms} = (1/15) λ
(Log angle axis)

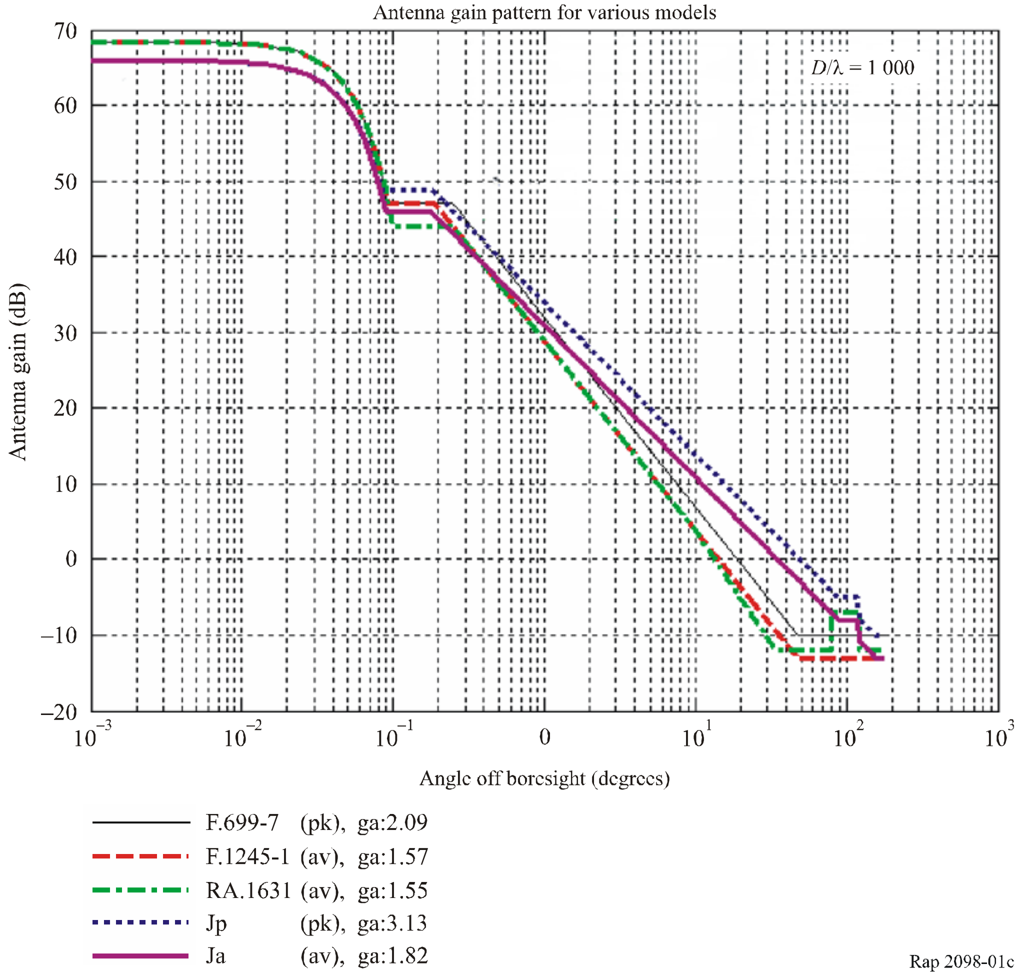


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FIGURE 2a

A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "average" antenna with surface error h_{rms} = (1/30) λ
(Linear angle axis)

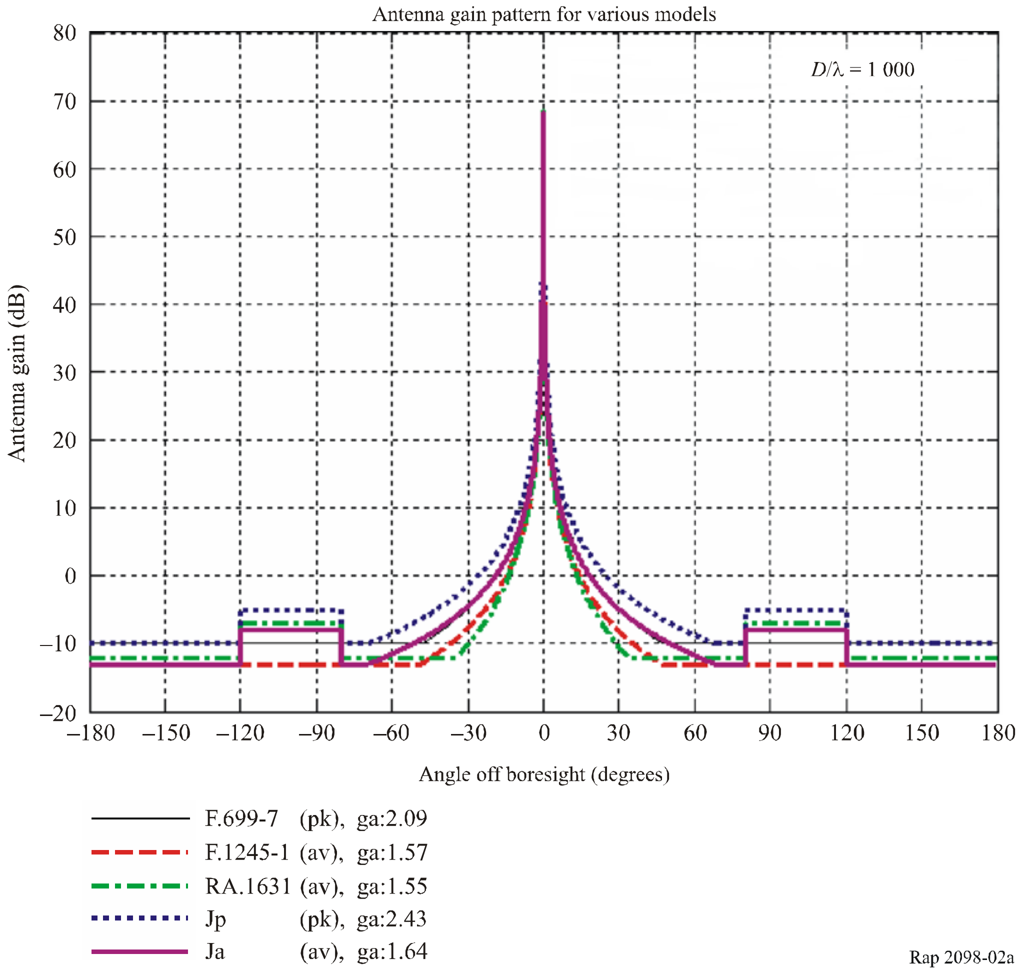

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FIGURE 2b

A comparison of gain pattern models for antenna diameter D = 1000 λ.
The Jp and Ja models are for "average" antenna with surface error h_{rms} = (1/30) λ
(Expanded linear angle axis)

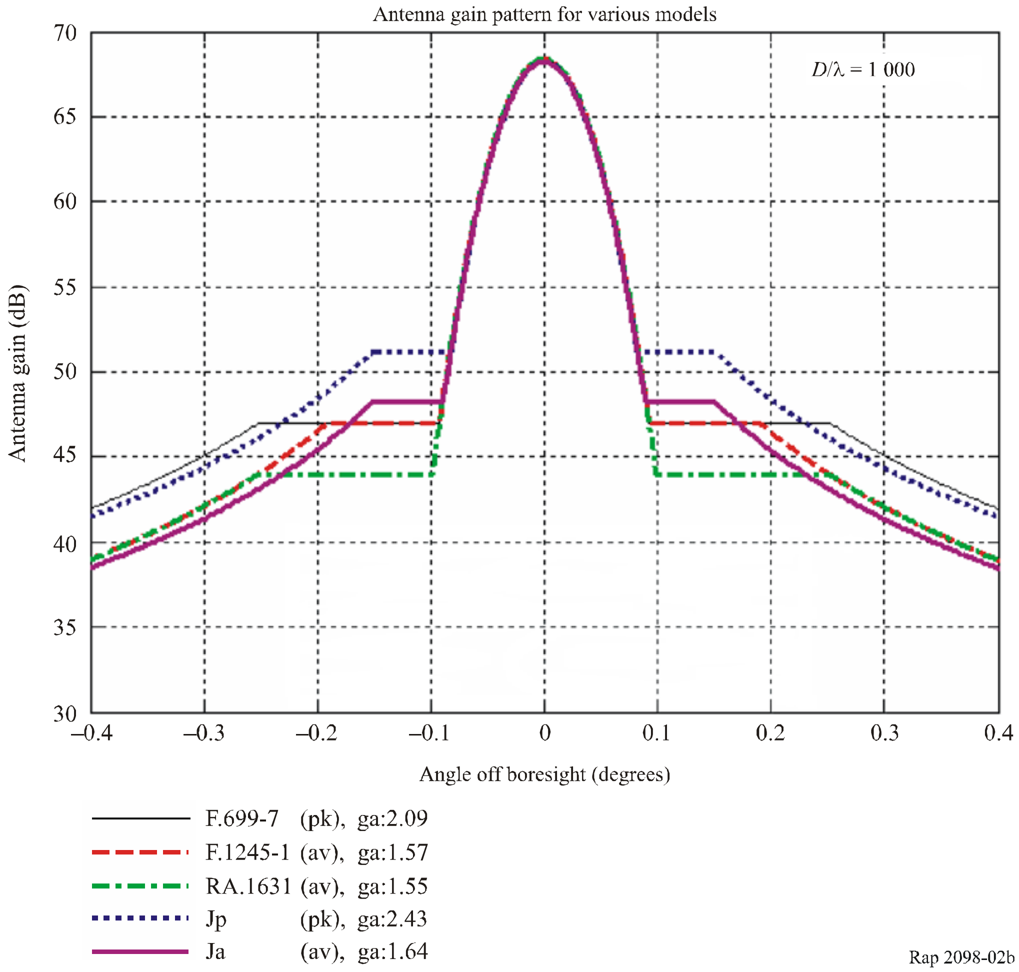

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FIGURE 2c

A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "average" antenna with surface error h_{rms} = (1/30) λ
(Log angle axis)

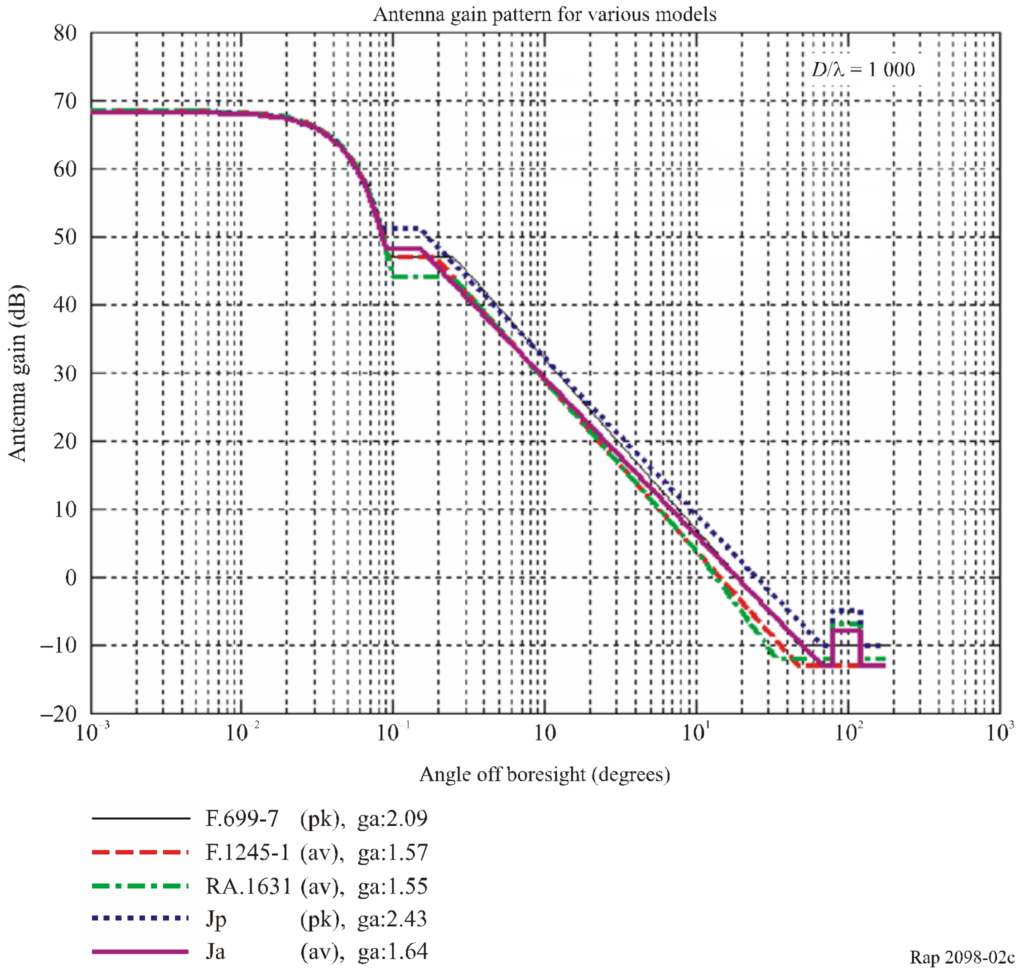

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FIGURE 3a

A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "good" antenna with surface error, h_{rms} = (1/60) λ
(Linear angle axis)

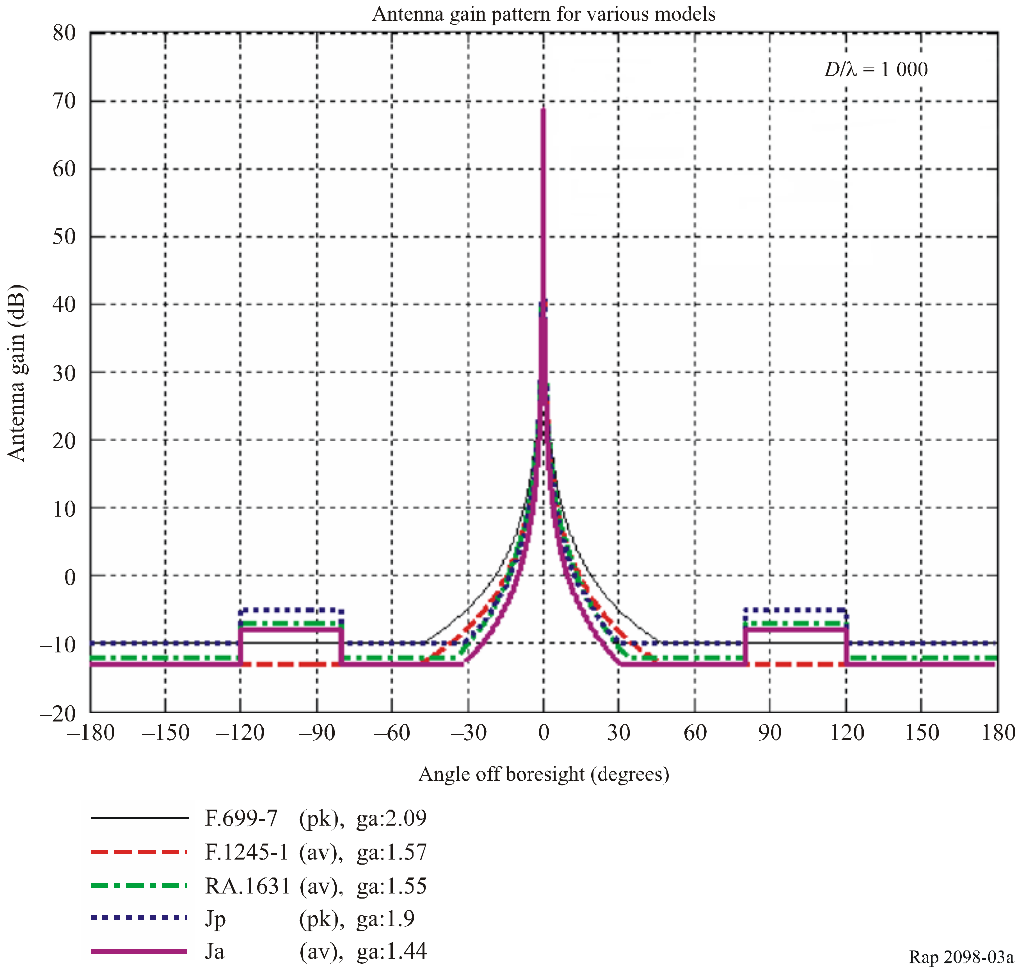


FIGURE 3b

A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "good" antenna with surface error, h_{rms} = (1/60) λ
(Expanded linear angle axis)

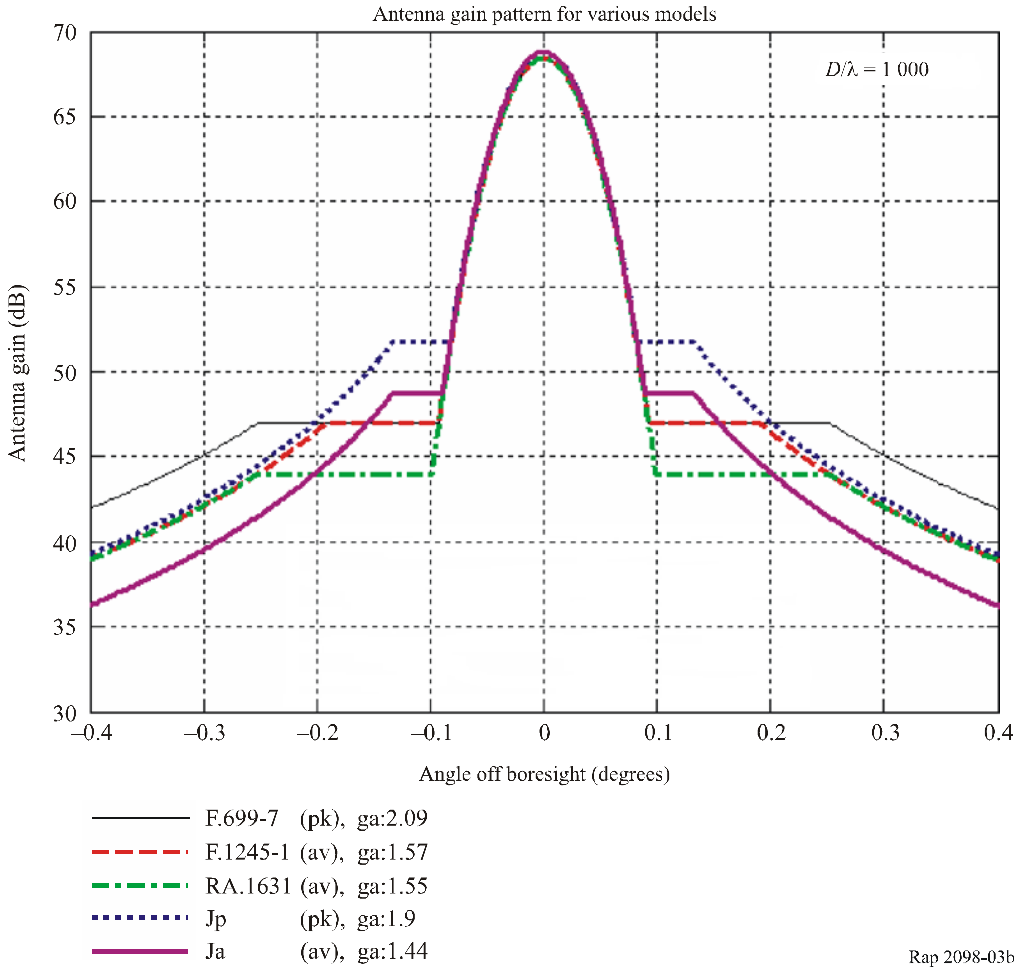
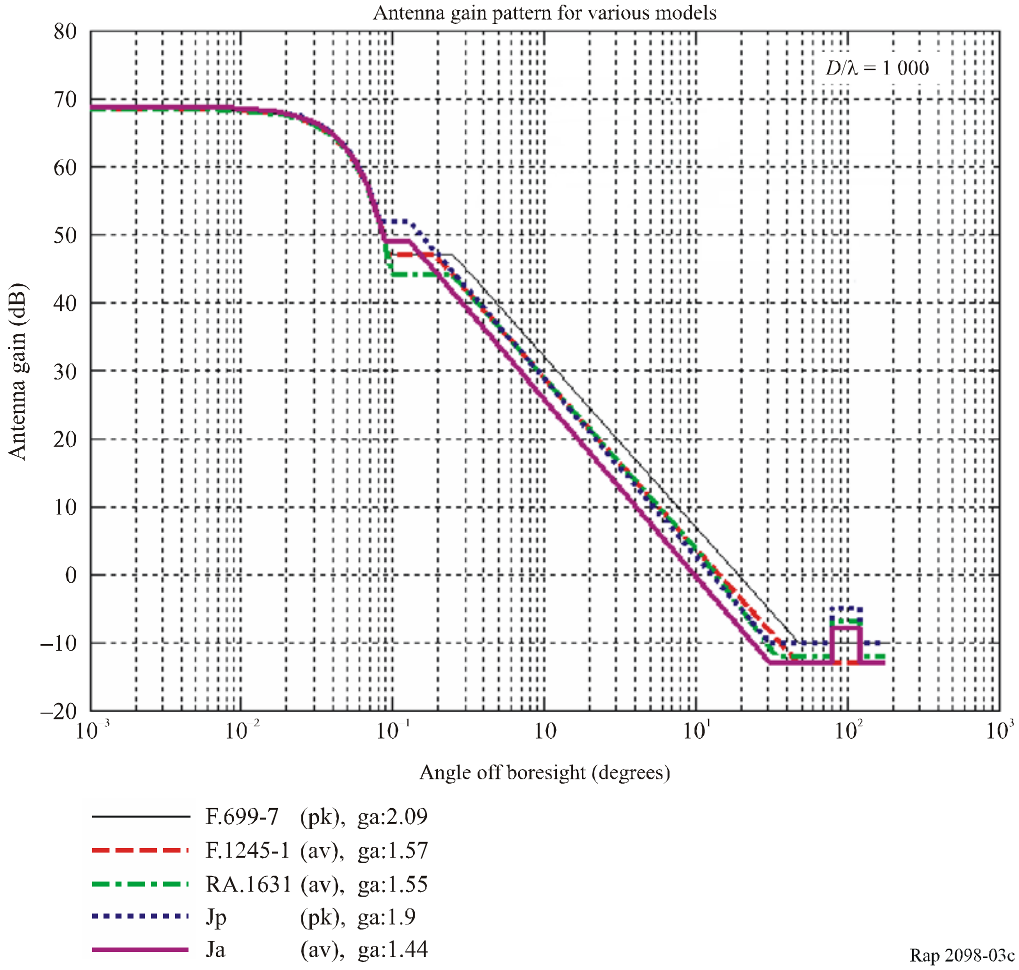


FIGURE 3c

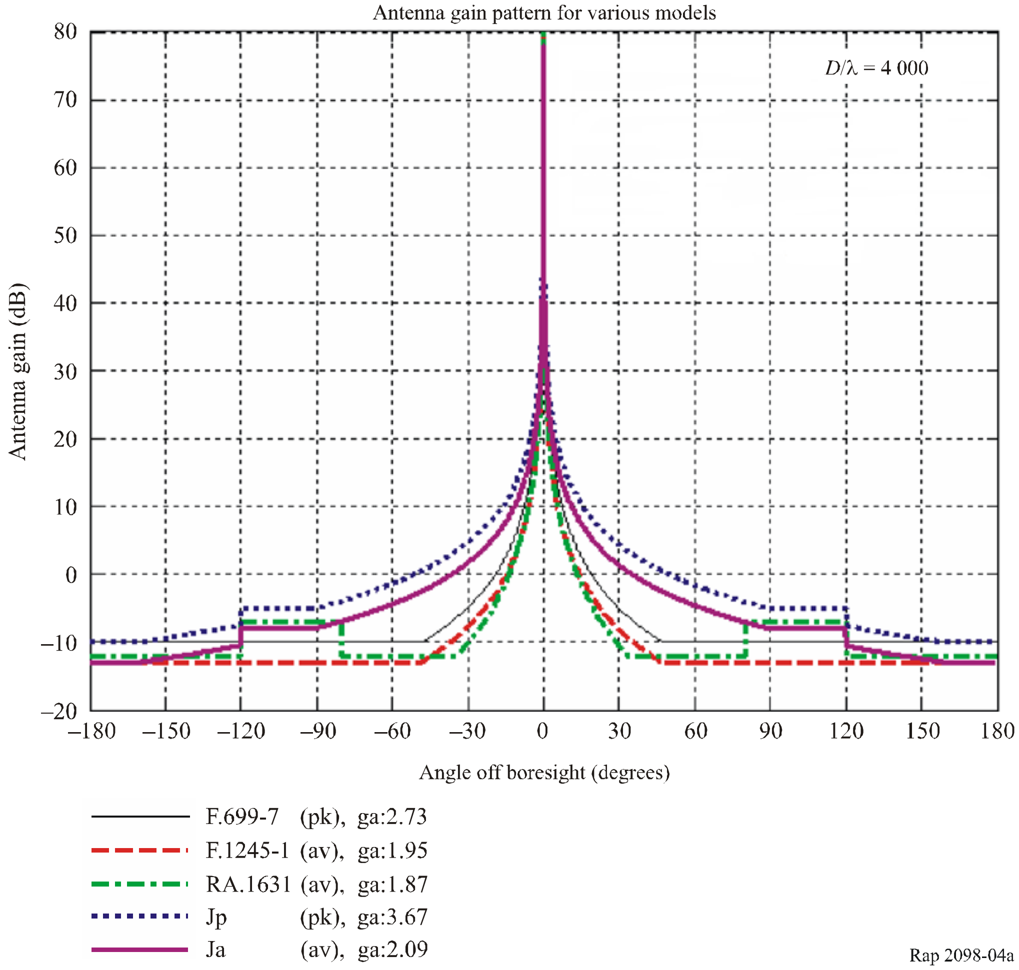
A comparison of gain pattern models for antenna diameter D = 1 000 λ.
The Jp and Ja models are for "good" antenna with surface error, h_{rms} = (1/60) λ
(Log angle axis)



Rap 2098-03c

FIGURE 4a

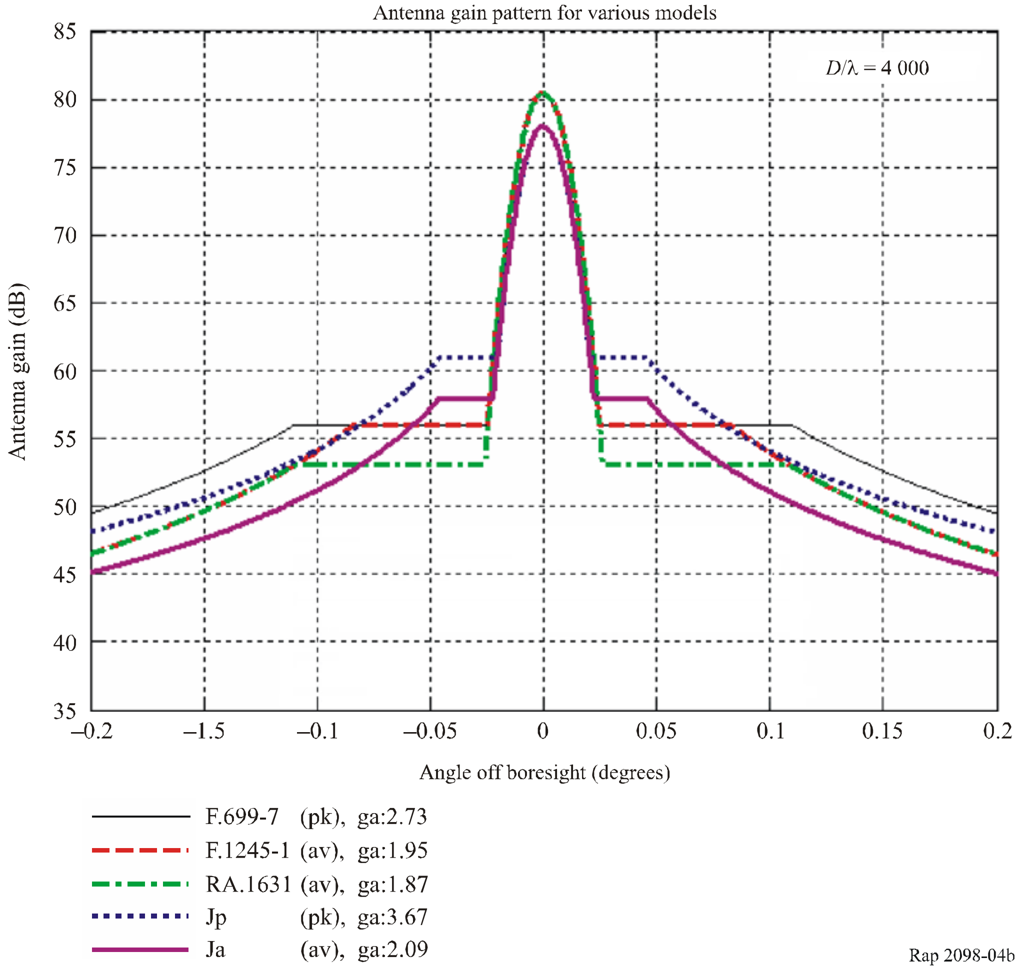
A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "poor" antenna with surface error h_{rms} = (1/15) λ
(Linear angle axis)



Rap 2098-04a

FIGURE 4b

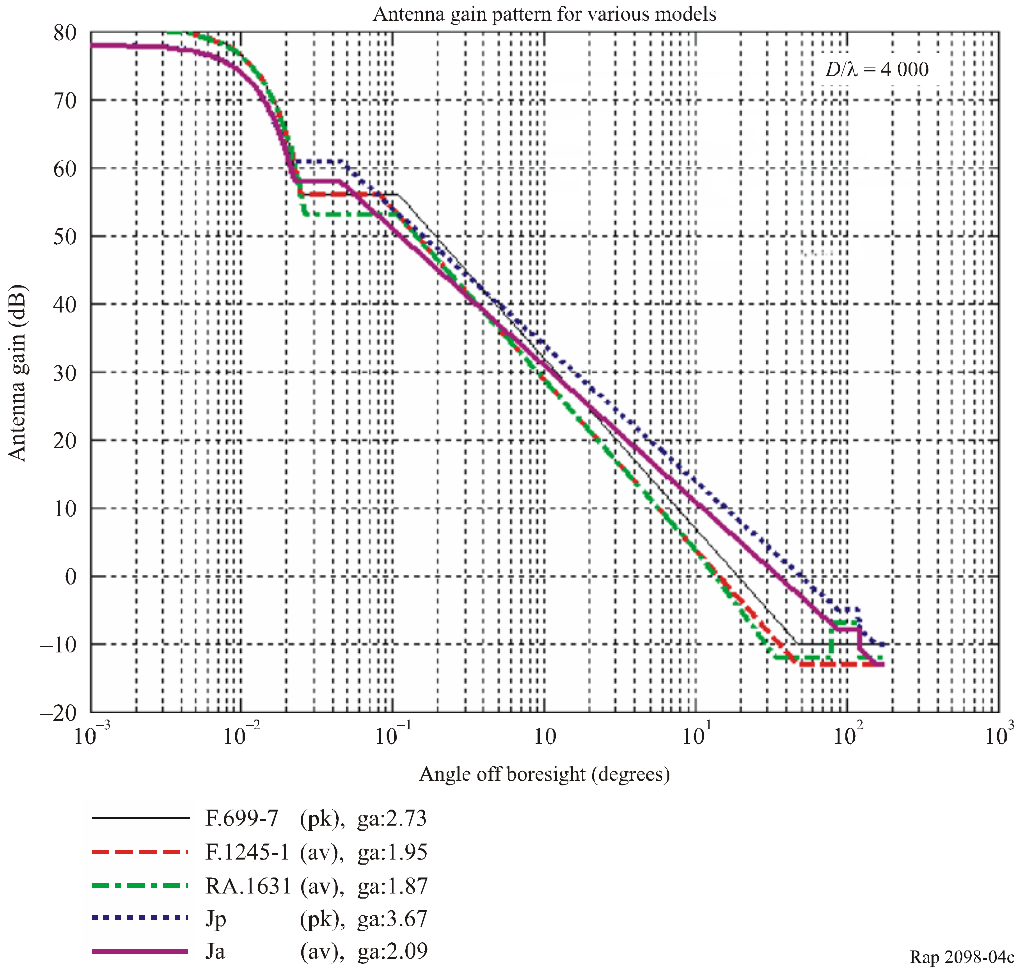
A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "poor" antenna with surface error h_{rms} = (1/15) λ
(Expanded linear angle axis)



Rap 2098-04b

FIGURE 4c

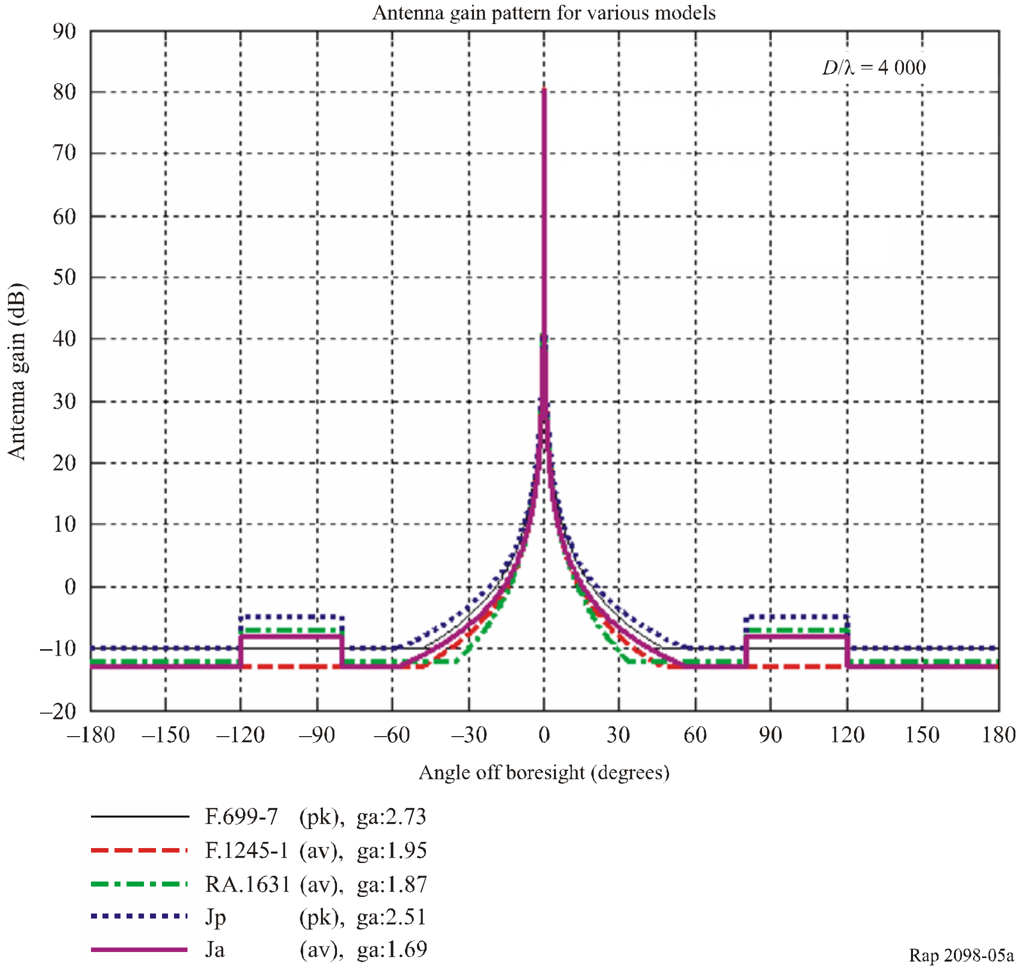
A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "poor" antenna with surface error h_{rms} = (1/15) λ
(Log angle axis)



Rap 2098-04c

FIGURE 5a

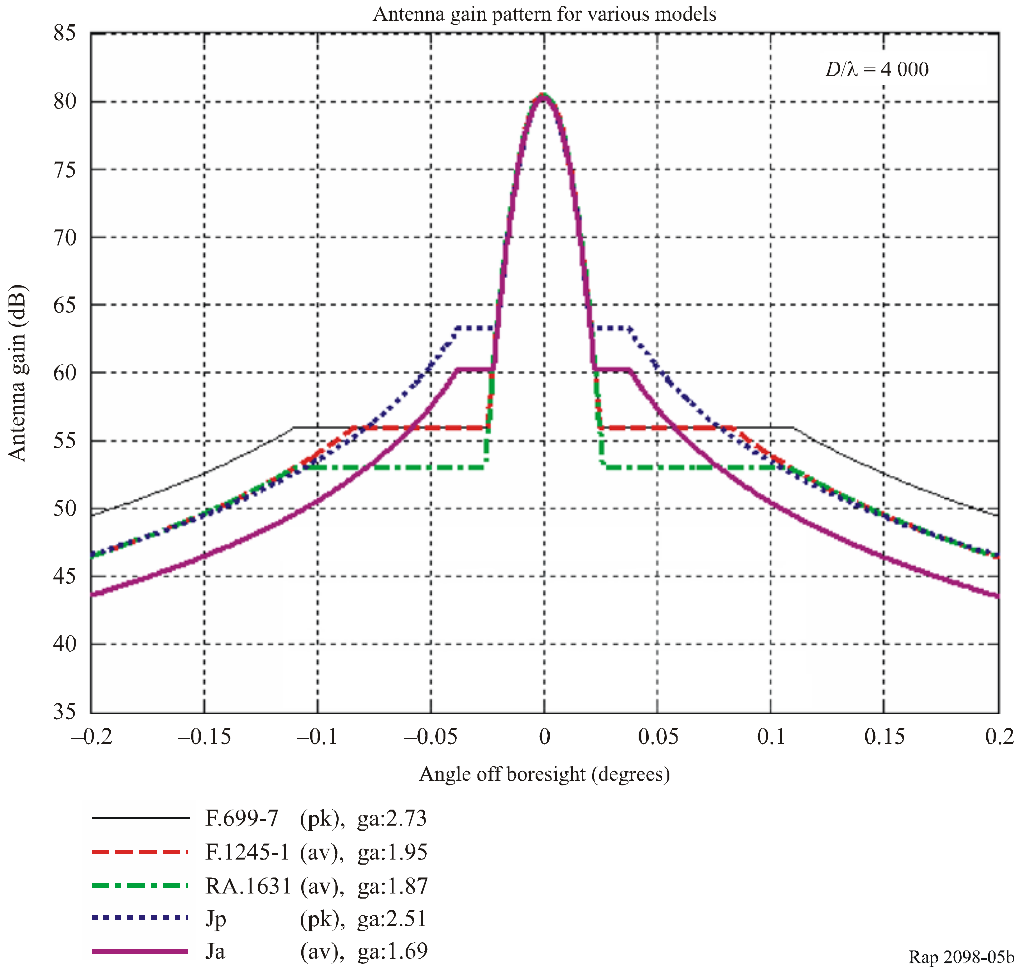
A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "average" antenna with surface error h_{rms} = (1/30) λ
(Linear angle axis)



Rap 2098-05a

FIGURE 5b

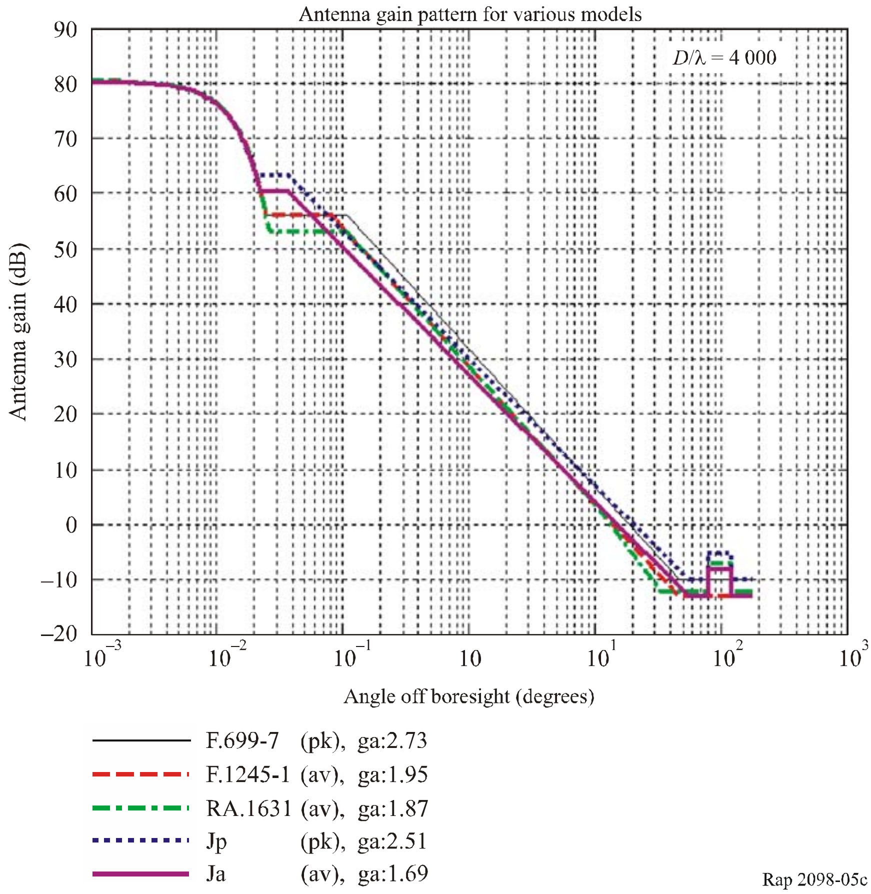
A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "average" antenna with surface error h_{rms} = (1/30) λ
(Expanded linear angle axis)



Rap 2098-05b

FIGURE 5c

A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "average" antenna with surface error h_{rms} = (1/30) λ
(Log angle axis)



Rap 2098-05c

FIGURE 6a

**A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "good" antenna with surface error h_{rms} = (1/60) λ
(Linear angle axis)**

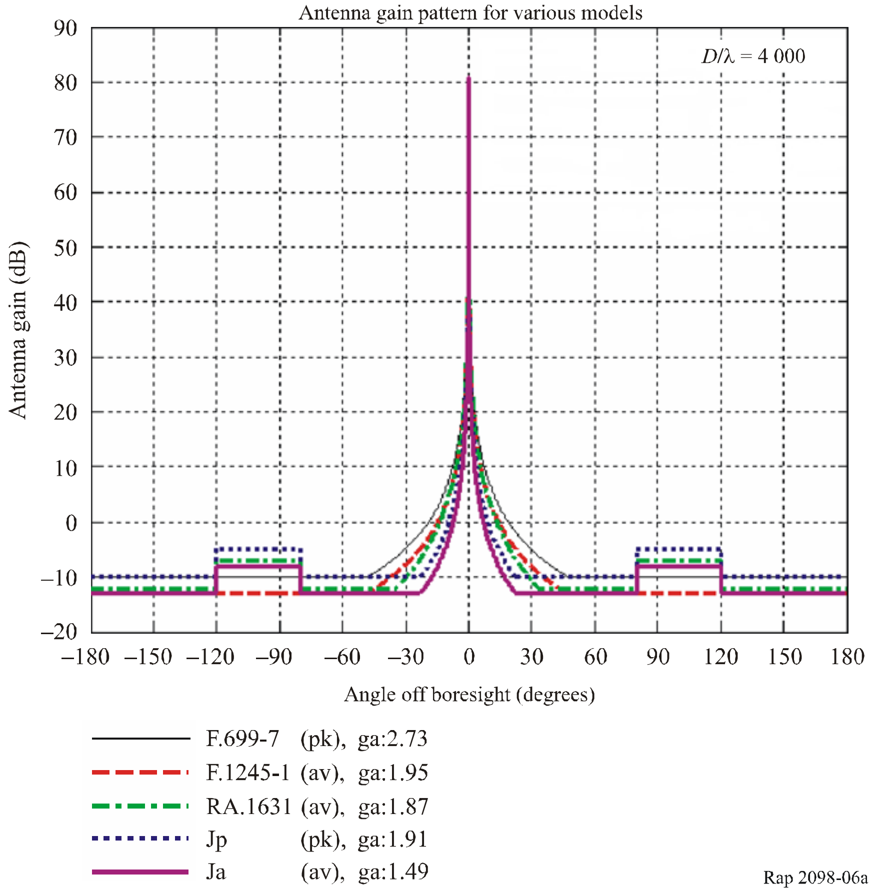


FIGURE 6b

A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "good" antenna with surface error h_{rms} = (1/60) λ
(Expanded linear angle axis)

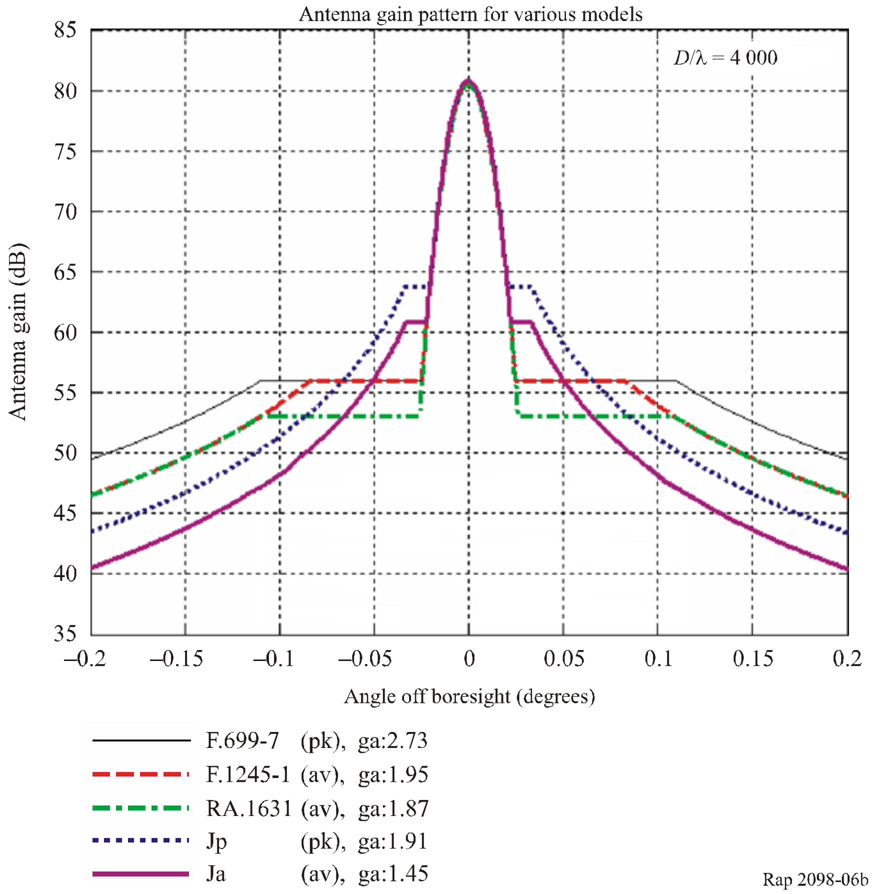
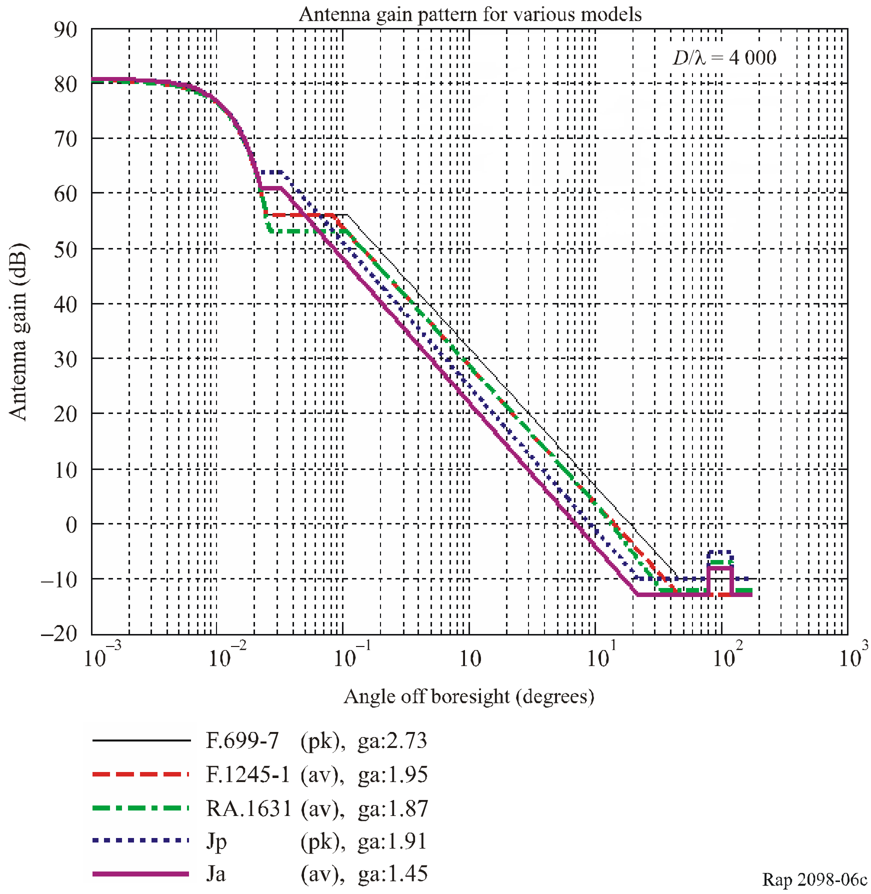


FIGURE 6c

A comparison of gain pattern models for antenna diameter D = 4 000 λ.
The Jp and Ja models are for "good" antenna with surface error h_{rms} = (1/60) λ
(Log angle axis)



Rap 2098-06c

FIGURE 7a

A comparison of averaged gain for different models, with Jp and Ja models for "poor" antenna with surface error h_{rms} = (1/15) λ (Averaged gain (dB))

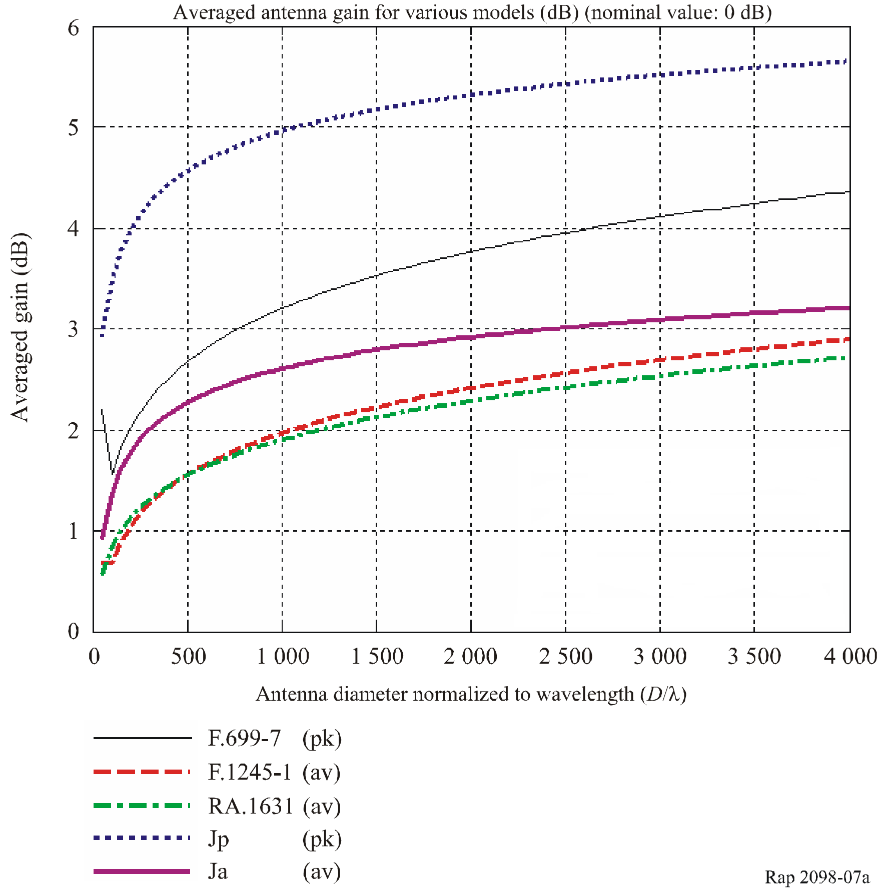
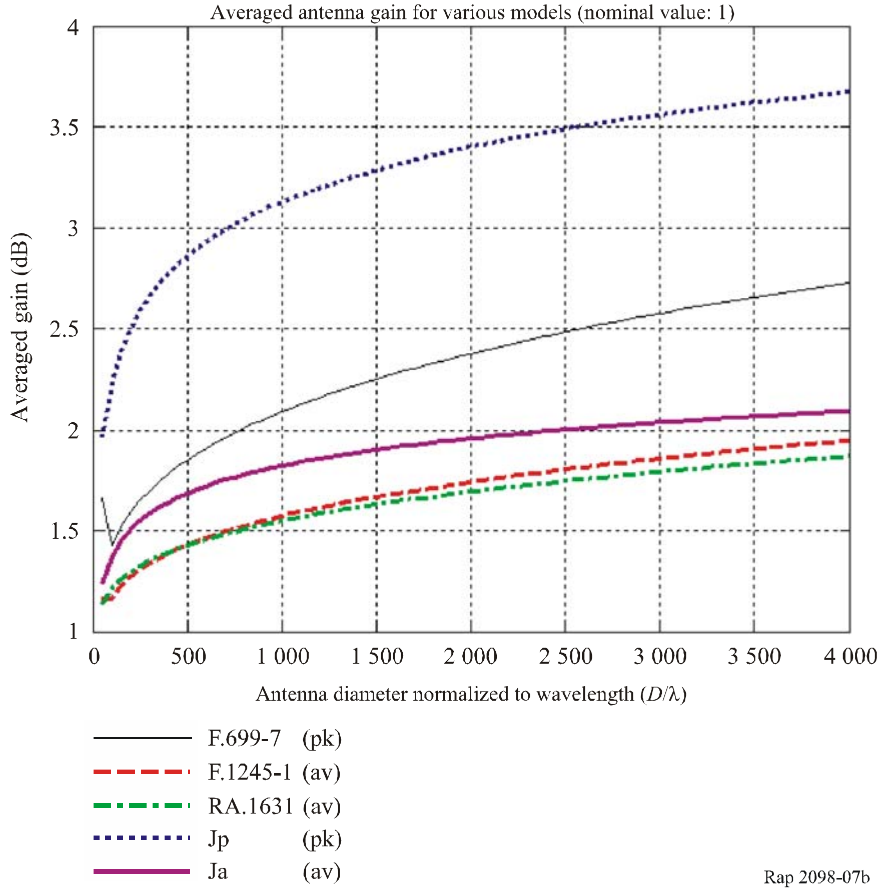


FIGURE 7b

A comparison of averaged gain for different models, with Jp and Ja models for "poor" antenna with surface error h_{rms} = (1/15) λ (Linear averaged gain)



Rap 2098-07b

FIGURE 8a

A comparison of averaged gain for different models,
with Jp and Ja models for "average" antenna with surface error h_{rms} = (1/30) λ
(Averaged gain (dB))

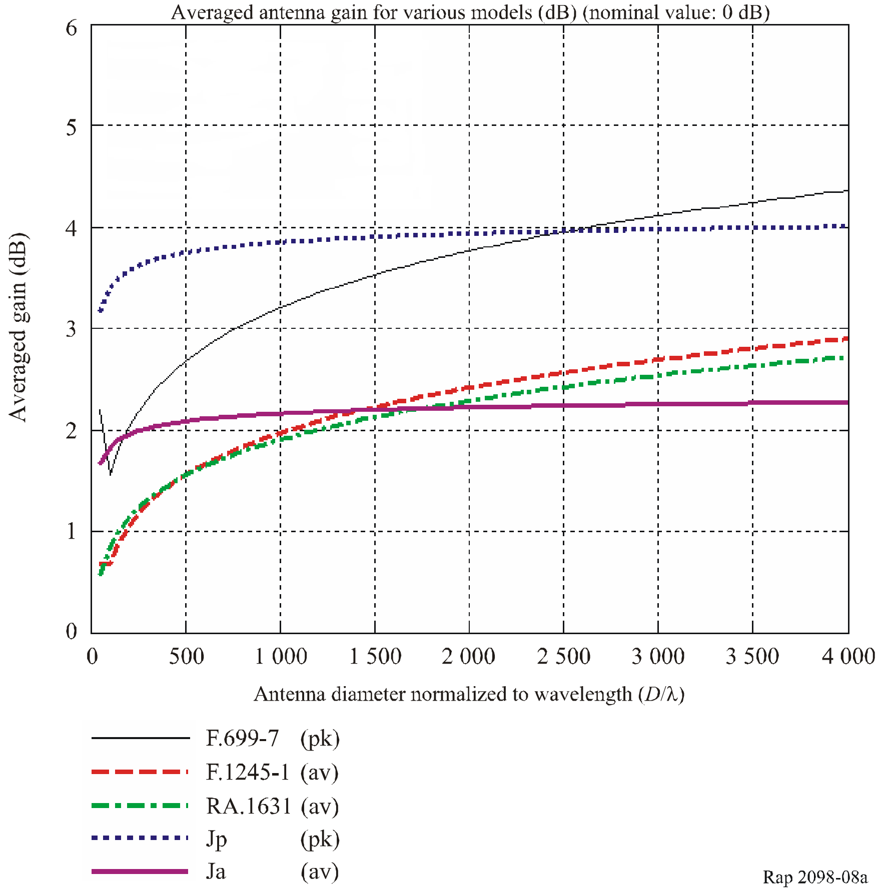


FIGURE 8b

A comparison of averaged gain for different models, with Jp and Ja models for "average" antenna with surface error h_{rms} = (1/30) λ (Linear averaged gain)

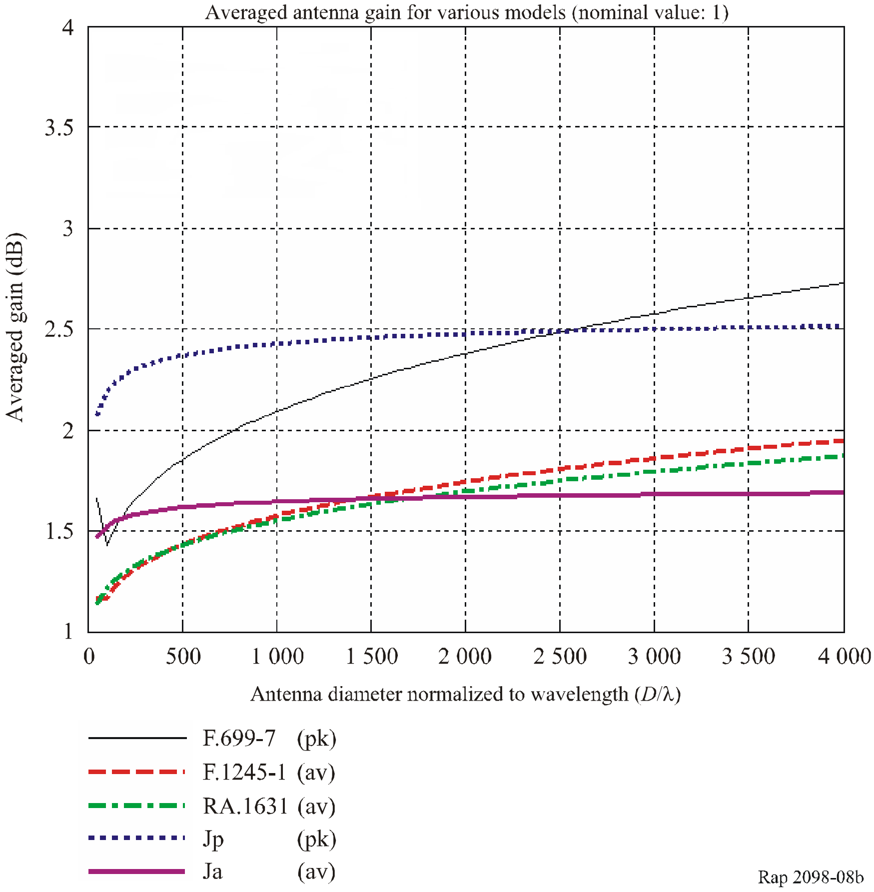


FIGURE 9a

A comparison of averaged gain for different models,
with Jp and Ja models for "good" antenna with surface error h_{rms} = (1/60) λ
(Averaged gain (dB))

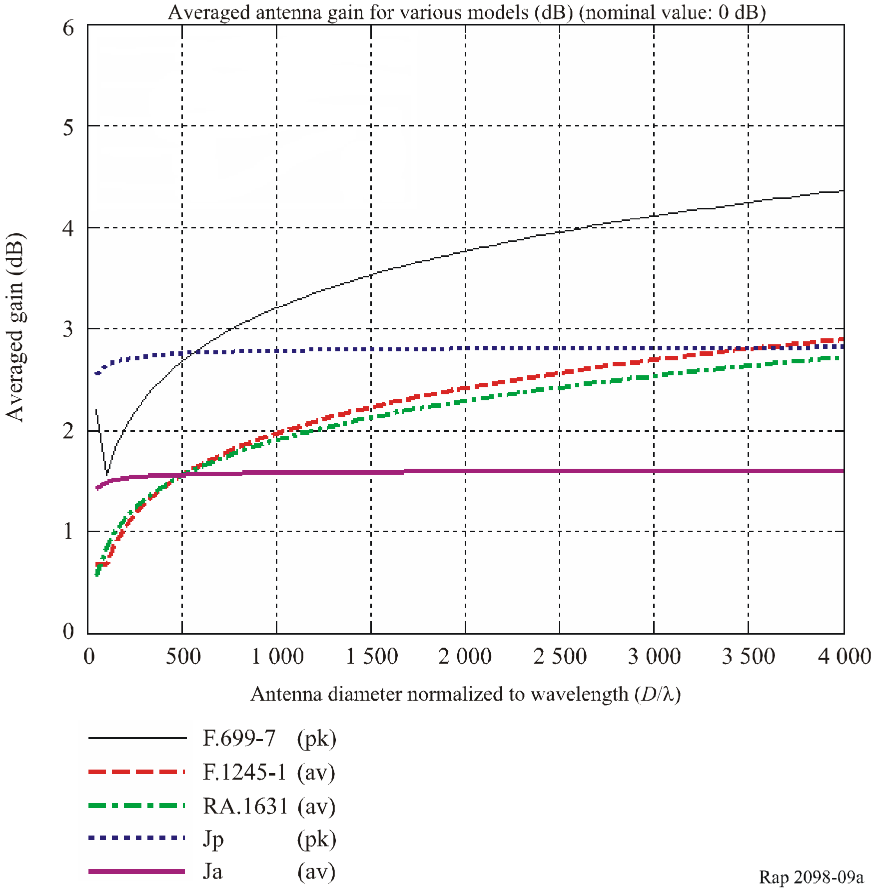


FIGURE 9b

A comparison of averaged gain for different models, with Jp and Ja models for "good" antenna with surface error h_{rms} = (1/60) λ (Linear averaged gain)

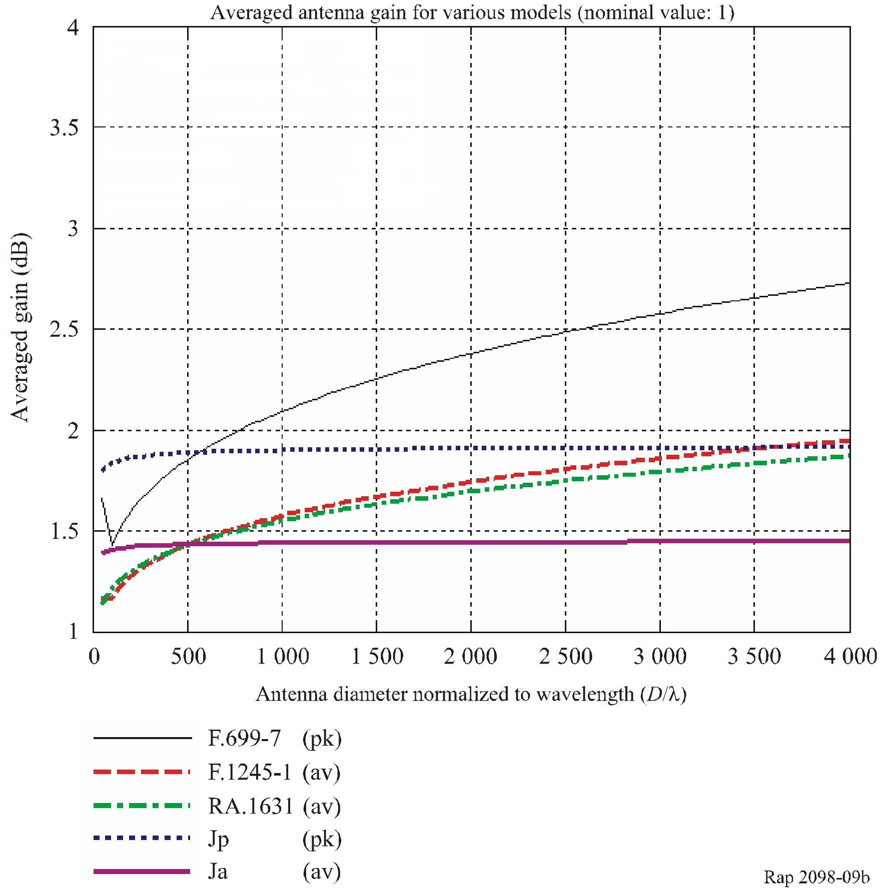


FIGURE 10a

**A comparison of averaged gain for different models,
with Jp and Ja models for a 34 m antenna with surface error h_{rms} = 0.25 mm
(Averaged gain (dB))**

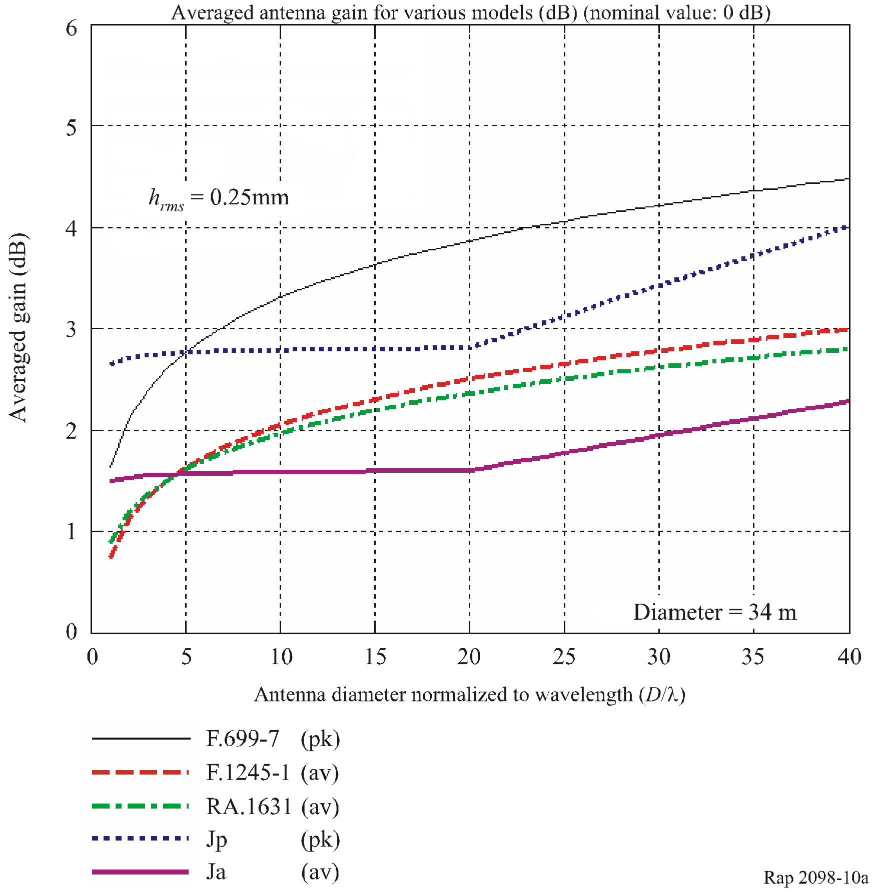
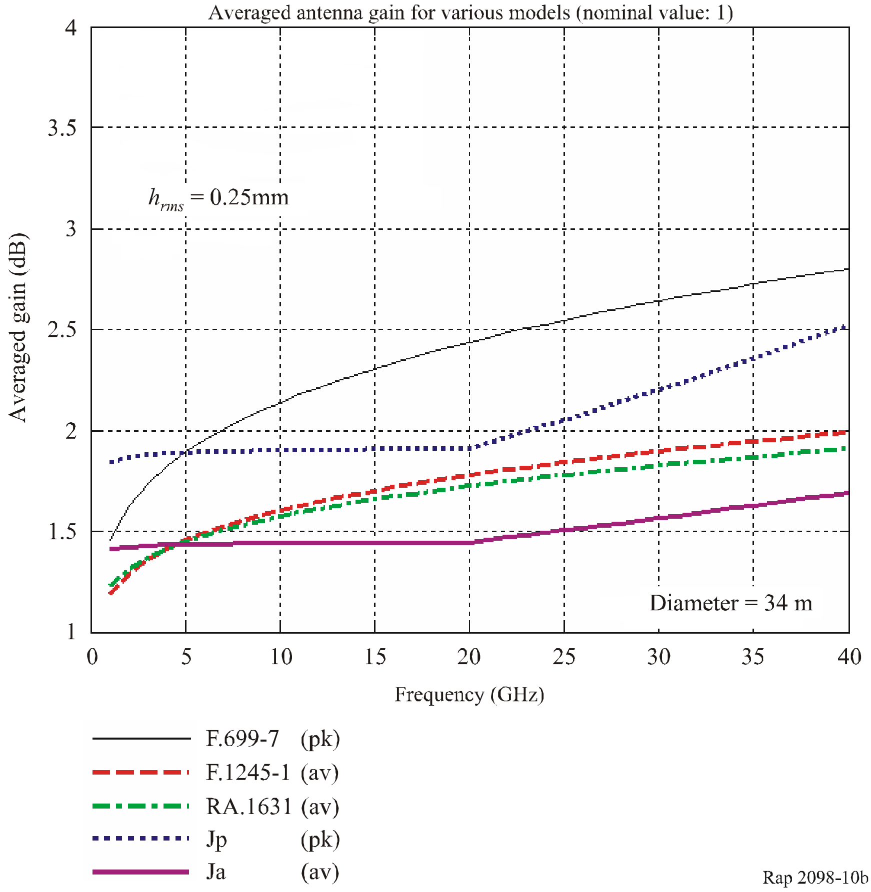


FIGURE 10b

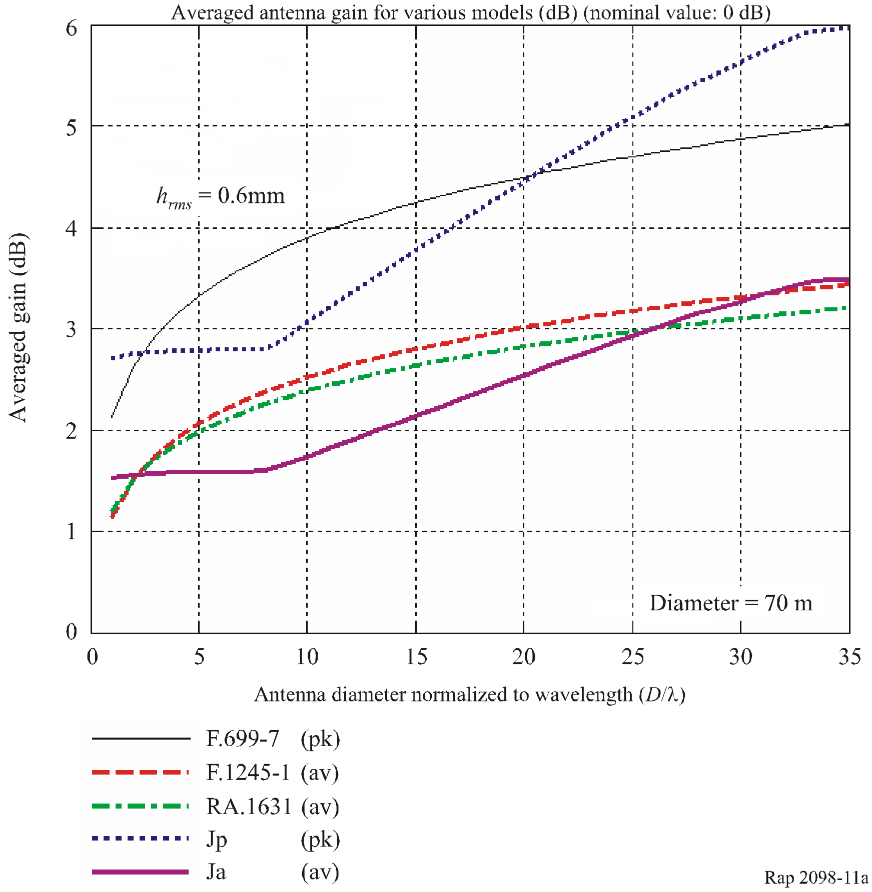
A comparison of averaged gain for different models, with Jp and Ja models for a 34 m antenna with surface error h_{rms} = 0.25 mm (Linear averaged gain)



Rap 2098-10b

FIGURE 11a

A comparison of averaged gain for different models, with Jp and Ja models for a 70 m antenna with surface error h_{rms} = 0.60 mm (Averaged gain (dB))



Rap 2098-11a

FIGURE 11b

A comparison of averaged gain for different models, with Jp and Ja models for a 70 m antenna with surface error h_{rms} = 0.60 mm (Linear averaged gain)

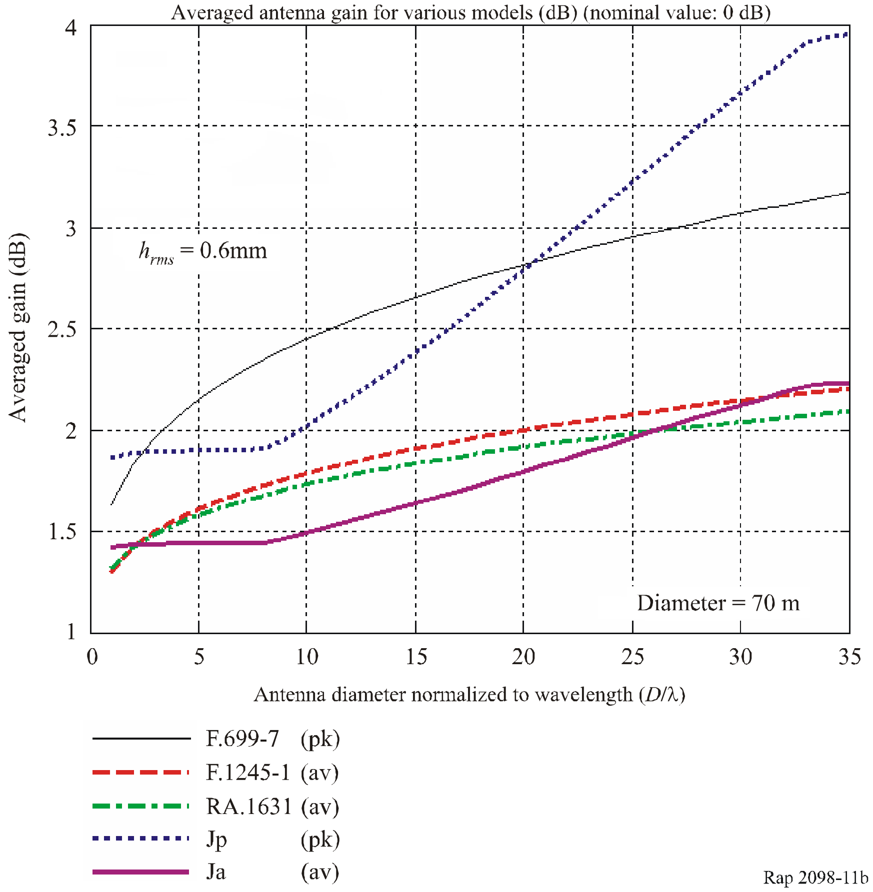
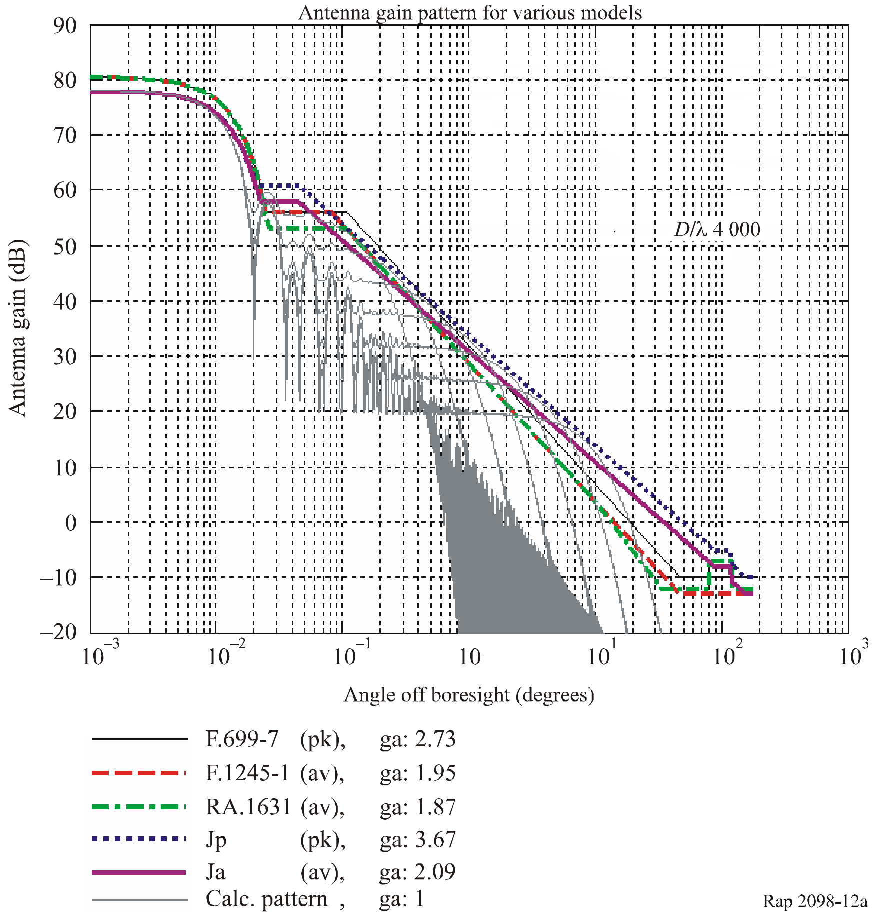


FIGURE 12a

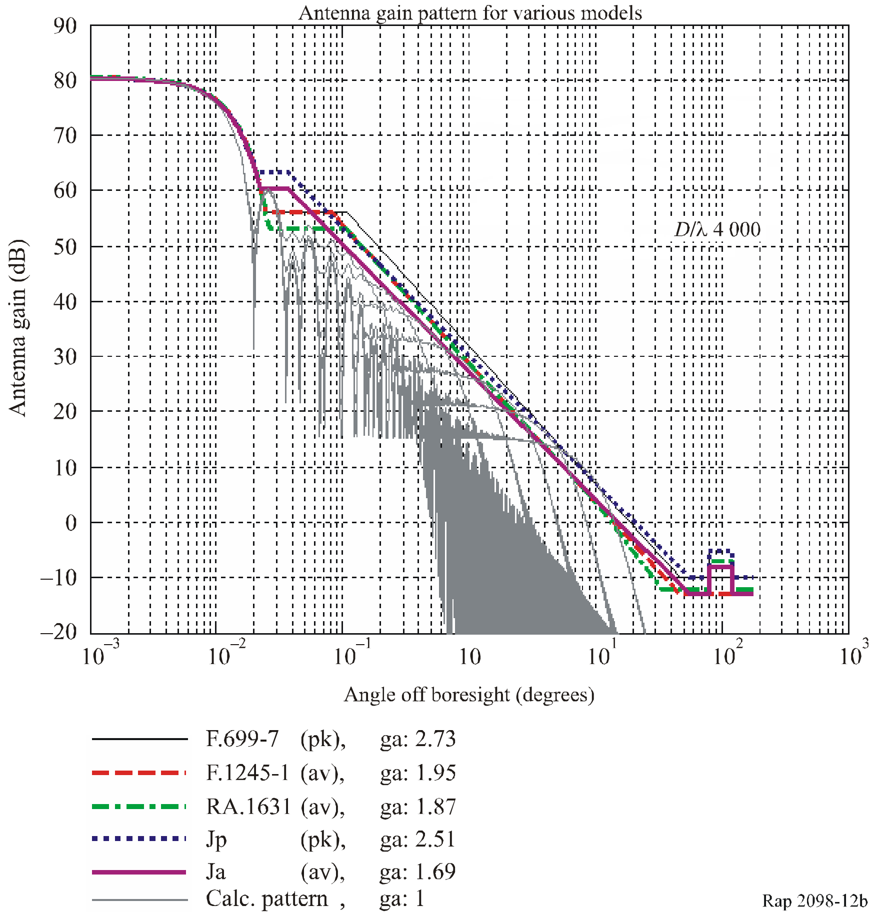
A comparison of averaged gain for different models, with Jp and Ja models for "poor" antenna with surface error h_{rms} = 1/15 λ. Includes calculated theoretical pattern with several different correlation lengths (c/λ = 2.5, 5, 10, 20, 40, 80, 160)



Rap 2098-12a

FIGURE 12b

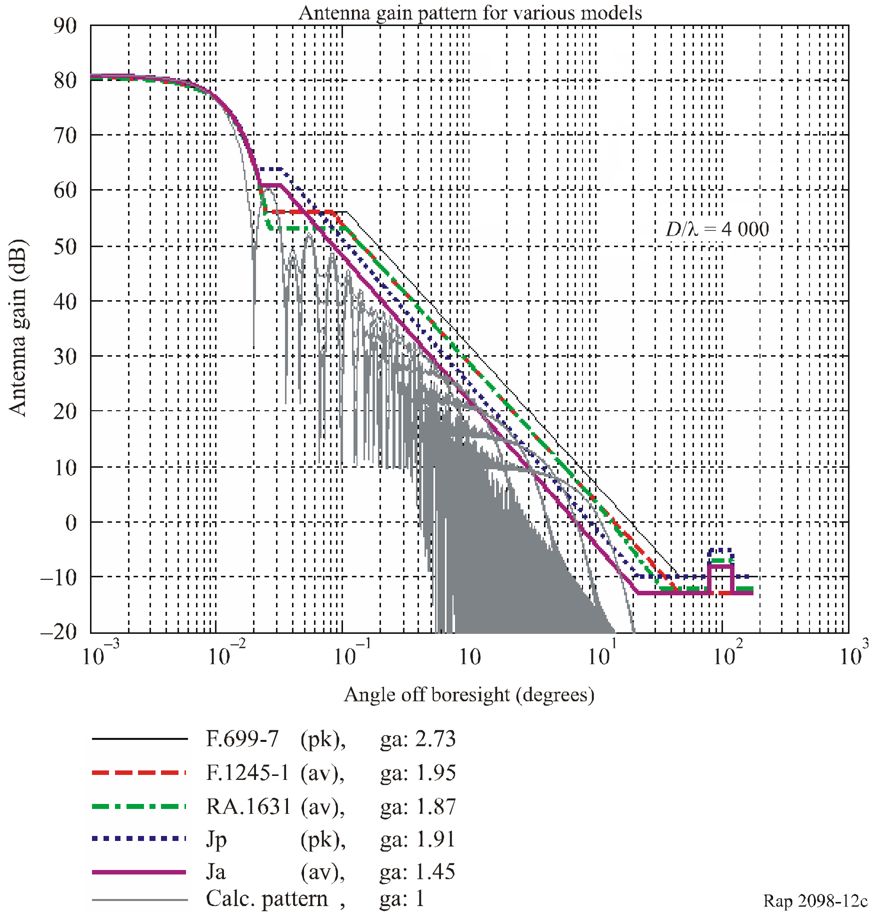
A comparison of averaged gain for different models,
with Jp and Ja models for "average" antenna with surface error h_{rms} = 1/30 λ.
Includes calculated theoretical pattern with several different correlation lengths
(c/λ = 2.5, 5, 10, 20, 40, 80, 160)



Rap 2098-12b

FIGURE 12c

A comparison of averaged gain for different models, with Jp and Ja models for "good" antenna with surface error h_{rms} = 1/60 λ. Includes calculated theoretical pattern with several different correlation lengths (c/λ = 2.5, 5, 10, 20, 40, 80, 160)



Rap 2098-12c