

# ANALYTIC MODELS FOR BI-STATIC SCATTERING FROM A RANDOMLY ROUGH SURFACE WITH COMPLEX RELATIVE PERMITTIVITY

Mostafa A. Karam<sup>1</sup>, Ryan S. McDonough<sup>2</sup>,

<sup>1</sup>ASRC Federal Technical Services, McLean, Virginia, 22101, USA. <sup>2</sup>NASA Glenn Research Center, Cleveland, Ohio, 44135, USA.

**Abstract** – This study provides explicit mathematical formulations for the bi-static scattering coefficient from a randomly rough surface with a complex relative permittivity based on the following analytic models: Small perturbation model (SPM), Physical optics model (PO), and Kirchhoff approximation model (KA). Then it addresses the two shortcomings associated with each of the three models: i) limited applicability domain, and ii) null predicted values for the cross-polarized bi-static scattering coefficients within plane of incidence. The plane of incidence contains both backscattering direction and forward (specular reflection) direction which are of interest to the spectrum community.

**Keywords** – Bi-static scattering coefficient, height correlation length, height variance, mean square slope, reflection coefficient

## 1. INTRODUCTION

Several natural and manmade surfaces affecting radio wave propagation can be treated as randomly rough surfaces with complex relative permittivity. Among those surfaces are:

- Earth surface including land surface and sea surface,
- Earth surface covers such as snow and sea ice,
- town and city buildings and structures at the HF frequencies and lower frequencies,
- and building walls at millimeter wave and higher frequencies, etc.

The bi-static scattering coefficient from the above surfaces is required for assessing several parameters which are of interest to the spectrum community such as [1-3]; these are:

- fading depth due to reflection from the Earth's surface [2],
- interference power due to reflection from the Earth's surface [1],
- radio noise due to microwave thermal emission from sea surface [3], etc.

Deriving analytical expressions for the bi-static scattering coefficients could enhance as well as extend the applicability domains of the existing prediction methods of the above parameters [1-3]. Based on the above, this study aims at: i) providing explicit expressions for the bi-static scattering coefficient based on three mathematical models widely used in assessing bi-static scattering from randomly rough surfaces, and ii) evaluating the capabilities of those models in predicting

cross-polarized bi-static scattering in the plane of incidence. The basis for the mathematical models in this study are:

- Small perturbation model (SPM),
- Physical optics model (PO), and
- Kirchhoff approximation model (KA).

The plane of incidence contains the incident direction and the normal direction to the surface. This plane contains the backscattering and the forward (specular reflection) directions.

The study is organized as follows. In Section 2, the randomly rough interface characteristics are introduced along with the polarizations of the incident and scattered fields, as well as the two components of the bi-static scattering coefficient: the coherent component and diffuse component. The analytical expression for the coherent component is also given in Section 2. Explicit expressions for the diffuse bi-static scattering coefficient component are developed in section 3, section 4 and section 5 based on SPM, PO and KA models respectively. Finally, a summary and conclusion for the study is provided in section 6.

## 2. PROBLEM FORMULATION

Consider a randomly rough surface separating two non-magnetic media: an upper medium with complex relative permittivity of unity (free space), and a lower medium with complex relative permittivity  $\epsilon_r$ . The surface height  $z(x, y)$ , which is aligned along the  $z$  direction, is randomly fluctuating around an average value of zero and with a constant variance  $\sigma^2$ .

$$\begin{aligned} \langle z(x, y) \rangle &= 0 \\ \langle |z(x, y)|^2 \rangle &= \sigma^2 \end{aligned} \quad (1)$$

The bracket  $\langle \rangle$  represents the ensemble average over the space.

The probability distribution of the surface heights is constant in space and its correlation function  $C(x', y')$  is given by Equation (2).

$$C(x', y') = \langle z(x, y)z(x + x', y + y') \rangle \quad (2)$$

The above correlation function is independent of  $x$  and  $y$ . Accordingly, it can be written as  $C(\rho)$  with

$$\rho = \sqrt{x'^2 + y'^2}.$$

In the roughness spectral domain  $\kappa$ , the surface correlation function  $C(\rho)$  has a spectral density function

$$W(\vec{\kappa}) = \left(\frac{\sigma}{2\pi}\right)^2 \int d\rho C(\rho) e^{j\vec{\kappa} \cdot \hat{\rho}}. \quad (3)$$

where  $\vec{\kappa}$  is the spatial wavenumber vector. For a Gaussian single-variant correlation function, Equation (3) reduces into

$$W(\vec{\kappa}) = \frac{\sigma^2}{4\pi} \exp\left\{-\frac{1}{4}\kappa^2 \ell^2\right\}. \quad (4)$$

In Equation (4),  $\ell$  is the surface correlation length. For a Gaussian correlation function, the surface height variance  $\sigma^2$  and the correlation length  $\ell$  may be used to obtain the surface mean square slope  $m$  as in Equation (5).

$$m^2 = 2(\sigma/\ell)^2 \quad (5)$$

Fig. 1 depicts the geometric configuration of the scattering from the randomly rough surface. Such a configuration is defined in a reference coordinate system  $(x, y, z)$ . Within this reference coordinate system, the surface is illuminated by an incident plane wave propagating along the  $\hat{k}_i(\theta_i, \phi_i)$  vector direction and is linearly polarized and defined by  $\hat{q}_i(q = v, h)$  where

$$\hat{k}_i = \sin \theta_i (\cos \phi_i \hat{x} + \sin \phi_i \hat{y}) - \cos \theta_i \hat{z},$$

$$\hat{h}_i = -\sin \phi_i \hat{x} + \cos \phi_i \hat{y},$$

and

$$\hat{v}_i = -\{\cos \theta_i (\cos \phi_i \hat{x} + \sin \phi_i \hat{y}) + \sin \theta_i \hat{z}\}. \quad (6)$$

A portion of the electromagnetic power illuminating the surface is scattered along a scattering direction  $\hat{k}_s(\theta_s, \phi_s)$  with polarization  $\hat{p}_s$  ( $p = v, h$ ) where

$$\hat{k}_s = \sin \theta_s (\cos \phi_s \hat{x} + \sin \phi_s \hat{y}) + \cos \theta_s \hat{z},$$

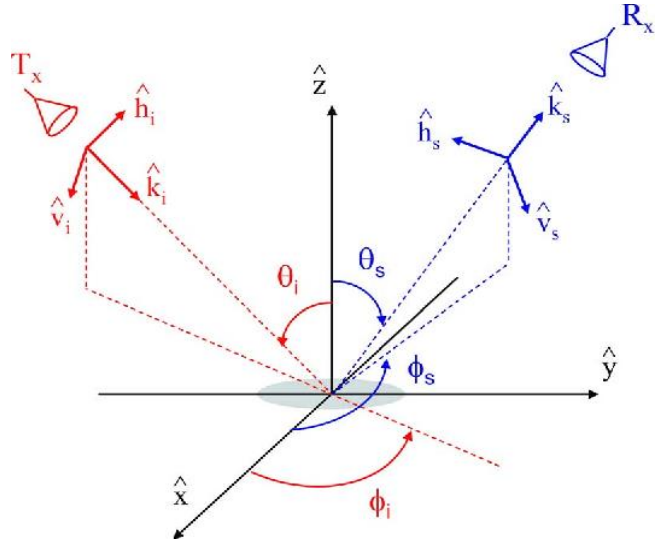
$$\hat{h}_s = -\sin \phi_s \hat{x} + \cos \phi_s \hat{y},$$

and

$$\hat{v}_s = \cos \theta_s (\cos \phi_s \hat{x} + \sin \phi_s \hat{y}) - \sin \theta_s \hat{z}. \quad (7)$$

In equations (6) and (7),  $v$  stands for vertical (parallel or TH) polarization, and  $h$  stands for horizontal (perpendicular or TE) polarization.

The bi-static scattering coefficient  $\gamma_{pq}(\hat{k}_s, \hat{k}_i)$  along the scattering direction  $\hat{k}_s$  is the fraction of power scattered along such a direction with polarization  $\hat{p}_s$  due to incident wave illuminating the surface along the  $\hat{k}_i$  direction with polarization  $\hat{q}_i$  (Fig. 1). The fraction of power is per unit solid angle and per unit area.



**Fig.1** - Geometric configuration of scattering from a randomly rough surface

Due to surface roughness, the bi-static scattering coefficient has two components: coherent component  $\gamma_{pq}^c(\hat{k}_s, \hat{k}_i)$ , and diffuse (non-coherent) component  $\gamma_{pq}^{dif}(\hat{k}_s, \hat{k}_i)$ .

$$\gamma_{pq}(\hat{k}_s, \hat{k}_i) = \gamma_{pq}^c(\hat{k}_s, \hat{k}_i) + \gamma_{pq}^{dif}(\hat{k}_s, \hat{k}_i) \quad (8)$$

The coherent component  $\gamma_{pq}^c(\hat{k}_s, \hat{k}_i)$  is co-polarized  $\{\gamma_{vv}^c(\hat{k}_s, \hat{k}_i)$  or  $\gamma_{hh}^c(\hat{k}_s, \hat{k}_i)\}$  and exists only along the forward (specular reflection) direction  $\{\theta_s = \theta_i, \text{ and } \phi_s = \phi_i\}$  as

$$\begin{aligned} \gamma_{pq}^c(\hat{k}_s, \hat{k}_i) &= \\ 4\pi |\gamma_{pp}^c(\theta_i)|^2 \exp\{-2k\sigma \cos \theta_i\} \delta(\theta_s - \theta_i) \delta(\phi_s - \phi_i) \delta_{pq}. \end{aligned} \quad (9)$$

In the above,  $r_{pp}(\theta_i)$  is the Fresnel reflection coefficient for  $p$  polarization as defined by:

$$r_{hh}(\theta_i) = \frac{\cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}},$$

and

$$r_{vv}(\theta_i) = \frac{\varepsilon_r \cos \theta_i - \sqrt{\varepsilon_r - \sin^2 \theta_i}}{\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i}}. \quad (10)$$

Moreover,  $\delta(\theta_s - \theta_i)$  and  $\delta_{pq}$  are the Dirac and Kronecker delta functions respectively:

$$\delta(\theta_s - \theta_i) = \begin{cases} 1, & \text{if } \theta_s = \theta_i \\ 0, & \text{otherwise} \end{cases} \quad (10a)$$

and

$$\delta_{pq} = \begin{cases} 1, & \text{if } p = q \\ 0, & \text{otherwise} \end{cases} \quad (10b)$$

Analytical expressions for the diffuse bi-static scattering coefficient components are provided below, based on the three different models with values of cross-polarized bi-static scattering coefficients

$$\gamma_{vh}^{dif}(\hat{k}_s, \hat{k}_i), \text{ and } \gamma_{hv}^{dif}(\hat{k}_s, \hat{k}_i)$$

within the plane of incidence based on each model.

### 3. DIFFUSE BI-STATIC COEFFICIENT BASED ON THE SMALL PERTURBATION METHOD (SPM)

The small perturbation method (SPM) applies for slightly rough surfaces having variance and correlation length governed by equations (35) and (36) defined in [4]. Those equations are recalled as Equation (11).

$$\begin{aligned} k\sigma < 0.3, \quad \text{and} \\ k\ell > 4.71 k\sigma. \end{aligned} \quad (11)$$

The diffuse bi-static scattering coefficient  $\gamma_{pq}^{SPM}(\hat{k}_s, \hat{k}_i)$  based on SPM can be written as (Equation (22) of [5], (21-67) of [6], (31) of [4], and (37) of [7])

$$\gamma_{pq}^{SPM}(\hat{k}_s, \hat{k}_i) = 8 (k^2 \cos \theta_s \cos \theta_i)^2 |g_{pq}|^2 W(k_{xy}). \quad (12)$$

where  $W(k_{xy})$  is the surface height spectral density function of Equation (4) calculated at roughness wavenumber  $\mathbf{k}$  equal to  $k_{xy}$ .

$$k_{xy} = k \sqrt{\sin^2 \theta_s + \sin^2 \theta_i - 2 \sin \theta_i \sin \theta_s \cos(\varphi_s - \varphi_i)} \quad (13)$$

Furthermore, the  $g_{pq}$  in Equation (12) are the SPM polarization factors with

$$g_{hh} = \frac{(\varepsilon_r - 1)}{(\cos \theta_s + \sqrt{\varepsilon_r - \sin^2 \theta_s})(\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i})} \cos(\varphi_s - \varphi_i),$$

$$g_{vh} = \frac{-(\varepsilon_r - 1)\sqrt{\varepsilon_r - \sin^2 \theta_s}}{(\varepsilon_r \cos \theta_s + \sqrt{\varepsilon_r - \sin^2 \theta_s})(\cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i})} \sin(\varphi_s - \varphi_i),$$

$$g_{hv} = \frac{(\varepsilon_r - 1)\sqrt{\varepsilon_r - \sin^2 \theta_i}}{(\cos \theta_s + \sqrt{\varepsilon_r - \sin^2 \theta_s})(\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i})} \sin(\varphi_s - \varphi_i),$$

and

$$g_{vv} = \frac{(\varepsilon_r - 1)(\varepsilon_r \sin \theta_i \sin \theta_s - \sqrt{\varepsilon_r - \sin^2 \theta_s} \sqrt{\varepsilon_r - \sin^2 \theta_i} \cos(\varphi_s - \varphi_i))}{(\varepsilon_r \cos \theta_s + \sqrt{\varepsilon_r - \sin^2 \theta_s})(\varepsilon_r \cos \theta_i + \sqrt{\varepsilon_r - \sin^2 \theta_i})}. \quad (14)$$

For Gaussian correlation functions, Equation (12) reduces to:

$$\gamma_{pq}^{SPM}(\hat{k}_s, \hat{k}_i) = (2k^2 \sigma \ell \cos \theta_s \cos \theta_i)^2 |g_{pq}|^2 \exp\left\{-\frac{k_{xy}^2 \ell^2}{4}\right\}. \quad (15)$$

Fig. 2 depicts the value for the backscattering coefficient based on SPM, Equation (15).

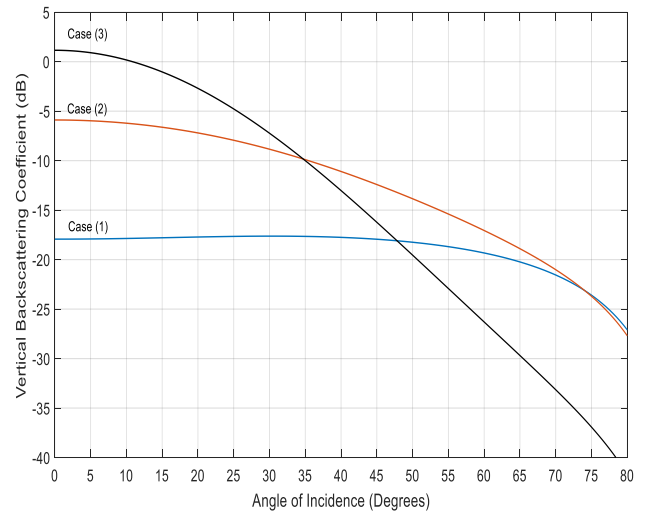


Fig. 2 - Vertically backscattering coefficient ( $\varepsilon_r=20$ )

Case (1):  $k\sigma=0.1$ ,  $k\ell=1.0$ , Case (2):  $k\sigma=0.2$ ,  $k\ell=2.0$ , Case (3):  $k\sigma=0.3$ ,  $k\ell=3.0$

In order to examine values of cross-polarized backscattering coefficients in the plane of incidence set

$$\varphi_s - \varphi_i = 0, \text{ or } \pm \pi. \quad (16)$$

Then introduce Equation (16) into Equation (14) and the resultant into Equation (12) yielding null values for the cross-polarized bi-static scattering coefficients within the plane of incidence. Those null values indicate that SPM is not capable of predicting the cross-polarized bi-static scattering coefficients within the plane of incidence.

When  $k\sigma$  increases, the validity conditions of SPM given in Equation (11) are not satisfied, and Equation (12) cannot be used. The next bi-static scattering coefficients to be used are those based on a physical optics model.

#### 4. DIFFUSE BI-STATIC SCATTERING COEFFICIENT BASED ON THE PHYSICAL OPTICS (PO) MODEL

The validity conditions of PO are given by Equation (17) of [4].

$$k\ell > 6, \text{ and } k\ell > 5.893k\sigma \quad (17)$$

The diffuse bi-static scattering coefficient  $\gamma_{pq}^{PO}(\hat{k}_s, \hat{k}_i)$  from randomly rough surface based on physical optics can be written as (Equation (12.55a) of [8], and (A.15) - (A.62) of [9]).

$$\gamma_{pq}^{PO}(\hat{k}_s, \hat{k}_i) = \left( \frac{|a_{pq}|k\ell}{2} \right)^2 \exp\{- (kq_z\sigma)^2\} I_0 \quad (18)$$

with

$$I_0 = \sum_{n=1}^{\infty} \frac{(kq_z\sigma_s)^{2n}}{n!n} \exp\left\{-\frac{(kxy\ell)^2}{4n}\right\},$$

$$k\sqrt{\frac{k_{xy}}{\sin^2\theta_s + \sin^2\theta_i - 2\sin\theta_i\sin\theta_s\cos(\varphi_s - \varphi_i)}},$$

and

$$q_z = \cos\theta_i + \sin\theta_s. \quad (19)$$

The above formulation of  $I_0$  is valid only for a Gaussian correlation function. The corresponding formulation for the exponential correlation function is given in Equation (A.63) of [9].

In addition, the  $a_{pq}$  are the PO polarization factors with:

$$a_{hh} = -r_{hh}(\cos\theta_i + \cos\theta_s)\cos(\varphi_s - \varphi_i),$$

$$a_{vh} = -r_{hh}(1 + \cos\theta_i\cos\theta_s)\sin(\varphi_s - \varphi_i),$$

$$a_{hv} = r_{vv}(1 + \cos\theta_i\cos\theta_s)\sin(\varphi_s - \varphi_i),$$

and

$$a_{vv} = -r_{vv}(\cos\theta_i + \cos\theta_s)\cos(\varphi_s - \varphi_i). \quad (20)$$

Fig. 3 depicts backscattering coefficient values based on PO, Equation (18).

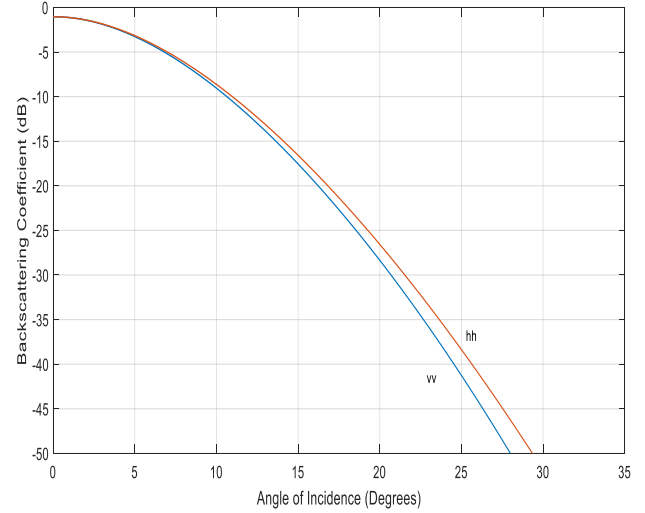


Fig. 3 – Backscattering coefficient based on physical optics model as a function of angle of incidence ( $k\sigma = 1.5$ , rms slope = 0.1,  $\epsilon_r = 1.6$ )

In order to examine values of cross-polarized bi-static scattering coefficients within the plane of incidence introduce Equation (16) into Equation (20) and the resultant into Equation (18). This indicates that the cross-polarized bi-static scattering coefficients based on PO have null values within the plane of incidence. The null values indicate that PO is not capable of predicting the cross-polarized bi-static scattering coefficients within the plane of incidence.

#### 5. DIFFUSE BI-STATIC SCATTERING COEFFICIENT BASED ON THE KIRCHHOFF APPROXIMATION

When the validity conditions of either SPM, Equation (11), or PO, Equation (17), are not satisfied, the Kirchhoff approximation may be used to obtain the bi-static scattering coefficients. The validity conditions for KA may be written as Equation (21) based on [4].

$$k\sigma > \frac{\sqrt{10}}{|\cos\theta_i + \cos\theta_s|}, \quad k\ell > 6 \quad (21)$$

Under the above conditions, the major contribution to the bi-static scattering coefficient along the scattering direction  $\hat{k}_s(\theta_s, \varphi_s)$  stems from around the local normal to the surface  $\hat{n}$

$$\hat{n} = \frac{q_x \hat{x} + q_y \hat{y} + q_z \hat{z}}{q} \quad (22)$$

with

$$q_x = (\sin\theta_s \cos\varphi_s - \sin\theta_i \cos\varphi_i), \quad (23)$$

$$q_y = (\sin\theta_s \sin\varphi_s - \sin\theta_i \sin\varphi_i), \quad (24)$$

$$q_z = (\cos \theta_s + \cos \theta_i), \quad (25)$$

and

$$q^2 = q_x^2 + q_y^2 + q_z^2. \quad (26)$$

Those contributions can be captured through the KA with applying the stationary phase method technique<sup>1</sup> yielding the diffuse bi-static scattering coefficient  $\gamma_{pq}^{KA}(\hat{k}_s, \hat{k}_i)$  (Equation (9) of [10]) given in Equation (27)

$$\gamma_{pq}^{KA}(\hat{k}_s, \hat{k}_i) = \frac{1}{2m_x m_y} \left| \frac{q}{q_z} \right|^4 |U_{pq}(\hat{k}_s, \hat{k}_i)|^2 \exp \left\{ -\frac{1}{2q_z^2} \left( \left\{ \frac{q_x}{m_x} \right\}^2 + \left\{ \frac{q_y}{m_y} \right\}^2 \right) \right\}. \quad (27)$$

where  $m_x$  and  $m_y$  are the slope variances along  $\hat{x}$  and  $\hat{y}$  direction respectively. For isotropic surface  $m_x = m_y = m$  with  $m$  given in Equation (5). Furthermore,  $U_{pq}(\hat{k}_s, \hat{k}_i)$  in Equation (27) are the KA polarization factors

$$U_{hh}(\hat{k}_s, \hat{k}_i) = \frac{(\hat{k}_s \cdot \hat{v}_i)(\hat{k}_i \cdot \hat{v}_s)r'_{hh} + (\hat{k}_s \cdot \hat{h}_i)(\hat{k}_i \cdot \hat{h}_s)r'_{vv}}{D_0^2}, \quad (28)$$

$$U_{vh}(\hat{k}_s, \hat{k}_i) = \frac{-(\hat{k}_s \cdot \hat{v}_i)(\hat{k}_i \cdot \hat{h}_s)r'_{hh} + (\hat{k}_s \cdot \hat{h}_i)(\hat{k}_i \cdot \hat{v}_s)r'_{vv}}{D_0^2}, \quad (29)$$

$$U_{hv}(\hat{k}_s, \hat{k}_i) = \frac{-(\hat{k}_s \cdot \hat{h}_i)(\hat{k}_i \cdot \hat{v}_s)r'_{hh} + (\hat{k}_s \cdot \hat{v}_i)(\hat{k}_i \cdot \hat{h}_s)r'_{vv}}{D_0^2}, \quad (30)$$

$$U_{vv}(\hat{k}_s, \hat{k}_i) = \frac{(\hat{k}_s \cdot \hat{h}_i)(\hat{k}_i \cdot \hat{h}_s)r'_{hh} + (\hat{k}_s \cdot \hat{v}_i)(\hat{k}_i \cdot \hat{v}_s)r'_{vv}}{D_0^2}, \quad (31)$$

and

$$D_0^2 = (\hat{k}_i \cdot \hat{v}_s)^2 + (\hat{k}_i \cdot \hat{h}_s)^2. \quad (32)$$

In addition,  $r'_{hh}$  and  $r'_{vv}$  in equations (28) - (31) are the Fresnel reflection coefficients of Equation (10) evaluated for the surface normal  $\hat{n}$  of Equation (22). The local incident angle  $\theta'_i$  associated with such a normal and required for calculating Fresnel reflection coefficients  $r'_{hh}$  and  $r'_{vv}$  can be evaluated as follows:

$$\cos \theta'_i = \hat{k}_i \cdot \hat{n} = q|q_z|/(2kq_z) \quad (33)$$

Furthermore, the vector scalar products reported in equations (28) - (32) can be obtained from the propagation vectors  $\hat{k}_i$ , and  $\hat{k}_s$ , the polarization vectors  $\hat{v}_i$ ,  $\hat{h}_i$ ,  $\hat{v}_s$ , and  $\hat{h}_s$  as in equations (6) - (7).

$$(\hat{k}_s \cdot \hat{v}_i) = -\sin \theta_s \cos \theta_i \cos(\varphi_s - \varphi_i) - \sin \theta_i \cos \theta_s \quad (34a)$$

$$(\hat{k}_s \cdot \hat{h}_i) = \sin \theta_s \sin(\varphi_s - \varphi_i) \quad (34b)$$

$$(\hat{k}_i \cdot \hat{v}_s) = \sin \theta_i \cos \theta_s \cos(\varphi_s - \varphi_i) + \sin \theta_s \cos \theta_i \quad (34c)$$

$$(\hat{k}_i \cdot \hat{h}_s) = -\sin \theta_i \sin(\varphi_s - \varphi_i) \quad (34d)$$

Fig. 4 depicts the values for the backscattering coefficient based on KA, Equation (27).

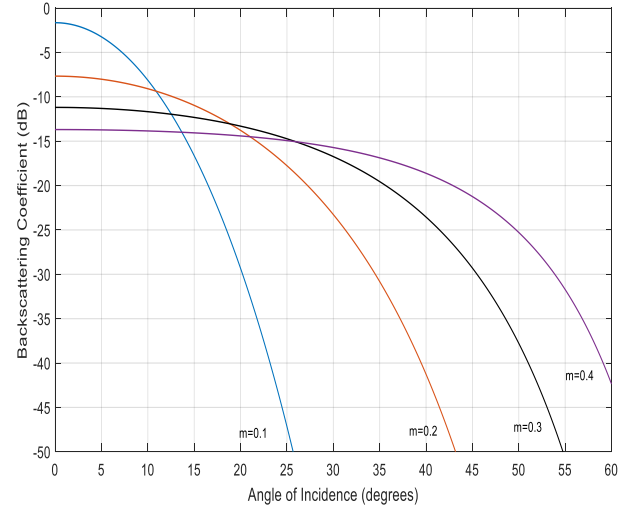


Fig. 4 - Backscattering coefficient as a function of angle of incidence ( $\epsilon_r=1.6$ ,  $m_x=m_y=m$ )

Now the values of the cross-polarized bi-static scattering coefficients within the plane of incidence are examined. For a scattering direction aligned within the plane of incidence, the identity, Equation (35), holds.

$$(\hat{k}_i \cdot \hat{h}_s) = (\hat{k}_s \cdot \hat{h}_i) = 0 \quad (35)$$

Introducing Equation (35) into Equation (32) yields

$$D_0^2 = (\hat{k}_i \cdot \hat{v}_s)^2. \quad (36)$$

Based on equations (35) and (36), for a scattering direction within the plane of incidence the cross-polarized KA polarization factors given in equations (29) and (30) reduce to:

$$U_{vh}(\hat{k}_s, \hat{k}_i) = 0 \quad (37)$$

and

$$U_{hv}(\hat{k}_s, \hat{k}_i) = 0. \quad (38)$$

Introducing equations (37) - (38) into Equation (27) yields null values for the cross-polarized bi-static scattering coefficients within the plane of incidence. The null values indicate that KA is not capable of predicting the cross-polarized bi-static scattering coefficients within the plane of incidence.

<sup>1</sup> The technique used in this section is also known as the geometric optics approach.

## 6. SUMMARY AND CONCLUSION

Analytical mathematical formulations for bi-static scattering coefficients from a randomly rough surface with complex relative permittivity were developed based on the following models: Small perturbation model (SPM), Physical optics model (PO), and Kirchhoff approximation (KA). Each formulation has its own applicability domain as given in equations (11), (17) and (21). Those applicability domains are provided in Table 1 for convenience.

**Table 1** - Validity domains of bi-static scattering models [1]

Bi-static scattering model	Validity range
SPM (Small perturbation model)	$k\sigma < 0.3$ , and $k\ell > 4.71k\sigma$
PO (Physical optics)	$k\ell > 6$ and $k\ell > 5.893k\sigma$
KA (Kirchhoff approximation)	$k\sigma > \frac{3.16227}{\cos \theta_s + \cos \theta_i}$ , $k\ell > 6$ , and, $k\ell > 4.17\sqrt{k\sigma}$

Furthermore, mathematical analysis shows that each of these models predicts null values for the cross-polarized bi-static scattering coefficients within the plane of incidence. In order to extend the applicability domain of the above three models, they can be integrated through a two scale scattering model [11]. The two scale scattering model has the property of producing non-null values for the cross-polarized coefficients within the plane of incidence.

Each of the three models can be used to predict the bi-static scattering coefficients of Earth surface within the validity domain of the model. In so doing, the complex relative permittivity of the Earth's surface is required. In the case of land surface (soil), the complex relative permittivity can be obtained from equations (37) – (40) of [12] in terms of frequency, land surface temperature, water content, and composition. In the case of sea surface, the complex relative permittivity may be obtained from equations (14) – (27) of [12] in terms of frequency, sea water temperature and sea salinity.

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