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**Considerations on the use of GNSS as a primary
time reference in telecommunications**



Summary

This technical report provides information relevant to optimal GNSS reception in telecommunication applications where highly accurate time recovery is critical. Unlike commonly used GNSS navigation applications, where position is the goal, the focus in telecommunications is on accurate time recovery with stationary receivers, which provide accurate times to equipment such as primary reference time clocks (PRTCs) and base stations in mobile networks.

NOTE – This is an informative ITU-T publication. Mandatory provisions, such as those found in ITU-T Recommendations, are outside the scope of this publication. This publication should only be referenced bibliographically in ITU-T Recommendations.

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Technical Report ITU-T GSTR-GNSS

Considerations on the use of GNSS as a primary time reference in telecommunications

1 Introduction

1.1 Scope

This technical report provides information relevant to optimal GNSS reception in telecommunication applications where highly accurate time recovery is critical.

Unlike commonly used GNSS navigation applications, where position is the goal, the focus in telecommunications is on accurate time recovery with stationary receivers, which provide accurate time to such equipment as primary reference time clocks (PRTC) and base stations in mobile networks.

This technical report is addressed to telecommunication operators, manufacturers, silicon vendors and test equipment vendors who are interested in a high-level view of the information and issues associated with GNSS reception. This includes general information; basic variables, parameters and equations; modes of operation; and the nature of the challenges and the methodology to mitigate them.

This document does not aim to collect or summarize the large amount of material continually published on the topic of GNSS reception, which includes scientific publications, doctoral theses, experimental test reports and articles in online encyclopedias.

One relevant area of current study not addressed in this technical report is the topic of GNSS vulnerability. Information on this can be found in a technical report from ATIS [2].

The objective of this report is to provide guidance towards designing and operating telecommunication GNSS-based clocks for accurate time-recovery applications.

1.2 High level description of GNSS systems

1.2.1 Systems overview

As of February 2020, there were 6 GNSS satellite constellations in orbit providing geolocation and time distribution:

- GPS, controlled by the US government, was the first one, launched in the late 1970s, and operational since the 1990s. It is still the most popular one and offers full coverage of the planet thanks to approximately 32 medium Earth orbit satellites in six different orbital planes. Military and civilian applications share five bands, the most commonly used being 1.57542 GHz (L1 signal) and 1.2276 GHz (L2 signal). The modulation technique is CDMA.
- GLONASS, controlled by the Russian government, was the second one, also launched in the late 1970s, and operational since the 1990s. It is the second most popular one and offers full coverage of the planet thanks to 24 medium Earth orbit satellites in three different orbital planes (better covering the poles than GPS). Military and civilian applications share five bands, the most commonly used being 1.602 GHz (L1 signal) and 1.246 GHz (L2 signal). The modulation technique is FM, but CDMA is being studied as well. Most GLONASS receivers also support GPS+GLONASS dual mode for enhanced operation.
- BeiDou, controlled by the Chinese government, was launched in the 2000s, and has been operational since the early 2010s. It is the second most popular one in Asia, and as of 2017 offers local coverage over China thanks to a mix of five geostationary and up to 30 medium

Earth orbit satellites. The network is expanding and targets global coverage in the future. Military and civilian applications share three bands, 1.561 GHz (B1 signal), 1.207 GHz (B2 signal), and 1.268 GHz (B3 signal). The modulation technique is CDMA. Most BeiDou receivers also support GPS+ BeiDou dual mode for enhanced operation.

- Galileo, controlled by the European Union, was launched in the 2010s, and not yet fully operational as of 2019. It will offer global coverage using up to 30 medium Earth orbit satellites in three different orbital planes. Publicly regulated and civilian applications share four bands, the most commonly used being 1.57542 GHz (E1) and 1.17645 GHz (E5a). The modulation technique is CDMA. Galileo receivers will also support GPS+Galileo dual constellation mode for enhanced operation.
- Indian Regional Navigation Satellite System (IRNSS), controlled by the Indian government, was launched in the 2010s, and fully operational as of 2017. It offers local coverage using up to three geostationary and four geosynchronous orbit satellites operational above India. Military and civilian applications share two bands, 1.176 GHz (L5 signal) and 2.492 GHz (S signal). The modulation technique is CDMA.
- The Quasi-Zenith Satellite System (QZSS), controlled by the Japanese government, was launched in 2010, and put into full operation in 2018. It offers local coverage with 1 geostationary satellite and 3 satellites in quasi-zenith orbit above Japan, Southeast Asia, and Australia. Military and civilian applications share the same bands and modulation technique as GPS. To reduce errors in satellite positioning, QZSS provides the Sub-meter Level Augmentation Service (SLAS) in the L1S signal and Centimeter Level Augmentation Service (CLAS) in the L6 signal. In the future, QZSS is planned to be operated with seven satellites at which point authentication messages will be provided with navigational messages as measures against spoofing attacks.

Below is a general figure for the full bandwidth utilization for different GNSS constellations:

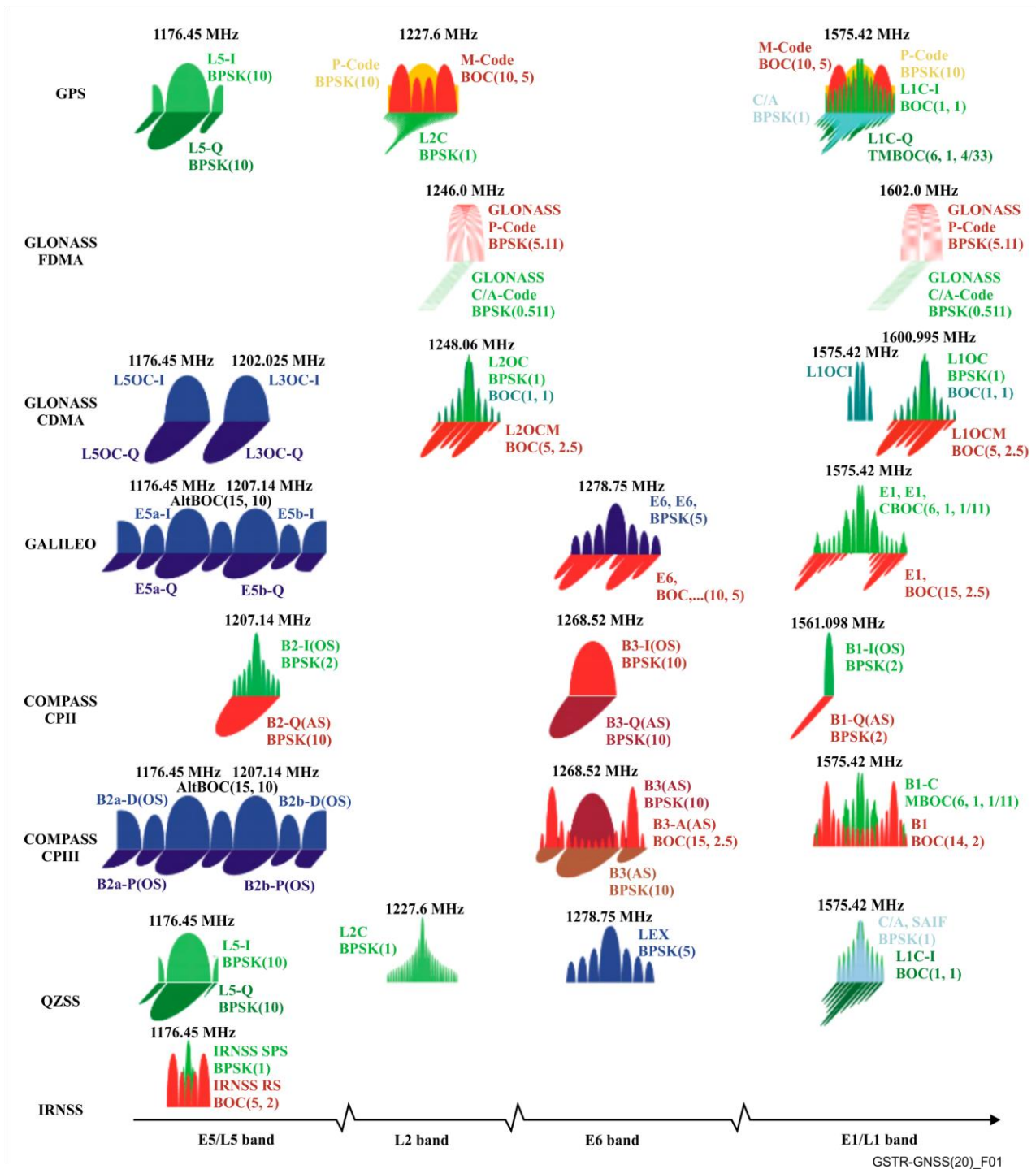


Figure 1 – GNSS constellations with different signals and frequency bands (as of 2017)

From the top to the bottom of the above figure, in order, are the following GNSS systems: GPS, GLONASS (FDMA and CDMA signals), Galileo, Compass (II and III phases, or BeiDou), QZSS, and IRNSS.

The key information for these constellations (civil signals) is summarized in the following table:

Table 1 – GNSS constellation key information (as of 2020)

System	Nominal constellation	Full operational capability	Number of operational satellites	Coverage	Civilian spectrum
GPS	24	1995	31 (2017)	Global	Current: L1 C/A, L2C Future: L1 C/A, L1C, L2C, L5
WAAS	3	IOC/2003 FOC/2008	3	Regional (USA)	Current: L1C/A Future: L1C/A, L5
GLONASS	24	1995 (GLONASS) 2010 (GLONASS-M)	24 (2017)	Global	Current: L1OF, L2OF, L3OC Future: L1/L2OF, L1/L2SF, L1/L2OC, L1/L2SC, L3OC *: some are CDMA signals different that GPS ones
SDCM	2 geostationary satellites	2014	3	Regional	L1 SCDM
GALILEO	24+3 (MEOs)	TBD	22 (2019)	Global	Current: E1 OS/SAR/PRS E5 OS/ SAR Future: E6 CS/PRS
EGNOS	3 (GEOs)	2009 for open service 2010 for safety-of-life service	2 (+2 test)	Regional (Europe)	Current: L1C/A Future (Egnos 2G): double band
COMPASS/ BeiDou	5 (GEOs)+ 30 (non GEOs)	TBD	23 (2017)	Global	1,559.052 ~ 1,591.788 MHz 1,166.22 ~ 1,217.37 MHz 1,250.618 ~ 1,286.423 MHz
GAGAN/ IRNSS/ NavIC	8	TBD	8	Regional (India)	GAGAN: L5, L1 IRNSS: S, L5, L1
MSAS	1 GEO		1 (MTSAT)	Regional (Japan) (Note: It is planned that MSAS will be terminated and taken over by QZSS)	L1
QZSS	1 (GEO) + 3 (QZO)	2018	4 (2018) 7 (TBD)	Regional (Japan)	L1 C/A, L1C, L1S (Sub-meter Level Augmentation Service), L2C, L5, L1-SAIF, L6 (Centimeter Level Augmentation Service)

1.2.1.1 Information delivered by GNSS satellites

Information delivered by GNSS satellites includes, for example, ephemeris, almanac, time and ionospheric corrections.

As satellites can be seen as reference beacons, the precise knowledge of their position and movement is crucial. The navigation messages incorporate the following.

- Almanac information: describing the complete constellation rough orbital information, as well as health information and ionospheric correction data, and UTC leap second information. This information is valid for months and allows a receiver to get a wide view of the constellation at start-up. However, modern receivers do not need to wait for the reception of the full almanac (up to 12.5 minutes) to commence start-up.
- Ephemeris information (as pseudo-Keplerian elements or satellite position and velocity): in which each satellite transmits its own accurate orbit predictions for a shorter-term view (up to four hours).
- Satellite clock information: describing the prediction of offset of the satellite's internal clock compared to the constellation system time. This is required for precise and accurate synchronization of all clocks of all visible satellites at the receiver.

NOTE – Assisted GPS (AGPS) allows a receiver to acquire all ephemeris and almanac information from a remote server over Internet much faster than from the GNSS satellite transmission, which takes 12.5 minutes (for GPS L1C and Galileo E1 signals). Details of the message transmission format are in interface control documents of each navigation satellite system.

The aim of the GNSS broadcast information is to enable the receiver to autonomously calculate the orbit position and clock of a satellite at the time of the transmission of the GNSS signal. This enables the receiver to estimate its position and to correct most of the systemic errors.

1.2.1.2 Renewal of information from navigation messages

Renewal of the navigation message content depends entirely on the strategy of each GNSS operator and is primarily driven by an availability of a satellite for upload from ground segment and secondarily by factors such as the stability of satellite orbits, clocks and predictions thereof.

1.2.2 How a navigation satellite system works

All GNSS operate in the same way, and are made of three segments:

- The *control* segment consists of a coordinated, hierarchical group of stations on various locations on the ground. They include ground atomic clocks acting as master clock to the clocks within satellites, as well as controlling or monitoring links to all satellites. Operations such as satellite enabling, orbit or time corrections, or uploading useful data (such as ephemeris and ionospheric model), are under the control of the operation control segment.
- The *space* segment consists of the satellites, under the management of the control segment, and relaying information broadly (time, ephemeris, status) to all receivers located on the surface of the Earth. The satellites include an energy source to allow multichannel radio transmissions to and from the control and user segments, as well as orbit adjustments. They also include high stability oscillators (atomic clocks on most constellations) that can be tuned by the control segment to keep optimal time accuracy.
- The *user* segment is made of millions of GNSS receivers, some of them mobile (smartphones or vehicles, for example), some of them static (differential GPS references or telecommunication clocks, for example). Each one is designed for a low-cost recovery of its location and time offset, while receiving the signals from multiple satellites. Some receivers allow dual band reception for one constellation, or even for multiple constellations.

1.3 Position and time recovery at the GNSS receiver

The core of GNSS receiver functionality can be viewed as a multiple channel correlator that compares internally generated pseudorandom sequences (replicas representing individual satellites) against RF samples retrieved from an antenna. The correlator outputs raw measurements, called pseudo-ranges, which represent a distance (or time) that the satellite signal travelled from a space vehicle to a receiver, but also includes the receiver's internal clock offset (from GNSS time) and other sources of noise.

For timing applications, the user can choose to first perform a self-survey stage, during which the GNSS receiver's antenna static position is considered as unknown, so the four unknowns to be solved are x_a , y_a , z_a (position of the antenna in the ECI frame) and t_{offset} (time offset of the receiver's local time vs GNSS epoch).

The system of four equations and four unknowns is complex to solve. There are many techniques described in the public literature. Among them are:

- a purely mathematical closed-form solution;
- other approaches, for example using iterative estimations based on Newton's method.

All methods need at least four satellites in view to solve the four equations and compute the four unknowns. When more than four are available, techniques described in clause 2 (SNR / Carrier-to-Noise power ratio masking, elevation masking, position dilution of precision (PDOP) masking, and time receiver autonomous integrity monitoring (T-RAIM) masking) allow for rejecting the faulty and unreliable ones, and further combining the results from the good satellites. A self-survey typically makes such estimations repetitively, for a few thousand seconds, before it averages the results and delivers the estimated static position.

After completion of the self-survey, the receiver's antenna position is considered as solved: x_a , y_a , z_a are no longer considered as unknowns but as input parameters. There may be monitoring techniques implemented to make sure that they remain valid or can be updated later. The known position can be used in the pseudo-range equation corresponding to each visible satellite. This is a simplified, first order equation to solve.

When more than one satellite is available, techniques described in clause 2 (SNR / Carrier-to-Noise power ratio masking, Elevation masking, PDOP masking, T-RAIM masking) allow rejecting the faulty and unreliable ones, and further combining the results from all good satellites. The GNSS receiver's antenna position is located on the surface of the earth, so that it must be checked that $x_a^2 + y_a^2 + z_a^2$ is close to the square of the Earth's radius, approximately (6371 km)².

For details on these computations, see Appendix IV.

1.3.1 GNSS positioning

For positioning purposes, solving the position variables consists of estimating the intersection of spheres around each satellite. Three satellites are needed to define a unique point on the surface of the Earth using trilateration techniques.

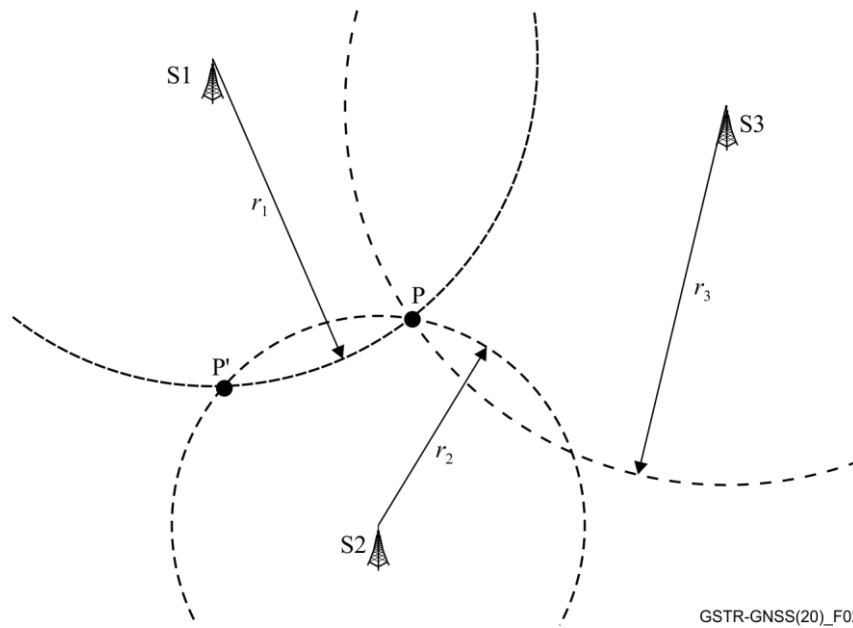


Figure 2 – Trilateration

Here, the minimum number of satellites (S_x) is 3, but if the receiver at point P does not have the correct time reference a priori, a fourth satellite is needed. (Note: For time recovery receivers used in a static position, this position is estimated as an unknown during self-survey, then converted into a known parameter. This simplifies the recovery equations, leaving only time as unknown.)

In theory, time can be estimated from a single satellite (even if, in real life, more than one is preferred to provide better accuracy).

1.3.2 GNSS time

For time processing, it is necessary to mention basic notions of timescales and how to process the time information in GNSS navigation messages.

Below is an example of timescales and parameters in the GPS system (other GNSS systems are comparable):

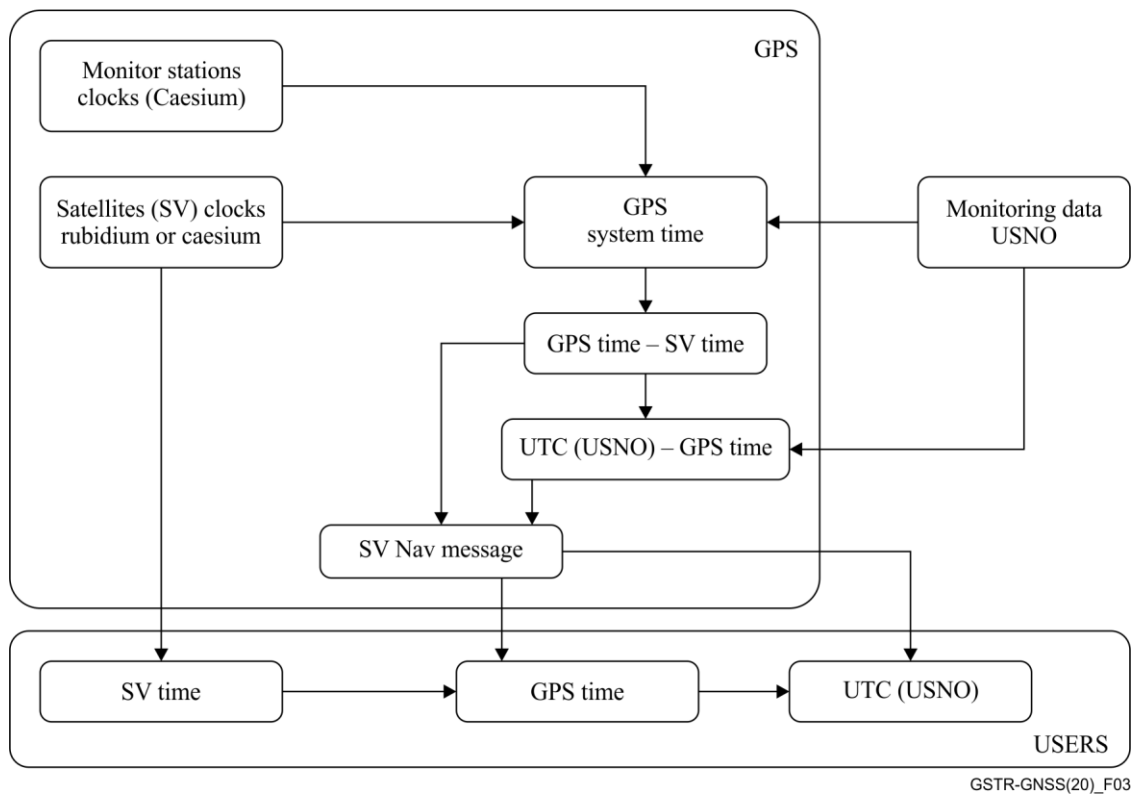


Figure 3 – Timescales and parameters in GPS system

Some basic information on GPS timescales:

- The GPS system uses its own timescale, named "GPS system timescale", abbreviated as "GPS time".
- The United States Naval Observatory (USNO) maintains the U.S. Department of Defense (DoD) reference for time and time interval. USNO has an ensemble of atomic clocks, which is used to derive a timescale called UTC (USNO). The clocks in the ensemble contribute to International Atomic Time (TAI) and Coordinated Universal Time (UTC). UTC (USNO) and UTC (NIST) are kept in very close agreement, typically to within 20 nanoseconds, and both can be considered official sources for time in the United States.
- "GPS time" is a paper timescale (in the same sense as UTC), as it is constructed from all the individual atomic clocks in the GPS system (space and ground segments), and it is linked by measurement to UTC (USNO), the DoD official reference timescale for the USA.
- "GPS time" and UTC (USNO) were aligned at the beginning of the GPS timescale epoch, which was at 00:00 UTC Sunday, 6 January, 1980.
- The "GPS time" broadcast format is as follows:
 - week number (10 bits), number of weeks since the beginning of the GPS time. Roll-over is 1024 weeks, which is slightly less than 20 years. Implementation specific solutions allow estimating the missing upper bits;
 - time of week (TOW) (19 bits), elapsed time in a week, a value with a resolution of 1.5 seconds.
- GPS time is a linear timescale; it does not follow the leap seconds as UTC does, the official international timescale. However, most GNSS systems (with the exception of GLONASS) provide leap second information, enabling the receiver to compute UTC time from GNSS time.

- Each satellite has its internal timescale, named Space Vehicle (SV) time.
- Thanks to monitoring stations and the processing of measurements in the master control station, each satellite receives from the ground segment by uplink channel the updated models of the differences SV time versus GPS time, and GPS time versus UTC (USNO).
- This modelling information is present as various parameters in GPS navigation messages embedded in each emitted GPS electromagnetic signal towards the Earth.

In brief, it is possible for a GPS receiver to process all these parameters from all received satellite signals to rebuild a local realization of GPS or UTC (USNO) timescale and produce a physical timing signal on a PPS output port.

2 Factors influencing the performance of a GNSS-based PRTC

The most common type of PRTC [10b] is one that distributes the time using radio signals from a GNSS system. However, the performance of a GNSS system is dependent on a range of issues, both under the control of and outside the control of the equipment vendor. Therefore, any vendor specification can only indicate what the equipment is capable of, rather than what performance the equipment will actually deliver in any given installation.

2.1 Installation and calibration

In measuring the performance of a GNSS-based PRTC, the following conditions should be verified as far as possible so that:

- The equipment is properly commissioned and calibrated for fixed offsets, such as antenna cable length and cable amplifiers. For example, an antenna cable will produce a delay of approximately 4 ns/m, depending on the cable type.
- Any 1PPS output signal asymmetry compensation contained within the PRTC (such as that described in clause A.1.2 of [10a]) is stable.
- The antenna has a clear view of the sky with minimal multipath distortion. This may be verified by recording the number of satellites visible throughout the measurement.

2.2 Equipment design and configuration

In measuring the performance of a GNSS-based PRTC, the following conditions should be verified as far as possible:

- The equipment has been designed to deliver accurate time (as opposed to accurate position), with the ability to lock position, with the position being locked before and during the test.
- The equipment has the ability to detect and mitigate timing-solution faults and errors on individual satellite contributions.
- The equipment is properly configured according to the current GNSS antenna installation conditions to assist in achieving enhanced timing performance (e.g., minimal acceptable signal to noise ratio, minimum elevation mask, and dilution of precision mask). More details on these options can be found in clause 4.

2.3 External elements

In measuring the performance of a GNSS-based PRTC, the following conditions should be verified as far as possible:

- The GNSS or radio distribution system is properly maintained and operated by the relevant authorities. This may be verified by checking the operational status bulletins issued by the relevant operating authorities.

In addition to these primary factors, there are some secondary conditions that may cause errors in the time measured by a GNSS system. These factors may be more difficult to quantify or to mitigate against. Secondary factors may include the following:

- Interference from ground level transmissions: While filters may be used to remove some ground level interference, they may not protect against local jamming. The presence of jamming may be verified by using interference detection equipment;
- atmospheric conditions such as thunderstorms, heavy rain or fog;
- solar interference such as sunspots and flares, which affect ionosphere delay;
- impedance matching in the antenna cable should be good enough to prevent significant power in a reflected signal reaching the receiver (see clause 3.8 and Appendix V).

NOTE 1 – The use of a dual-band GNSS receiver (e.g., GPS L1 / L2, Galileo E1 / E5a, GLONASS L1 / L2, and BEIDOU B1 / B2) can help to mitigate ionosphere delays, see Appendix IV.

NOTE 2 – The GNSS signal, because of its frequency, will experience negligible attenuation due to water vapour (wet atmosphere part for signal attenuation even in sunny weather). In the case of clouds, rain, fog, and snow, which contain a water component that is not in the vapour phase (i.e., in the solid or liquid phase), the GNSS signal strength will be slightly degraded, even in the case of deposition of water/snow/ice on the antenna radome.

For more detail on the sources of time error in the GNSS time distribution network, see clause 3.

3 Sources of time error in GNSS time distribution

GNSS satellite clock time is obtained by receiving space signals into a GNSS receiver. The error on GNSS signal transmission must be corrected in order to obtain high-precision satellite clock time. As shown in Figure 4, these errors can be divided to three categories: satellite-related errors, signal propagation-related errors, and receiver-related errors, according to different sources. According to statistical calculations, the accuracy of one-way timing can be as much as 20 ns after correcting for such errors.

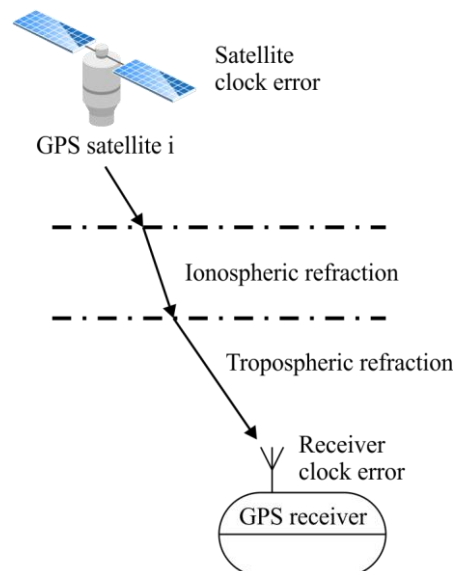


Figure 4 – GPS satellite one-way timing

3.1 Satellite ephemeris error

The ephemeris error refers to the deviation between the calculated satellite position and the actual position of the satellite, and is caused by errors in the broadcast ephemeris or other orbital information errors. Because the satellites are affected by many kinds of perturbations before their signals reach the earth surface, it is difficult for the ground monitoring stations to determine the influence of these perturbations sufficiently, which can cause satellite orbit error. In addition, the quality of the monitoring system, including the number and spatial distribution of the tracking stations, orbital parameters, and the model used in the orbital calculation will affect the ephemeris accuracy and result in ephemeris errors.

3.2 Satellite clock error

The satellites of a GNSS system have high-precision atomic clocks, but there is frequency offset, frequency drift and other effects constituting variation between these clocks and GNSS standard time. Frequency offset and frequency drift will change over time, resulting in non-synchronous deviation between satellite clock time and GNSS system time, that is, satellite clock error. As satellite position is a function of time, GNSS observations are based on precision measurements. The satellite clock error will cause an error in the pseudo-range, resulting in a time error for a GNSS receiver's local time.

3.3 Doppler shift

Since satellites are always moving, it is necessary to take into account the Doppler shift due to the relative velocity between the satellite and the receiver, as well as the additional propagation delay due to satellite position change when making accurate time and frequency measurements using the satellite.

3.4 Relativistic effect

The relativistic effect [5] is the phenomenon caused by a difference of speed and difference of Earth gravitational potential between the satellite clock, which is in motion in space, and the receiver clock. Satellite clock frequency correction can be pre-modified, but the satellite orbits are elliptical (i.e., the eccentricity of each orbit is non-zero) and their operating data are changing with time; therefore, even if the satellite clock frequency is premodified, there are still residuals from the relativistic influence.

3.5 Ionospheric delay

The ionospheric delay is caused by free electrons and positive ions in the path between the satellite and the user; the neutral atmosphere is ionized into a plasma composed of a large number of positive ions and free electrons. When the GNSS electromagnetic wave signal traverses the ionosphere, the plasma will cause reflection, refraction and scattering of this electromagnetic wave signal, which affects the propagation speed. In quiet ionospheric conditions, the resulting additional delay ranges from a few nanoseconds to 20 to 25 ns for GNSS signals.

There are some alternatives to address the uncertainty associated with the ionospheric delay:

- i. Receive GNSS without addressing ionospheric uncertainty. This depends on the assumption that the impact of space weather is acceptable. These receivers are vulnerable to space weather and can exhibit time error excursions beyond PRTC limits. Given space weather exhibits long (typically 11-year cycles) and the last maximum in 2013 was unusually mild, there is a vulnerability that may not be understood clearly by the user community.
- ii. Correct for ionospheric delay using a model. Single frequency receivers can use a model to provide an approximate correction for ionospheric delay. For example, most single band GPS receivers apply a correction using the Klobuchar model [1]. Updated parameters for this

model are broadcast in the GPS navigation message. The Klobuchar model typically reduces the time error due to ionospheric delay by 50%.

- iii. Use regional correction services like SBAS, which provide improved accuracy of the ionosphere delay model.
- iv. Mitigate with primary atomic clocks. This is an effective strategy that utilizes diurnal filtering and space weather detection to prevent adjusting the local atomic timescale to incorrect time. This strategy can be used effectively in ePRTC [10c] equipment and can be used either in conjunction with multiband receivers or with single frequency receivers.
- v. Utilize multiband GNSS receivers to actively compensate for ionospheric time delay. The obvious advantage is the autonomous correction of space weather induced errors. Given the continual correction capability, this will allow for a more robust and accurate class of PRTC equipment with a somewhat less demanding need for local oscillator support.

For further details on ionospheric delay see Appendix II.

3.6 Tropospheric delay

The tropospheric delay generally refers to the additional propagation delay caused by the refraction of the electromagnetic wave signal in the neutral atmosphere. The non-dispersive delay of the signal propagation in the troposphere causes the electromagnetic wave propagation path to be longer than the geometric distance. There is also a close relationship between tropospheric refraction and surface climate, atmospheric pressure, humidity and temperature, and finally the tropospheric delay is also related to the elevation angle of the satellite. As an example, for GPS the effect on the timing of tropospheric delay of zenithal propagation is about 7.59 ns and the effect of tropospheric delay at an elevation angle of 10 degrees is about 66 ns [6]. If the meteorological parameters such as temperature, humidity, barometric pressure and vapour pressure are used to correct the tropospheric delay in the observation station, the impact on timing can be reduced to sub-nanosecond.

3.7 Limited open sky area

If the open sky area where navigation satellite signals can be received directly is limited by obstacles near the receiver's antenna, too few line-of-sight (LOS) satellites are visible overhead. Time synchronization accuracy is degraded due to the geometry of the visible satellites overhead, with an increased dilution of precision (DOP) value.

3.8 Multipath effects

In the process of GNSS signal reception, GNSS satellite signals are reflected by obstacles near the receiver's antenna (such as a building or any structure behaving as a mirror), causing multiple delayed reflected signals interfering with the direct signal from the satellite. This multiple reflection (multipath) causes deviation between the observed time value and the true one [4]. This is a major source of error and affects the accuracy of GNSS time estimation. Multipath causes deviation of the pseudo-range measurements and consequently phase of the derived timing output signal thereby failing to achieve optimal time synchronization accuracy and precision. Multipath is related to the environment around the receiver, and it is difficult to correct it by a model. The best approach to mitigation of multipath is to choose the appropriate (open sky) antenna position if possible.

Multipath signals from visible (LOS) and non-visible (NLOS) satellites affect the accuracy of time synchronization and position measurements. For the signals from a visible (LOS) satellite, the stronger, direct signal is received first, before the relatively weaker multipath signal, so the effects can be cancelled effectively through signal processing in the correlator circuit of the GNSS receiver. Conversely, since there is no direct signal from a non-visible (NLOS) satellite, the effects can only be cancelled by not using (filtering) the satellite signal for time synchronization. As the requirements

on accuracy and precision of the GNSS receiver increase, this latter effect becomes particularly noticeable and can become a significant cause of degradation in time synchronization precision and accuracy.

3.9 Multiple antenna cable reflections

The GPS code is embedded in the signal using biphasic modulation of the carrier. The receiver locks to the signal by locally generating the C/A code for the particular satellite and performing servo lock to maximize correlation with the received code. With no multipath reflections, the correlation code has a simple maximum peak. However, each reflected signal distorts the correlation code, pulling the lock point and creating a bias. With multipath reflections before the antenna, the arrival phases of the reflected signals change as the satellite moves overhead. However, due to imperfect impedance matching at the receiver and antenna, reflected signals can enter the receiver after bouncing from the receiver to the antenna and back to the receiver, transiting the antenna cable two times more than the direct signal. For further information on this effect, see Appendix V.

3.10 Receiver internal error

The internal time measurement link of the timing receiver may be defined as a processing link from the time in the signal received at the antenna (as the starting point) to the receiver's output 1PPS pulse (as the end point).

In its processing of time, the GNSS timing receiver internal error contains two components:

- A) A systemic constant error, that is, the link delay caused by internal propagation on cables and electronic implementation, as well as intermediate calculation from the antenna to the 1PPS pulse generation. This can be calibrated by design and/or installation, and this can be corrected to a great extent.
- B) A random error mainly refers to the timing error caused by the receiver noise, resolution and receiver clock bias. For example, the delivery of a synchronous 1PPS signal on an output, clocked with a certain frequency is subject to a certain phase resolution. GNSS receivers routinely estimate this rounding effect on every pulse and advertise its value on the management link.

4 Mitigation of time error in a GNSS-based PRTC

GNSS timing receivers and PRTC devices may provide a set of configuration parameters, which can be utilized to configure the receiver to ignore satellites that are expected to deteriorate the performance. These are:

1. SNR / Carrier-to-Noise power ratio masking
2. Elevation masking
3. PDOP / Position Dilution of Precision masking
4. T-RAIM / Time Receiver Autonomous Integrity Monitoring masking
5. Cable delay correction

4.1 SNR masking

The quality of received GNSS-satellite signals are often reported by the GNSS receiver as carrier-to-noise power ratio (C/N_0) values. Low C/N_0 values can result from low-elevation satellites, partially obscured signals (due to dense foliage for example) or reflected RF signals (multipath).

A signal contaminated with multipath can degrade the position and timing solutions. Such signals are most commonly found in urban environments with many tall buildings and a preponderance of

reflective materials. The combination of direct and reflected signals at the receiver tend to have low C/N_0 values, since the multipath reflected signals mask the true shape of the original signal and hence make it difficult for the receiver to detect it in the noise. Non-line-of-sight (NLOS) signals without direct signals present can substantially degrade performance. The NLOS signals are received with relatively low C/N_0 values.

When the GNSS antenna has a clear view of the sky, a minimal C/N_0 threshold can be set to exclude satellites that are likely affected by multipath. However, it should be noted that unpredicted interference may cause the C/N_0 value to be below the threshold and hence cause total loss of GNSS signal.

4.2 Elevation masking

Signals from low-elevation satellites are typically of poorer quality than those from higher elevation satellites. These signals travel farther through the ionospheric and tropospheric layers and undergo distortion due to certain atmospheric conditions. In addition, signals from satellites with very low elevation are very likely contaminated with quite a lot of multipath.

Excluding satellites with very low elevation from GNSS fix and timing computations can reduce the likelihood of the potential errors discussed above.

4.3 PDOP/TDOP masking

Position/Time dilution of precision (P/TDOP) is a measure of the error caused by the geometric relationship of the satellites used in the position solution. High DOP occurs when the geometry is such that a small error in the measured distance between the receiver and a satellite results in a larger error in the location and/or time reported by the receiver. Sets of tracked satellites that are tightly clustered or aligned in the sky have a high PDOP and contribute to lower position accuracy.

The DOP indicates the confidence level of a position fix. Low DOP values indicate a high confidence level, while high DOP values indicate a low confidence level.

4.4 T-RAIM masking

Time receiver autonomous integrity monitoring (T-RAIM) allows, from GNSS receivers in position-lock mode, computing the distribution of time recovery from all visible individual satellites, and to reject the outliers. Choosing the value of the mask value is a trade-off between reduced satellite availability and increased integrity level of the time estimate (more details are provided in Appendix III).

4.5 Cable delay correction

The position estimated by the GNSS receiver is that of the antenna, even if there is a long cable between this antenna and the receiver, and between the receiver and user. Cables between the antenna and receiver, between the receiver and user, as well as other electronic circuits inside the antenna, receiver and user: all contribute to adding a delay in the reception process. This total delay will bias the 1PPS to arrive "too late" at the user application.

The above-mentioned delays cannot be solved in the equations by the receiver since they are common for all satellites. If not manually configured into the user application (or the receiver), it will add a time error to the user application. It is therefore recommended that the user configurable interface offers a compensation parameter for these various delays. It is possible to merge the cable delays into a single parameter. Compensation can be done by actively adjusting the time of deliverance of the 1PPS pulse, or by adjusting the data that says what time the pulse arrives.

5 Operational schemes for mitigation of time error in GNSS time distribution

5.1 Multi-constellation and multi-frequency receiver

The ability of a GNSS receiver to handle multiple frequencies from multiple constellations in the calculation of position is essential to mitigate various errors. Using multi-frequency receivers is one of the most effective ways to remove ionospheric delay from the PVT calculation. Ionospheric delay varies with frequency, so it impacts various GNSS signal carriers differently. By comparing the delays of two GNSS signals, L1 and L2, for example, the receiver can correct for the impact of ionospheric delays.

The new and modernized wideband signals in the L5/E5 band provide inherent noise and multipath mitigation capabilities. When receivers combine L5/E5 capabilities with the ability to remove ionospheric delay using the dual-frequency combination, significant improvements in both measurement and positioning accuracy can be achieved. Multi-frequency receivers also provide more immunity to interference. If there is narrowband interference in the L2 frequency band, a multi-frequency receiver could still track L1 and L5 signals (provided those are interference free) to ensure continuous operation of the receiver. Such an operation mode is however suboptimal as, for example, there is significantly fewer operational GNSS satellites in the L5 band: all Galileo satellites provide E5 signals, yet GPS L5 signals are still marked unhealthy to date (September 2019).

A multi-constellation receiver can access signals from several constellations: GPS, GLONASS, BeiDou and Galileo for example. The use of other constellations in addition to GPS, results in there being a larger number of satellites in the field of view, which has the following benefits:

- improved position and time accuracy;
- improved availability in difficult environmental conditions;
- improved spatial distribution of visible satellites, resulting in improved dilution of precision.

5.2 Differential GNSS

A commonly used technique for improving GNSS performance is differential GNSS. In differential GNSS, the position of a fixed GNSS receiver, referred to as a base station, is determined to a high degree of accuracy using conventional surveying techniques. Then, the base station broadcasts its measurements to other GNSS receivers, called rovers. The fundamental assumption is that the base station and rovers are not far from each other (tens of kilometres) and hence have similar satellites in view. The signals from those common-in-view satellites also experience similar propagation conditions, and therefore when rovers apply base station measurements (corrections) to their own measurements, all common sources of error (e.g., satellite orbit and clock errors, ionosphere delays) are removed to a great extent.

Differential positioning requires a data link between the base station and rovers, if corrections need to be applied in real-time.

5.3 Satellite-based augmentation systems

For applications where the cost of a differential GNSS system is not justified, or if the rover stations are spread over too large an area, a satellite-based augmentation system (SBAS) may be more suitable for enhancing position accuracy. SBAS systems are geosynchronous satellite systems that provide services for improving the accuracy, integrity and availability of basic GNSS signals. Accuracy is enhanced through the transmission of wide-area corrections for GNSS range errors. Integrity is enhanced by the ability of the SBAS network to quickly detect satellite signal errors and send alerts to receivers that they should not track the failed satellite. Signal availability can be improved if the SBAS transmits ranging signals from its satellites.

5.4 Precise point positioning (PPP)

PPP is a positioning technique that removes or models GNSS system errors to provide a high level of PVT accuracy from a single receiver. A PPP solution accuracy depends on GNSS satellite clock and orbit correction quality. In general, the better the corrections the better the final accuracy of a PPP solution. Corrections can be delivered to the end user via GNSS (or other) satellite broadcast or over the Internet. These corrections are then used by the receiver to provide accuracies in decimetre-level or better with no base station required.

A typical PPP solution requires quite a long period of time to converge to decimetre accuracy (10-30 minutes) in order to resolve ambiguities of the carrier phase pseudo-ranges. The actual accuracy achieved, and the convergence time required is dependent on the multipath conditions, and the quality of the corrections and how they are applied in the receiver. High quality geodetic receivers achieve millimetre-level accuracies.

Similar in structure to an SBAS system, a PPP system provides corrections to a receiver to increase position accuracy. However, PPP systems typically provide a greater level of accuracy and charge a fee to access the corrections. PPP systems also allow a single correction stream to be used worldwide, while SBAS systems are regional.

5.5 GNSS data post-processing

For many applications, such as airborne surveys, corrected GNSS positions are not required in real-time. For these applications, raw GNSS satellite measurements are collected and stored for processing post-mission. Post-processing does not require real-time transmission of differential correction messages. This simplifies the hardware configuration greatly.

During post-processing, base station data can be used from one or more GNSS receivers. Multi-base processing helps preserve high accuracy over large project areas, which is a common occurrence for aerial applications. Depending on the project's proximity to a permanently operating GNSS network, base station data can often be freely downloaded, eliminating the need for establishing your own base station(s). Moreover, it is possible to process without any base station data through PPP, which utilizes downloaded precise clock and ephemeris data.

Post-processing applications offer a great deal of flexibility. Applications can involve stationary or moving base stations, and some support integration with customer or third-party software modules. Post-processing applications may be designed to run on personal computers, accessible through simple-to-use graphical user interfaces.

Post-processing generally results in a more accurate, comprehensive solution than is possible in real-time.

References

- [1] International Telecommunication Union (2010), *Handbook on Satellite Time and Frequency Transfer and Dissemination* <<http://www.itu.int/pub/R-HDB-55/en>>, ITU, ISBN: 9261133010.
- [2] ATIS-0900005 (2017), *GPS Vulnerability*, ATIS Technical Report.
- [3] Qifeng Xu.(2001), *Space geodesy*, People's Liberation Army Publishing House.
- [4] Jiyu Liu (2008), *Positioning principles and methods of GPS satellite navigation*, Science Press.
- [5] Mertikas S.P. (1985), *Error Distributions and Accuracy Measures in Navigation: An Overview*. M.Sc.E. thesis, Department of Surveying Engineering Technical Report No. 113, University of New Brunswick, Fredericton, New Brunswick, Canada.

- [6] Lewandowski W (1989). *Positioning of GPS Antennas in time-keeping laboratories of north American, Proc. of the 43rd Annual symposium on frequency control*, 218-224.
- [7] Ziqin Wei, Maorong Ge. (1998), *Mathematical Model of GPS Relative Positioning*, *Surveying and Mapping Press*, p. 56-57.
- [8] Cove K. (2005), *Improvements in GPS Tropospheric Delay Estimation with numerical weather prediction. M.Sc.E. thesis, department of geodesy and geomatics engineering technical report No. 230, University of New Brunswick, Fredericton, New Brunswick, Canada, pp.98.*
- [9] Status Update on the Quasi Zenith Satellite System (2019), *The 4th Japan-EU Satellite Positioning Public-Private Roundtable, Mar. 14.*
- [10] ITU-T Recommendations:
- [10a] Recommendation ITU-T G.8271/Y.1366 (2020), *Time and phase synchronization aspects of telecommunication networks.*
- [10b] Recommendation ITU-T G.8272/Y.1367 (2018), *Timing characteristics of primary reference time clocks.*
- [10c] Recommendation ITU-T G.8272.1/Y.1367.1 (2016), *Timing characteristics of enhanced primary reference time clocks.*
- [11] Interface Control Document (ICD) references:
- [GPS] Navstar GPS Space Segment /Navigation User Interface (2019), IS-GPS-200K, May
- [Galileo] European GNSS (Galileo) Open Service Signal-in-Space Interface Control Document (2016), Galileo-OS-SIS-ICD, December.
- [GLONASS] Interface Control Document Navigational signals L1, L2, (2008), ICD-GLONASS-eng-v5.1.
- [BeiDou] BeiDou Navigation Satellite System Signal In Space Interface Control Document Open Service Signal B1C (Version 1.0) (2017), BDS-SIS-ICD-B1C-1.0, December.
- [QZSS] Quasi-Zenith Satellite System Interface Specification Satellite Positioning, Navigation and Timing Service (2018), IS-QZSS-PNT-003, November.
- [IRNSS] Indian Regional Navigation Satellite System Signal in Space ICD for Standard Positioning Service (2017), IRNSS-SPS-ICD-1.1-2017, August.
- [EGNOS] EGNOS Open Service (OS) Service Definition Document (2017), EGNOS-OS-SDD, October.

Appendix I

Cable delay effects and correction in a GNSS receiver

I.1 Presentation of the main components and hypothesis

The GNSS reception system is composed of:

- an antenna, collecting all radio signals from satellites;
- (optionally) a cable between the antenna and the GNSS receiver;
- a GNSS receiver, terminating the radio signals, solving position and time variables, and delivering a 1PPS to the user, along with time information = $k \cdot 1s$;
- (optionally) a cable between the GNSS receiver and the user;
- a user, receiving the 1PPS.

The goal is to define which variables are solved by the receiver, and how to calibrate the cables and other delays so that the 1PPS is delivered "on time" to the user.

NOTE – The discussion only focuses on identifying a few unknowns (which position is solved, how cable and other local propagation delays are compensated), and purposely ignores or simplifies other thoroughly studied challenges (see [Jespersen] for example):

- The effects of relativity, Doppler, ionospheric and tropospheric errors, multipath are considered as solved.
- The general frame for position considered is ECI (Earth Centred Inertial coordinate), thus ignoring the effects of the earth rotation. Note that this is not the frame generally transmitted from GNSS for satellite positions.
- Also, for additional simplicity it can be considered that the receiving system (antenna + receiver + user) is at a static position and that this position has been resolved. Only time recovery is of interest.

I.2 Main equation and variables

This document mentions the notion of pseudo-range, which is a key one in the context of solving position and time by a GNSS receiver.

The distance between each satellite [i] and the antenna can be computed in different ways

$$d_i = \sqrt{(x_i - x_a)^2 + (y_i - y_a)^2 + (z_i - z_a)^2} = c * (t_{ri} - dt_{ar} - t_i) = c * ((t'_{ri} - t_{offset}) - dt_{ar} - t_i)$$

where:

- t_i is the satellite "i" emission time in GPS epoch time. This information is carried over the radio signal from satellites to receiver and therefore known
- t'_{ri} is the observed reception time in the receiver's local time
- t_{ri} is the unknown reception time in the GPS epoch time
- t_{offset} is the unknown real time offset of the receiver's local time vs GPS epoch:
 - $t_{offset} = t'_{ri} - t_{ri}$.
 - t_{offset} is the difference between the time read simultaneously on the receiver in its local time, and the time read on a device that is purely aligned to GPS time.

- Example 1: If the receiver's clock starts counting from value 0 second today in 2017, it is "late" and t_{offset} will be negative. Its value will be the number of seconds since GPS epoch (Jan 6, 1980) = -315 million of seconds (also includes nanoseconds).
- Example 2: If the receiver's time is almost good but too early by 1000 ns, $t_{offset} = +1000$ ns. When t is read as $k*1s$, t' is simultaneously read as already $k*1s + 1000$ ns s on the receiver.

dt_{ar} is the real propagation delay between the antenna phase centre and receiver correlator (strictly positive, including antenna electronics, receiver electronics, and a cable if present). This is an unknown value, but it can be compensated by $configured_dt_{ar}$, as discussed below. This delay applies the same for both GPS epoch time and receiver's local time

c is the speed of radio signals, supposedly known and perfect

x_i, y_i, z_i describe the position of the satellite "i" in the ECI frame when the signal was sent (at t_i in the GPS epoch). This information is carried over the radio signal from satellites to receiver and therefore known

x_a, y_a, z_a describe the position of the antenna in the ECI frame when the signal was received (at $t_{ri} - dt_{ar}$ in the GPS epoch). Either known or unknown value, as discussed below.

I.3 Solving for the unknowns

The pseudo-range is by definition, for satellite "i"

$$pri = \sqrt{(x_i - x_a)^2 + (y_i - y_a)^2 + (z_i - z_a)^2} + c * (t_{offset} + dt_{ar}) = c * (t'_{ri} - t_i)$$

The system is normally able to solve the four unknowns as soon as four satellites are in view, where the unknowns are:

- x_a, y_a, z_a the position of the antenna (unless it has been manually configured or estimated in the past and the receiver is operating in "position lock" mode)

$estimated_t_{offset} = (t_{offset} + dt_{ar})$, the time error of the receiver + the propagation delay between the antenna and receiver. In the above equation, dt_{ar} is common to all satellites equations and cannot be isolated from t_{offset} . The $estimated_t_{offset}$ by the receiver will be overestimated by this dt_{ar} cable delay.

The receiver's job is to deliver a 1PPS pulse when it estimates that t (in the GPS epoch) = $k*1s$, which means when $t' = k*1s + (estimated_t_{offset} - configured_dt_{ar})$, i.e., when $t' = k*1s + (t_{offset} + dt_{ar} - configured_dt_{ar})$.

If the propagation delay between the antenna and receiver is correctly configured, then $dt_{ar} - configured_dt_{ar} = 0$ and the 1PPS is delivered at $t' = k*1s + t_{offset}$, which means $t = k*1s$ exactly.

If, however, the cable delay is not configured, then $configured_dt_{ar} = 0$, and the $estimated_t_{offset}$ is overestimated by dt_{ar} , and 1PPS is delivered when $t' = k*1s + (t_{offset} + dt_{ar})$, which means $t = k*1s + dt_{ar}$: too late by dt_{ar} .

Note that in some deployment scenarios, there is a second cable which also adds delay on the 1PPS received by the user: dt_{ru} , the propagation delay between the receiver and user.

If a one-way 1PPS output signal is delivered along with time information " $t = k \cdot 1s$ " by the receiver "on time" and received at the user with a delay of dt_{ru} , the user will receive it "too late". If the user aligns its local clock to the 1PPS received "too late", its own local 1PPS will be "too late" by the same amount.

Conclusion

Solving the GNSS equations estimates the position of the antenna. Any configuration interface proposing to manually enter the position should consider the antenna's position, not the position of the receiver or the user.

The time the user receives as a 1PPS has the following cumulative positive delays that need to be accounted for:

- antenna electronics and any processing,
- any cable from the antenna to the receiver,
- receiver electronics and processing,
- and any cable from the receiver to the user.

Compensation can be done by actively adjusting the time of deliverance of the pulse, or by adjusting the data that says what time the pulse arrives.

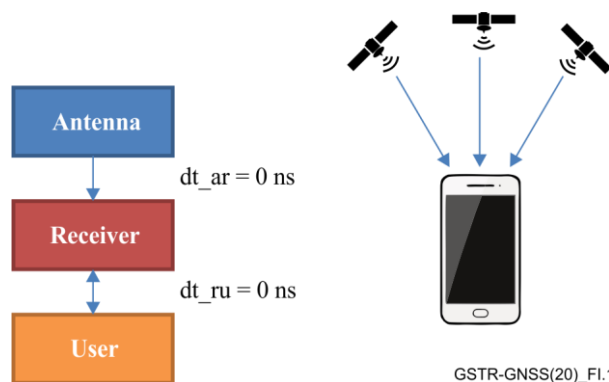
The value of this time compensation cannot be estimated by the GNSS receiver, and must be manually estimated:

- either by knowing the cable length and velocity, as well as other delays in the antenna and receiver "by design", thus computing the total propagation delay,
- or using reflection tools to measure the round-trip delay of the cable.

Then, the user's management system must offer configuration parameters to manually configure these time correction values.

The section below details four deployment cases.

Case 1 = co-located antenna + receiver + user



This case is typically a mobile phone, where all functions are collocated within a few centimetres.

All signals from all satellites are received and solved in the receiver without any delay.

The solved position and `time_offset` is shared with the user without any delay.

Obviously, the solving for all unknowns ($x_r, y_r, z_r, t_{offset}$) corresponds to the one shared by antenna + receiver + end user.

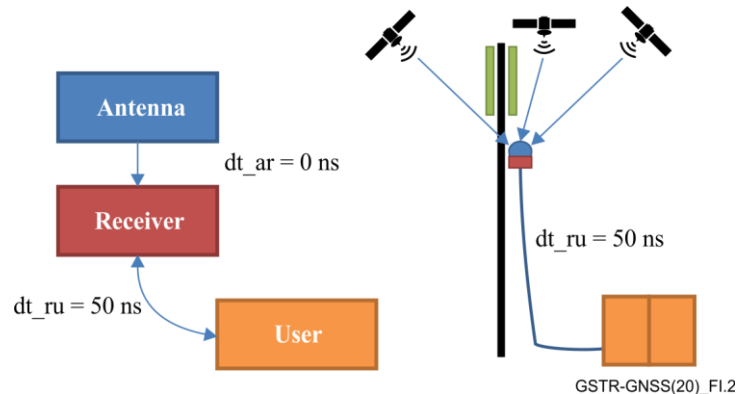
Example:

If $t_{\text{offset}} = t' - t = 1000 \text{ ns}$, which means that the GNSS receiver is "too early" by 1000 ns.

Considering that configured_dt_ar is correctly set to 0 ns, that dt_ar is really 0, then $\text{estimated_t_offset}$ is 1000 ns.

The receiver's job is to deliver a 1PPS pulse when it estimates that t (in the GPS epoch) = $k*1$ which means when $t' = k*1\text{s} + (\text{estimated_t_offset} - \text{configured_dt_ar})$, i.e., when $t' = k*1\text{s} + (t_{\text{offset}} + \text{dt_ar} - \text{configured_dt_ar})$. As $\text{dt_ar} - \text{configured_dt_ar} = 0$ and the 1PPS is delivered at $t' = k*1\text{s} + t_{\text{offset}}$, which means $t' = k*1\text{s}$ exactly.

Case 2 = co-located antenna + receiver, distant user



This case is typically a base station with integrated receiver including the antenna on a mast.

All signals from all satellites are received and solved in the receiver without any delay ($\text{dt_ar} = 0$).

If a one-way 1PPS output signal is delivered by the receiver "on time" and received at the user with a delay of dt_ru , the user will receive it "too late". If the user aligns its local clock to the 1PPS received "too late", its own local 1PPS will be "too late" by the same amount.

So, this dt_ru cable delay needs to be compensated by some configuration parameter:

- either advancing the 1PPS signal in the receiver and advertising $t = k*1\text{s}$
- or keeping the 1PPS signal in the receiver and advertising $t = k*1\text{s} + \text{configure_dt_ru}$
- or simply let the user know that the information received from the receiver (1PPS + advertised $t = k*1\text{s}$) is received too late by a value of configure_dt_ru , and that local time should be delayed by the same configure_dt_ru .

Example:

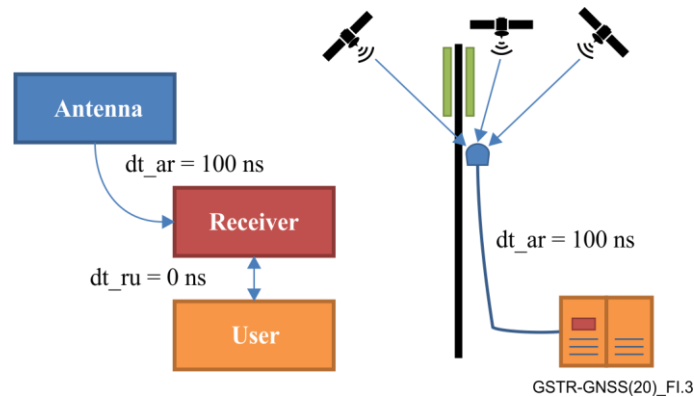
If 1PPS is delivered "on time" by the GNSS receiver, and $t_{\text{ru}} = 50 \text{ ns}$.

When GPS 1PPS and time information " $t = k*1\text{s}$ " is sent at $t = k*1\text{s}$ by the receiver, it is received at $t = k*1\text{s} + 50\text{ns}$ at the user. The user will align its local time to that 1PPS and believe it is only $k*1\text{s}$, while it is already $k*1\text{s} + 50 \text{ ns}$.

If $\text{configure_dt_ru} = 50 \text{ ns}$, as expected, the user time will be increased by 50 ns, and the 1PPS delivered by the user will therefore be delivered 50 ns earlier.

Please note that alternatively, if the receiver is sharing time over a two-way protocol (such as IEEE1588), this delay is automatically corrected and does not need cable delay compensation.

Case 3 = distant antenna, co-located receiver + user



This case is typically a base station with antenna on the mast, and receiver inside the base station.

All RF signals from all satellites are received at the antenna, then follow the same delay t_{ar} till they are captured in the receiver.

In the pseudo-range equation, compared to case 1, this is delaying each "tri" by the same value t_{ar} , and therefore is solved as an overestimated t_{offset} than the real one. It causes a positive bias on the estimated t_{offset} , while the estimated position x,y,z is still the one of the antenna (where all signals are collected in a straight line path from the satellites) like in case 1.

If not corrected by the configured dt_{ar} , this positive bias on the estimated t_{offset} makes the receiver deliver the 1PPS too late.

Example:

Considering:

- $t_{offset} = t' - t = 1000 \text{ ns}$, which means that the GNSS receiver is "too early by 1000 ns".
- $dt_{ar} = 100 \text{ ns}$, the real propagation delay between antenna and receiver

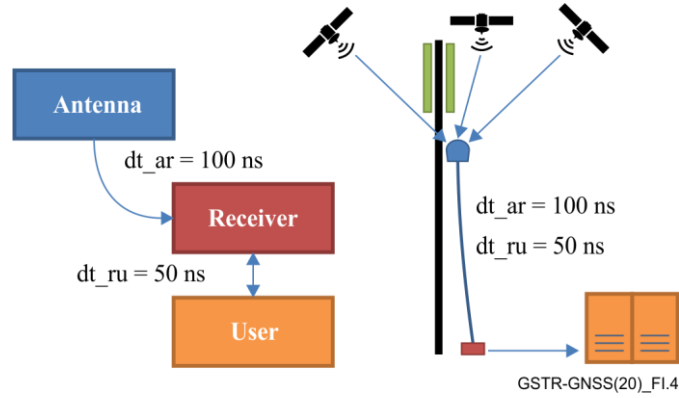
Considering that configured dt_{ar} is incorrectly set to 0 ns, that dt_{ar} is really 100 ns, then estimated t_{offset} is 1100 ns.

The receiver's job is to deliver a 1PPS pulse when it estimates that t (in the GPS epoch) = $k*1s$, which means when $t' = k*1s + (\text{estimated_}t_{offset} - \text{configured_}dt_{ar})$, i.e., when $t' = k*1s + (1100 \text{ ns})$, which means $t = k*1s + 100 \text{ ns}$ exactly: too late by 100 ns.

Now, considering that configured dt_{ar} is correctly set to 100ns, that dt_{ar} is really 100 ns, then estimated t_{offset} is 1000 ns, fully matching the real t_{offset} .

The receiver's job is to deliver a 1PPS pulse when it estimates that t (in the GPS epoch) = $k*1s$, which means when $t' = k*1s + (\text{estimated_}t_{offset} - \text{configured_}dt_{ar})$, i.e., when $t' = k*1s + (1000 \text{ ns})$, which means $t = k*1s$ ns exactly: on time.

Case 4 = distant antenna, receiver and users



This case is typically a base station with antenna on the mast, and receiver outside the base station. For example, a GNSS receiver is shared between several base stations.

This case is a mix of cases 2 and 3.

All RF signals from all satellites are received at the antenna, then they follow the same delay t_{ar} until they are captured in the receiver.

If not compensated by an appropriately configured- t_{ar} , td_{ar} will cause a 100 ns "too late" 1PPS transmission by the receiver.

If not compensated by an appropriately configured- t_{ur} , td_{ru} will cause a 50 ns "too late" 1PPS reception at the user.

I.4 Determining position with non-simultaneous satellite signals

It is possible to solve for the unknowns even if the four satellites are not simultaneously in view, assuming the receiver is stationary. This is described here:

It is necessary to determine the fixed coordinates of a stationary receiver for optimum timing. Ideally, one would like to make all measurements simultaneously, from satellites with a good geometric spread in order to minimize the geometric dilution of precision (GDOP). However, with systems in urban canyons it is not always possible to achieve a good GDOP. An alternative is to make measurements serially in time, using a stable clock to bring the measurements together as if they were made simultaneously. For positioning with simultaneous measurements, one needs at least four pseudo-ranges, ρ_k , to solve for the user's coordinates x , y , z and the user's time offset from system time t . For a pseudo-range, ρ , we have

$$\rho^2 = (S_x - x)^2 + (S_y - y)^2 + (S_z - z)^2 + (ct)^2 \quad (\text{I.1})$$

for satellite S with coordinates S_x , S_y , S_z , and with c the speed of light, ignoring the Sagnac effect and effects of the ionosphere and troposphere. For a measurement $m1$ made when the local clock has changed its time offset from t to $t + \Delta t$, the pseudo-range, ρ_1 is

$$\rho_1^2 = (S_{x1} - x)^2 + (S_{y1} - y)^2 + (S_{z1} - z)^2 + (c(t + \Delta t))^2 \quad (\text{I.2})$$

If one knows the rate offset of the local clock from system time, the term Δt , then equation (I.2) takes on the form of the general pseudo-range (I.1). Hence, it is sufficient to know the rate offset well enough to account for the change in the local clock time offset. For example, if one knows the clock rate to 1 part in 10^{12} , the pseudo-range error after 1000 s will be only 1 ns, or 30 cm. If a local clock

is available that is measured remotely against a GNSS using a two-way protocol such as PTP, it might be possible to measure this rate offset with sufficient accuracy.

The problem remains that there must be satellite measurements made at some point with enough geometric diversity to limit the GDOP.

References

- [Jespersen] Jespersen J.L., Weiss M., Davis D.D., Allan D.W. (1980), *Global Positioning System for accurate time and frequency transfer and for cost-effective civilian navigation*, *Proc. IEEE PLANS Conf.*, available from <https://tf.nist.gov/general/pdf/496.pdf>.

Appendix II

Ionospheric delay, its effect on GNSS receivers, and mitigation of these effects

The use of two frequencies by GNSS receivers allows the receiver to detect and correct up to 90% for ionospheric effects. The dual-frequency measurements (for instance, L1 and L2 bands for GPS constellation, E1 and E5a for Galileo constellation, E1 and E2 bands for GLONASS constellation, and similar for BEIDOU) enable those receivers to correct the error, because one frequency will refract differently from another and the difference can be used to determine the effect on the signal being used.

If the receiver is able to manage linear operation on pairs of GNSS signals measurements, it is possible to estimate very precisely the ionospheric propagation induced delay error. In this case, the receiver can subtract this error from measurements.

Note that the impact on the GNSS receivers of ionospheric events is expected to be very similar within one geographic region, so if all the single-frequency (for instance: L1) receivers in this region follow the same phase error, then the relative phase error from this effect will be the same on the output of all PRTCs. In this case, the impact from an ionosphere event is not significant if only the relative phase requirement is relevant.

Mixing single frequency (for instance: L1) with dual frequency (for instance: L1/L2) PRTCs should be done in a controlled way in order to get the related benefits. Effectively, in a certain geographic area, all single frequency GNSS receivers will all deliver non-corrected time, while all dual frequency ones will deliver the corrected one. This will result in relative time error between the two groups of receivers. As a consequence, clusters requiring tight relative time accuracy should be carefully supplied with the same type of receivers (if possible, all dual frequency for best absolute time error, otherwise all single frequency ones, but avoiding a mix of them).

However, it is possible that clusters with single frequency receivers and clusters with dual frequency receivers may work independently, in parallel.

In a GNSS receiver the measured uncorrected distance from the receiver to a satellite is referred to as a pseudo-range, R . It can be expressed as:

$$R = r + c\delta t_r + \delta R_i + c\delta t_s \quad (\text{II.1})$$

Here r is actual distance from the receiver to the satellite; δt_r is the error of the receiver clock, which we wish to determine; δt_s represents known clock errors of the satellite, which are broadcast in the satellite message; c is the speed of light in vacuum; and δR_i is the error in pseudo-range due to the ionosphere. Not included are other smaller errors, such as those due to the delay in the troposphere. If R_i were known, then the offset of the receiver clock, δt_r , could be determined.

The ionosphere is a plasma, due to ionization of gas molecule by radiation from the sun. The plasma causes the ionosphere to act as a dispersive medium with respect to signals, such that to first order [Klouchar], [Crawford], [Shohet].

$$\omega^2 = c^2 k^2 + \omega_p^2 \quad (\text{II.2})$$

Here ω is the angular frequency of a signal propagating in the ionosphere; k is the corresponding wavenumber, and ω_p is the plasma frequency. Under the assumption that the electron mass is much less than the ion mass (which is true for the ionosphere), ω_p is given by

$$\omega_p = \sqrt{\frac{n_e e^2}{m_e \epsilon_0}} \text{ [rad/s] (SI (MKS) units) [Shohet]} \quad (\text{II.3})$$

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m_e}} \text{ [rad/s] (cgs units) [Nicholas].} \quad (\text{II.3a})$$

In Eq. (II.3), n_e is the number density of electrons in electrons/m³, e is the electron charge in Coulombs ($e = 1.602 \times 10^{-19}$ C), m_e is the rest mass of the electron in kg ($m_e = 9.109 \times 10^{-31}$ kg), and ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}$ C² s² kg⁻¹ m⁻³). In Eq. (II.3-a), n_e is the number density of electrons in electrons/cm³, e is the electron charge in statcoulombs ($e = 4.803 \times 10^{-10}$ statcoulombs), and m_e is the rest mass of the electron in g ($m_e = 9.109 \times 10^{-28}$ g)

The group velocity, v_g , for an electromagnetic wave in the ionospheric plasma is

$$v_g = \frac{d\omega}{dk}. \quad (\text{II.4})$$

Solving Eq. (II.2) for ω and differentiating gives

$$v_g = \frac{kc^2}{\sqrt{k^2 c^2 + \omega_p^2}} = \frac{kc^2}{\omega^2}. \quad (\text{II.5})$$

Solving Eq. (II.2) for k and substituting into Eq. (II.5) gives

$$v_g = c\sqrt{1 - (\omega_p/\omega)^2} \quad (\text{II.6})$$

If $\omega \gg \omega_p$, Eq. (II.6) may be approximated by expanding it to first-order in $(\omega_p/\omega)^2$. The result is

$$v_g = c \left[1 - \frac{\omega_p^2}{2\omega^2} \right] \quad (\text{II.7})$$

Eq. (II.6) shows that the group velocity is less than c by the amount $c\omega_p^2/(2\omega^2)$. The error in the pseudo-range, δR_i , is equal to this quantity multiplied by the propagation time for the signal through the ionosphere. The result is that when $\omega \gg \omega_p$, δR_i is inversely proportional to the signal frequency squared (i.e., it is proportional to $1/\omega^2$). Since the plasma frequency of the ionosphere is on the order of 1 MHz [Crawford], this certainly holds for GNSS satellite signals.

When there are two pseudo-ranges from the same satellite at two different frequencies, the fact that δR_i is inversely proportional to ω^2 means that

$$(\delta R_{i1}) f_1^2 = (\delta R_{i2}) f_2^2 \quad (\text{II.8})$$

In Eq. (II.8), we have used the frequency in Hz, rather than the angular frequency in rad/s. Then

$$\delta R_{i1} = \delta R_{i2} \frac{f_2^2}{f_1^2} \quad (\text{II.9})$$

The error in the pseudo-range due to ionospheric effects is given by the difference between the actual pseudo-range (R_{i1} or R_{i2}) and the pseudo-range in the absence of ionospheric effects (R_0). Then

$$R_1 - R_0 = (R_2 - R_0) \frac{f_2^2}{f_1^2} \quad (\text{II.10})$$

Therefore, the pseudo-range in the absence of ionospheric effects can be determined by solving Eq. (II.10) for R_0 ; the result is

$$R_0 = \frac{f_2^2 R_2 - f_1^2 R_1}{f_2^2 - f_1^2} \quad (\text{II.11})$$

From Eq. (II.1), the receiver clock correction is

$$\delta t_r = \frac{R_0 - r}{c} - \delta t_s \quad (\text{II.12})$$

References

- [Crawford] Crawford F. (1968), *Berkeley Physics Course, Vol. 3 WAVES*, Mc Graw-Hill, Inc., New York, USA.
- [Klouchar] Klouchar, J. (1987), *Ionospheric Time-Delay Algorithms for Single-Frequency GPS Users. IEEE Transactions on Aerospace and Electronic Systems (3)*, pp. 325-331.
- [Nicholas] Krall, N.A. and Trivelpiece A.W. (1973), *Principles of Plasma Physics*, McGraw-Hill.
- [Shohet] Juda Leon Shohet (1971), *The Plasma State*, Academic Press.

Bibliography

- [b-Giffard-1999] Giffard R. (1999), *Estimation of GPS ionospheric delay using L1 code and carrier phase observables, 31st Annual Precise Time and Time Interval Meeting*.

Appendix III

Time receiver autonomous integrity monitoring

GNSS receivers used for timing can employ time receiver autonomous integrity monitoring, or T-RAIM, algorithm to improve the robustness of the receiver. This is the timing application equivalent to the RAIM algorithms used for positioning applications. One condition must be met before T-RAIM can operate in a timing receiver: The position of the receiver must be known before the GNSS solution to which T-RAIM will be applied. Most timing applications involve stationary equipment, so once the position is known, it can be reused for subsequent timing solutions. The position can be entered at a surveyed position, or the timing receiver can have an initial position averaging mode. In the latter case many (for example 10 000) GNSS position fixes are averaged to remove noise. The receiver then transitions into timing mode and uses the stored position. This position can also be stored in non-volatile memory and used after a power cycle. In this case the operator will need to clear the position when moving the receiver.

Once the receiver enters timing mode each satellite is treated as an independent source of time. T-RAIM algorithms are performed for each GNSS solution as shown below:

1. For each satellite in view compute Δt_i , the receiver clock offset indicated by satellite i .
2. Compute average $\langle \Delta t \rangle$, over all satellites in view, for a single GNSS solution.
3. If the number of satellites < 3 , skip to step 6.
4. For each satellite, compute $|\Delta t_i - \langle \Delta t \rangle|$, so that the receiver clock offset from each satellite can be compared to the average offset.
5. If $|\Delta t_i - \langle \Delta t \rangle| > threshold$ for any of the satellites, then remove the satellite with the largest absolute difference from the solution and start over at step 1.
6. The receiver clock offset for this GNSS solution is $\langle \Delta t \rangle$

T-RAIM can remove bad satellites from the GNSS solution until there are two satellites left, if the time errors are all different. If the time errors are similar, T-RAIM can remove all bad satellites, as long as they are in the minority. Once a satellite is removed from a solution, then it may be quarantined for a period of time, for example one hour. The value of the threshold for identifying bad satellites must be chosen depending on the accuracy required for the application. There is a trade-off between the probability of failing to identify a bad satellite and generating false positives due to random variation. For example, a threshold of $1 \mu\text{s}$ might be used. Note that different GNSS receivers which implement T-RAIM, may have minor variations with respect to the algorithm described in this appendix.

Appendix IV

Solving GNSS equations to establish position and time

IV.1 Initial equation

The distance between each satellite i and the antenna can be computed using the pseudo-range equation for each satellite:

$$d_i = \sqrt{(x_i - x_a)^2 + (y_i - y_a)^2 + (z_i - z_a)^2} = c \cdot ((tr'_i - t_{offset}) - \Delta t_{ar} - t_i) \quad (IV.1_i)$$

This can be simplified to:

$$d_i = \sqrt{(x_i - x_a)^2 + (y_i - y_a)^2 + (z_i - z_a)^2} = c \cdot (\Delta t_i - t_{offset}) \quad (IV.2_i)$$

where:

t_i is the satellite i emission time in GPS epoch time. This information is carried over the radio signal from satellites to receiver and therefore known

tr'_i is the observed reception time in the receiver's local time

t_{offset} is the unknown real time offset of the receiver's local time relative to the GPS epoch.

The quantity t_{offset} is the difference between the time read simultaneously on the receiver in its local time, and the time read on a device that is purely aligned to GPS time.

Example 1: If the receiver's clock starts counting from value 0 second on 6 January 2018, it is "late" and t_{offset} will be negative. Its value will be the number of seconds since GPS epoch (Jan 6, 1980) $\cong -1.2$ billion of seconds.

Example 2: If receiver's time is almost good but too early by 1000 ns, $t_{offset} = +1000$ ns. When t is read as $k \cdot 1$ s, t' is simultaneously read as already $k \cdot 1$ s + 1000 ns on the receiver.

Δt_{ar} is the propagation delay between the antenna and receiver (strictly positive, including antenna electronics, receiver electronics, and a cable if present). In this appendix, it is considered as well-calibrated, known and perfect by the GNSS receiver.

Δt_i is noted as $(tr'_i - \Delta t_{ar} - t_i)$ to simplify the equations.

c is the speed of radio signals, supposedly known and perfect.

x_i, y_i, z_i describe the position of the satellite "i" in the ECI frame when the signal was sent (at t_i in the GPS epoch). This information is carried over the radio signal from satellites to receiver and therefore known.

x_a, y_a, z_a describe the position of the antenna in the ECI frame when the signal was received (at $tr_i - \Delta t_{ar} - t_{offset}$ in the GPS epoch), either a known or unknown value, as discussed below.

NOTES:

Relativistic effects are not covered in this appendix.

During self-survey, the GNSS receiver's antenna static position is considered as unknown, so the four unknowns to be solved are x_a, y_a, z_a and t_{offset} .

Once self-survey is completed, t_{offset} remains the only unknown to solve (but the position can be monitored and challenged).

The GNSS receiver's antenna position is located on the surface of the earth, so that it must be checked that $x_a^2 + y_a^2 + z_a^2$ is close to the square of the Earth radius approximately (6371 km)².

Squaring Eq. (IV.2_i) produces:

$$d_i^2 = (x_i - x_a)^2 + (y_i - y_a)^2 + (z_i - z_a)^2 = c^2 \cdot ((dt_i - t_{offset}))^2 \quad (IV.3_i)$$

IV.2 Different techniques to solve the equations with 4 unknowns

As shown above, the system of four equations with four unknowns is complex to solve. There are many techniques described in the literature. Some of these are:

- a purely mathematical closed-form solution;
- other approaches, for example using iterative estimations based on Newton's method.

All methods need at least four satellites in view to solve the four equations and compute the four unknowns. When more than four satellites are visible, techniques described in this technical report (e.g., SNR/carrier-to-noise power ratio masking, elevation masking, PDOP masking, T-RAIM masking) allow rejecting the faulty or redundant ones, and allow for checking and averaging the results from any group of four good satellites. A self-survey typically makes such estimations repetitively over a period of a few thousand seconds, before it averages the results and delivers the estimated static position.

A high-level view of the mathematical closed-form solution:

- Step 1: From the four equations Eq. (IV.3_i), replace three of them by the difference between the remaining equation and each of the three equations, thus removing all quadratic terms for the four unknowns (x_a , y_a , z_a , and t_{offset}).
- Step 2: Solve these three linear equations and find x_a , y_a and z_a as linear functions of t_{offset} .
- Step 3: Replace x_a , y_a and z_a , in Eq. (IV.3_1), by the above functions of t_{offset} obtained in Step 2, making it a second-order equation with only t_{offset} unknown.
- Step 4: Solve the second-order equation, and select the t_{offset} value that makes the right-hand side of Eq. (IV.1) and Eq. (IV.2) positive. In this step, t_{offset} is determined.
- Step 5: Compute x_a , y_a and z_a from t_{offset} . Check that this position is on the surface of the Earth. In this step x_a , y_a and z_a are determined in the same reference frame as the satellite position, i.e., ECI (Earth Centred Inertial coordinates).
- Step 6: Using the known time and rotation of the Earth, convert x_a , y_a and z_a into longitude/latitude/elevation for a human-friendly display.

This conversion needs to carefully consider the imperfect rotation of the Earth. Models representing this are available from the International Earth Rotation and Reference Systems Service (IERS). It should also consider the departure of the shape of the Earth from a perfect sphere and apply geodesic conversion. Popular conversions are the Helmert and Molodensky-Badekas transformations.

IV.3 Solving the equation with one unknown

After completion of the self-survey, the receiver's antenna position is considered as determined, and x_a , y_a and z_a are no longer considered as unknowns, but as input parameters. There are monitoring techniques to make sure that they remain valid, and can be enhanced later, but this is not described here.

The known position can be used in Eq. (IV.2_i), corresponding to each visible satellite. This is now a simplified, first order equation to solve.

When more than one satellite is available, techniques described in this technical report (e.g., SNR/carrier-to-noise power ratio masking, elevation masking, PDOP masking, T-RAIM masking)

allow rejecting the faulty or redundant ones, and allow for checking and averaging the results from any good satellite.

IV.4 Detailed computation for the solution of the four equations

- **Step 1:** From the four equations Eq. (IV.3_i), replace three of them by the difference between the remaining equation and each of the three equations, thus removing all quadratic terms for the four unknowns (x_a , y_a , z_a , and t_{offset}). This is done as follows:

$$x_i^2 + x_a^2 - 2 \cdot x_i \cdot x_a + y_i^2 + y_a^2 - 2 \cdot y_i \cdot y_a + z_i^2 + z_a^2 - 2 \cdot z_i \cdot z_a = c^2 \cdot \Delta t_i^2 + c^2 \cdot t_{offset}^2 - 2 \cdot c^2 \cdot \Delta t_i \cdot t_{offset} \quad (\text{IV.4}_i)$$

After creating the three differences between the first equation and each of the other three, the system becomes:

$$\text{Eq. (IV.5}_1) = \text{Eq. (IV.3}_1): (x_1 - x_a)^2 + (y_1 - y_a)^2 + (z_1 - z_a)^2 = c^2 \cdot ((\Delta t_1 - t_{offset}))^2$$

$$\text{Eq. (IV.5}_2) = \frac{1}{2} \cdot (\text{Eq. (IV.4}_1) - \text{Eq. (IV.4}_2)) :$$

$$(x_2 - x_1) \cdot x_a + (y_2 - y_1) \cdot y_a + (z_2 - z_1) \cdot z_a = c^2 \cdot t_{offset}(\Delta t_2 - \Delta t_1) + \frac{1}{2}[(x_2^2 + y_2^2 + z_2^2 - c^2 \Delta t_2^2) - (x_1^2 + y_1^2 + z_1^2 - c^2 \Delta t_1^2)]$$

$$\text{Eq. (IV.5}_3) = \frac{1}{2} \cdot (\text{Eq. (IV.4}_1) - \text{Eq. (IV.4}_3)) :$$

$$(x_3 - x_1) \cdot x_a + (y_3 - y_1) \cdot y_a + (z_3 - z_1) \cdot z_a = c^2 \cdot t_{offset}(\Delta t_3 - \Delta t_1) + \frac{1}{2}[(x_3^2 + y_3^2 + z_3^2 - c^2 \Delta t_3^2) - (x_1^2 + y_1^2 + z_1^2 - c^2 \Delta t_1^2)]$$

$$\text{Eq. (IV.5}_4) = \frac{1}{2} \cdot (\text{Eq. (IV.4}_1) - \text{Eq. (IV.4}_4)) :$$

$$(x_4 - x_1) \cdot x_a + (y_4 - y_1) \cdot y_a + (z_4 - z_1) \cdot z_a = c^2 \cdot t_{offset}(\Delta t_4 - \Delta t_1) + \frac{1}{2}[(x_4^2 + y_4^2 + z_4^2 - c^2 \Delta t_4^2) - (x_1^2 + y_1^2 + z_1^2 - c^2 \Delta t_1^2)]$$

- **Step 2:** Solve these three equations and find x_a , y_a and z_a as linear functions of t_{offset} .

In doing this, the above system can be represented as a matrix and vector:

$$\mathbf{A} \mathbf{p}_a = c^2 \cdot t_{offset} \cdot \Delta \mathbf{t} + \Delta \mathbf{M} \quad (\text{IV.6})$$

where:

\mathbf{A} is the matrix of the satellite positions

$$\begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{bmatrix}$$

\mathbf{p}_a is the unknown antenna position vector

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

$\Delta \mathbf{t}$ is the vector of the time differences

$$\begin{bmatrix} \Delta t_2 - \Delta t_1 \\ \Delta t_3 - \Delta t_1 \\ \Delta t_4 - \Delta t_1 \end{bmatrix}$$

$\Delta \mathbf{M}$ is the vector of the satellite pseudo-range differences

$$\begin{bmatrix} \frac{1}{2}[(x_2^2 + y_2^2 + z_2^2 - c^2 \cdot \Delta t_2^2) - (x_1^2 + y_1^2 + z_1^2 - c^2 \cdot \Delta t_1^2)] \\ \frac{1}{2}[(x_3^2 + y_3^2 + z_3^2 - c^2 \cdot \Delta t_3^2) - (x_1^2 + y_1^2 + z_1^2 - c^2 \cdot \Delta t_1^2)] \\ \frac{1}{2}[(x_4^2 + y_4^2 + z_4^2 - c^2 \cdot \Delta t_4^2) - (x_1^2 + y_1^2 + z_1^2 - c^2 \cdot \Delta t_1^2)] \end{bmatrix}$$

The traditional resolution of this equation consists in multiplying by the inverted matrix:

$$\mathbf{p}_a = \mathbf{A}^{-1} \cdot \Delta \mathbf{t} \cdot c^2 \cdot t_{offset} + \mathbf{A}^{-1} + \Delta \mathbf{M} \quad (\text{IV.7})$$

So finally, the system gets simplified, with x_a , y_a and z_a as a linear function of t_{offset} :

$$\text{Eq. (IV.8_1) = Eq. (IV.3_1): } (x_1 - x_a)^2 + (y_1 - y_a)^2 + (z_1 - z_a)^2 = c^2 \cdot ((\Delta t_1 - t_{offset}))^2$$

$$\text{Eq. (IV.8_2): } x_a = k_a \cdot t_{offset} + k_b$$

$$\text{Eq. (IV.8_3): } y_a = k_c \cdot t_{offset} + k_d$$

$$\text{Eq. (IV.8_4): } z_a = k_e \cdot t_{offset} + k_f$$

NOTE – $k_a \dots k_f$ are functions of the known quantities and are not provided in detail here.

– **Step 3:** replace x_a , y_a and z_a by the above formula using t_{offset} in Eq. (IV.3_1), making it a second order equation with only t_{offset} unknown.

$$\begin{aligned} & (x_1 - k_a \cdot t_{offset} - k_b)^2 + (y_1 - k_c \cdot t_{offset} - k_d)^2 + (z_1 - k_e \cdot t_{offset} - k_f)^2 = c^2 \cdot \\ & ((\Delta t_1 - t_{offset}))^2 \end{aligned} \quad (\text{IV.9})$$

This is equivalent to:

$$\begin{aligned} & (k_a^2 + k_c^2 + k_e^2 - c^2) \cdot t_{offset}^2 \\ & + 2 \cdot [k_a \cdot (k_b - x_1) + k_c \cdot (k_d - y_1) + k_e \cdot (k_f - z_1) + c^2 \cdot \Delta t_1] \cdot t_{offset} \\ & + [(k_b - x_1)^2 + (k_d - y_1)^2 + (k_f - z_1)^2] - c^2 \cdot \Delta t_1^2 \\ & = 0 \end{aligned} \quad (\text{IV.10})$$

or:

$$A \cdot t_{offset}^2 + B \cdot t_{offset} + C = 0 \quad (\text{IV.11})$$

where

$$\begin{aligned} A &= (k_a^2 + k_c^2 + k_e^2 - c^2), \\ B &= 2 \cdot [k_a \cdot (k_b - x_1) + k_c \cdot (k_d - y_1) + k_e \cdot (k_f - z_1) + c^2 \cdot \Delta t_1], \\ C &= ((k_b - x_1)^2 + (k_d - y_1)^2 + (k_f - z_1)^2 - c^2 \cdot \Delta t_1^2) \end{aligned}$$

Step 4: Solve the second order equation, Eq. (IV.11). The solution is:

Provided

$$B^2 - 4 \cdot A \cdot C > 0 \quad (\text{IV.12})$$

the solution is

$$t_{offset} = \frac{-B \pm \sqrt{B^2 - 4 \cdot A \cdot C}}{2A} \quad (\text{IV.13})$$

Select the one t_{offset} value that makes the right part of Eq. (IV.1) and Eq. (IV.2) positive. In this step, t_{offset} is determined.

- **Step 5:** Compute x_a , y_a and z_a from t_{offset} using Eq. (IV.8_2..4). Check that this position is on the surface of the Earth. In this step, x_a , y_a and z_a are determined relative to the ECI reference frame.
- **Step 6:** For display only, compute longitude, latitude and elevation above sea level from x_a , y_a , z_a .

This last computation is not described here.

Appendix V

The effect of multiple reflections within the antenna cable

The GPS code is embedded in the signal using biphasic modulation of the carrier. The receiver locks to the signal by locally generating the C/A code for the particular satellite and performing servo lock to maximize correlation with the received code. With no multipath reflections, the correlation code has a simple maximum peak. However, each reflected signal distorts the correlation code, pulling the lock point and creating a bias. With multipath reflections before the antenna, the arrival phases of the reflected signals change as the satellite moves overhead. However, due to imperfect impedance matching at the receiver and antenna, reflected signals can enter the receiver after bouncing from the receiver to the antenna and back to the receiver, transiting the antenna cable two times more than the direct signal. Figure V.1 illustrates this.

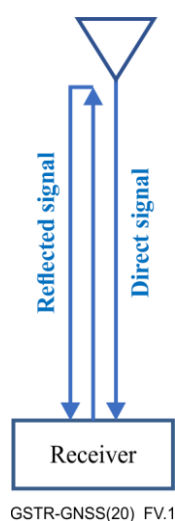


Figure V.1 – A reflected signal in an antenna cable

Because the code edges occur in the carrier, a change in the apparent lock point of the receiver can cause a non-linear delay change in the apparent time produced by the receiver. This problem is discussed in the two references listed below.

This effect can cause large changes in the apparent delay of the receiver system without any apparent warning, due to small changes in the electrical length of the reflected signal. A change in the apparent electrical delay of the antenna cable of 115 ps can cause a time change in the receiver of 50 ns. The change in the reflected signal in the cable can happen slowly over time and appear to simply be a change in the time of the reference clock.

As shown in Figure V.2 (copied from the second reference), with the bad luck of the wrong phase and cable length for the reflected signal, even with the reflected signal 25 dB below the direct signal, the change of $\frac{1}{4}$ the L1 cycle, 115 ps, produces a change in the arrival time of the reflected signal of $\frac{1}{2}$ a cycle, which in turn can produce a change in the apparent delay of the receiver of about 50 ns. This can be seen in the figure where the error changes from the top of the envelope to the bottom, due to a change in half a cycle of the carrier.

This 50 ns change in the timing error happens if the total delay of the reflected signal is about 0.5 of the pseudorandom noise chip, or about 500 ns, which happens if the one-way delay is about 250 ns.

This is not an uncommon length for an antenna cable. If, for example, the speed of propagation in the cable is 60% the speed of light, this would correspond to a cable of length 50 m.

A chip is a pulse of a direct-sequence spread spectrum code used in code division multiple access channel access techniques. The chip is typically a rectangular pulse of +1 or -1 amplitude, which is multiplied by a data sequence (similarly +1 or -1 representing the message bits) and then by a carrier waveform to make the transmitted signal. The chips are therefore just the bit sequence out of the code generator; they are called chips to avoid confusing them with message bits.

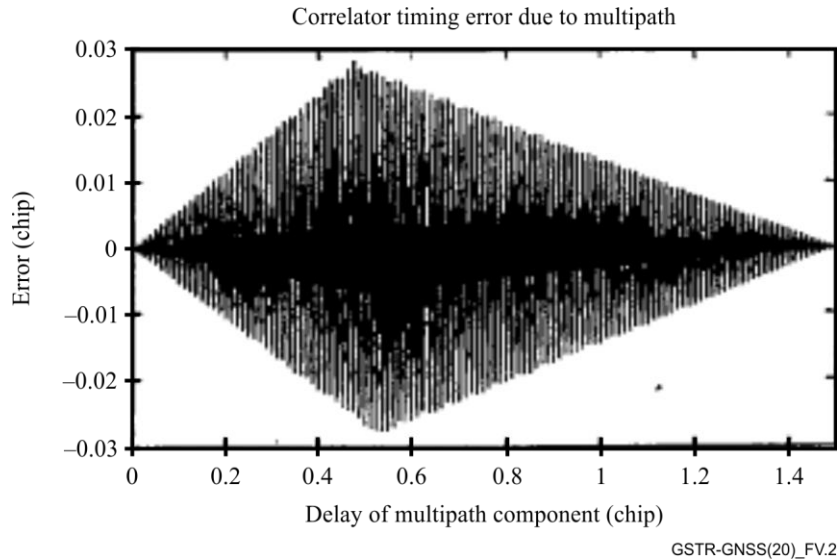


Figure V.2 – GPS correlator timing error due to a multipath signal 25 dB below the direct signal as a function of the delay

Even with a reflected signal 35 dB below the direct signal, a change in delay of almost 20 ns is possible. Figure V.3 (also taken from the second reference) illustrates this.

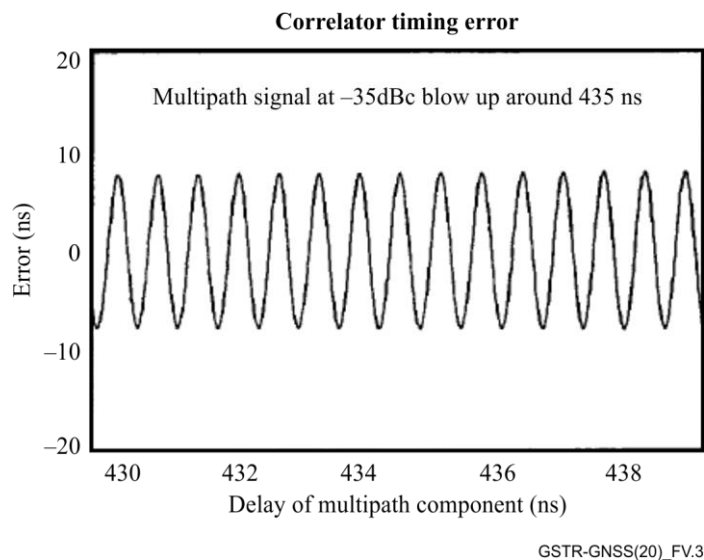


Figure V.3 – GPS correlator timing error due to a multipath signal 35 dB below the direct signal. The window shows the interval of maximum variation

Thus, it is important that the impedance matching of the antenna cable with the antenna and receiver terminations be controlled to limit the power of a reflected signal.

References

- [Ascarrunz] Ascarrunz F.G., Parker T.E., Jefferts S.R. (1999), *Group-delay errors due to coherent interference*, *Proc. European Frequency and Time Forum*, available online from <https://tf.nist.gov/general/pdf/1313.pdf> .
- [Weiss] Weiss M.A., Ascarrunz F.G., Parker T., Zhang V., Gao X. (1999), *Effects of antenna cables on GPS timing receivers*, *Proc. European Frequency and Time Forum*, available online from <https://tf.nist.gov/general/pdf/1384.pdf>.

Appendix VI

Satellite common-view

VI.1 Satellite common-view

Satellite common-view refers to two places anywhere on earth receiving the same navigation satellite signal at the same time, which can eliminate the common error on the two propagation paths and realize time comparison between the two places.

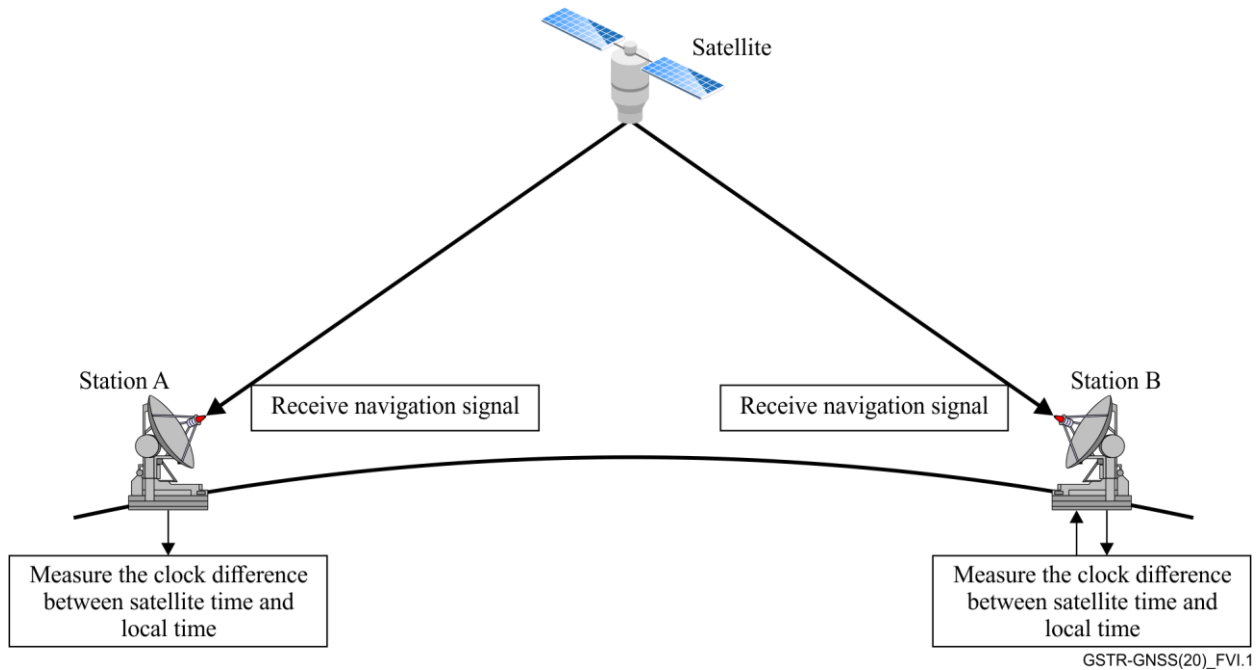


Figure VI.1 – Principle of satellite common-view

The basic principle of satellite common-view is shown in the above figure. When the difference between station A and station B needs to be calculated, the two stations need to observe the same satellite S . $T(A)$ is the time of station A, $T(B)$ is the time of station B, $T(S)$ is the satellite time, $d(A)$ is the delay between station A and satellite S and $d(B)$ is the delay between station B and satellite S ; the time difference between station and satellite for A and B is as below:

$$\Delta T_{AS} = T(A) - T(S) - d(A)$$

$$\Delta T_{BS} = T(B) - T(S) - d(B)$$

Then the time deviation between station A and station B can be calculated as below:

$$\Delta T_{AB} = \Delta T_{AS} - \Delta T_{BS} = (T(A) - T(B)) - (d(A) - d(B))$$

It can be seen from the above equation that satellite common-view can eliminate the influence of satellite clock error and most of the path delay, which will improve the accuracy of the relative clock difference between the two places and realize high precision time comparison.

VI.2 Satellite common-view for monitoring

According to the satellite common-view principle, the same satellite needs to be observed at the same time. Therefore, monitoring with satellite common-view requires the following data: satellite, system time, satellite number and time offset.

After the synchronization network is established, any two nodes can be monitored by satellite common-view, as is shown below.

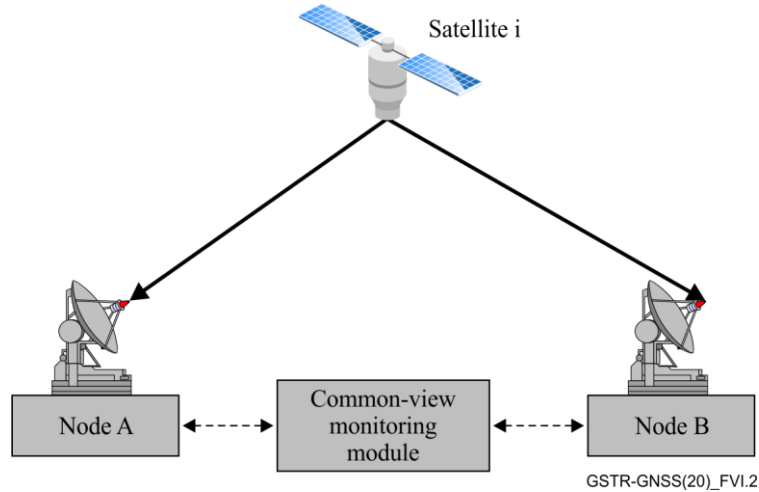


Figure VI.2 – Any two nodes can be monitored by satellite common-view at the same time

GNSS satellites continuously broadcast microwave signals every moment, and the main signal components of these signals are ranging signals, used to measure the distance to the satellite, and navigation messages. The navigation messages include ephemeris data, used to calculate the position of each satellite in orbit, the information about the time and status of the entire satellite constellation, called the almanac, and other data such as satellite system time, clock correction parameters and ionospheric delay correction parameters.

When node A receives the GPS signal, the satellite receiver will parse the signal. The pseudo-range ρ_{iA} is acquired according to the ranging signal measurement, and since the form of the ranging signal of each satellite is unique, the satellite number information can be obtained. Meanwhile, the satellite receiver obtains the satellite system time and other information used to calculate the time offset from the navigation message, such as the coordinates of the satellite position resolved from the ephemeris data, thereby calculating the time offset by the following formula:

$$c \cdot \delta t_A = \rho_{iA} - \sqrt{(x_i - x_A)^2 + (y_i - y_A)^2 + (z_i - z_A)^2} + c\delta t_i - D_{iA}^{ion} - D_{iA}^{trop} - D_{iA}^{other}$$

where:

- c is the speed of light.
- ρ_{iA} is a pseudo-range (see clause I.3 in Appendix I), and the satellite receiver acquires it by processing a ranging signal.
- The node A coordinate position is (x_A, y_A, z_A) . This data is known after positioning in advance and the Kalman filtering process.
- The satellite coordinate position (x_i, y_i, z_i) is obtained by the ephemeris information in the navigation message.

- δ_i is the satellite clock difference. This value can be obtained from the satellite clock correction parameters in the navigation message.
- The ionospheric delay D_{iA}^{ion} and tropospheric delay D_{iA}^{trop} can be obtained from the ionospheric delay correction parameters in the satellite navigation message or related mathematical models. The GNSS receiver may have its own mathematical model for processing, or other methods applied to process D_{iA}^{ion} or D_{iA}^{trop} .
- D_{iA}^{other} refers to other delays, which can be corrected according to relevant mathematical models or other means.

Finally, the time offset between the local time of node A resolved from the signal of satellite i and the GNSS system time can be obtained from the GNSS receiver. Thus, for node A, three pieces of data which is necessary for satellite common-view detection are obtained: satellite system time, satellite number and time offset. Similarly, for node B, the corresponding three data can also be obtained.

Node A and node B resolve the satellite system time, satellite number and time offset (calculated once per second) and perform data processing (such as performing least squares linear fitting smoothing on 60 data per minute to eliminate observation noise). The above information is then sent to the common-view monitoring module once a minute through the terrestrial network. This module can exist independently out of node A and node B, or can be integrated in the node.

After receiving the message information sent by the two nodes, the common-view monitoring module will analyse the satellite system time, satellite number and corresponding time offset information in the message, so that the time offset information of node A and node B under the same satellite system time and the same satellite number will be further processed (i.e., subtracted), to obtain the time offset value between the two nodes. Thus, the time offset and frequency offset between the two nodes can be monitored.

Appendix VII

The effect of multipath within the receiver signal processing

VII.1 Visible (LOS) satellite signals

Multipath signals accompany direct signals from visible satellites as shown in Figure VII.1. The peak timing position of the correlator output is almost the same as that for the direct signal, so time synchronization accuracy is barely affected.

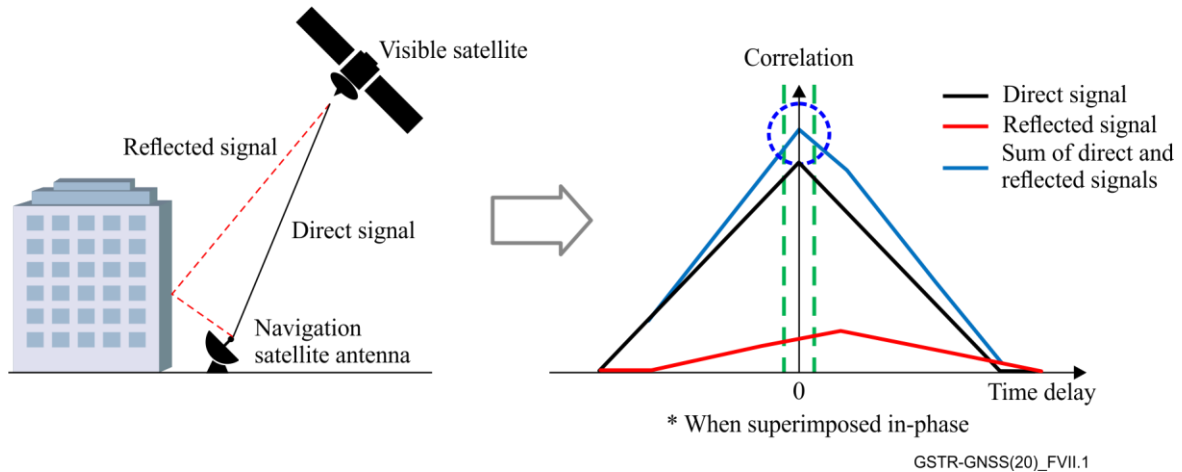


Figure VII.1 – Effect of multipath signals of visible (LOS) satellites

In the LOS case there are signal processing techniques to isolate the direct signal from the reflected signal.

VII.2 Non-visible (NLOS) satellite signals

Multipath signals from non-visible (NLOS) satellites are without accompanying the direct signal as shown in Figure VII.2. The peak timing position of correlator output is different from that of the direct signal due to propagation delay, with a large effect on time synchronization accuracy.

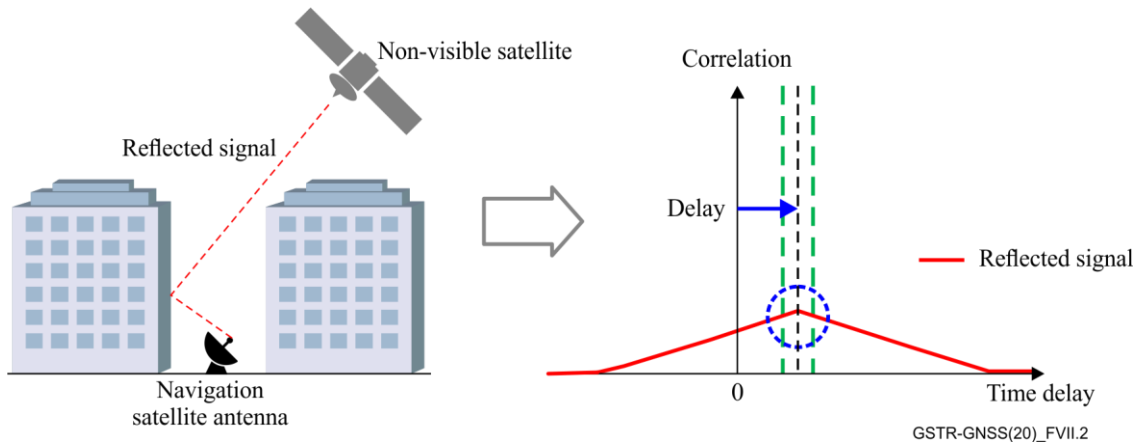


Figure VII.2 – Effect of multipath signals of non-visible (NLOS) satellites

VII.3 PRTC time error measurement methods in multipath environments

Figures VII.3 and VII.4 show one way of testing time error performance in a non-ideal GNSS signal reception environment. Such conditions are predominantly in urban canyon areas where buildings limit open sky visibility and therefore, direct satellite signals are negatively affected by multipath signals generated by surrounding structures.

A 3D model of an urban environment may be built, and then a 3D ray-trace simulator used to calculate all the possible paths from a GNSS satellite to the reception point. The ray-tracing model determines where the direct signal is obscured, and where it is reflected or diffracted by buildings or other objects in the 3D structural model.

The ray tracing model is used as input to the GNSS simulator to enable it to generate both the direct and reflected or diffracted signals. The simulator needs to generate multiple signal replicas per satellite to reproduce the direct and multipath signals based on the calculated multipath profile. The process of generating multiple satellite signal replicas can be, for example, implemented with an array of signal transmitters or software-defined radio.

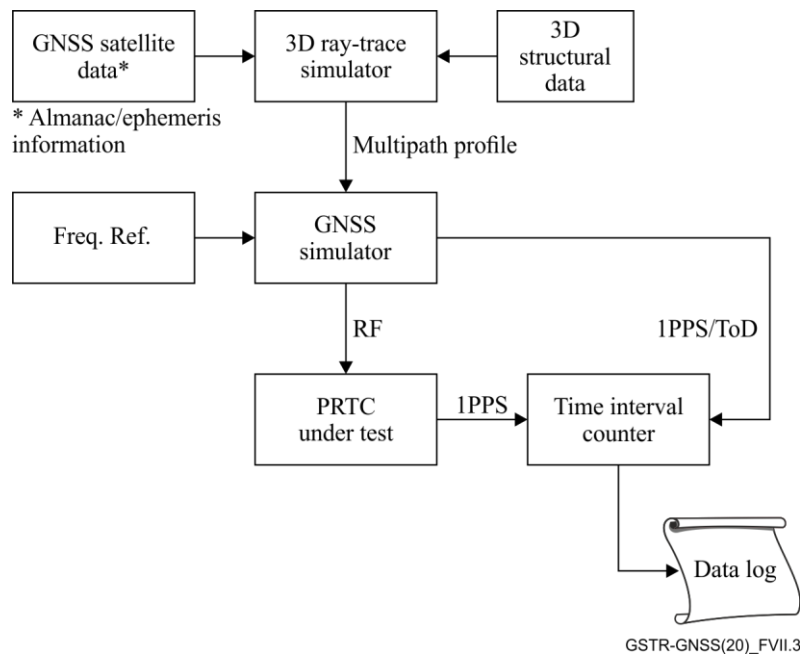


Figure VII.3 – Comparing time accuracy of a PRTC against a GNSS simulator

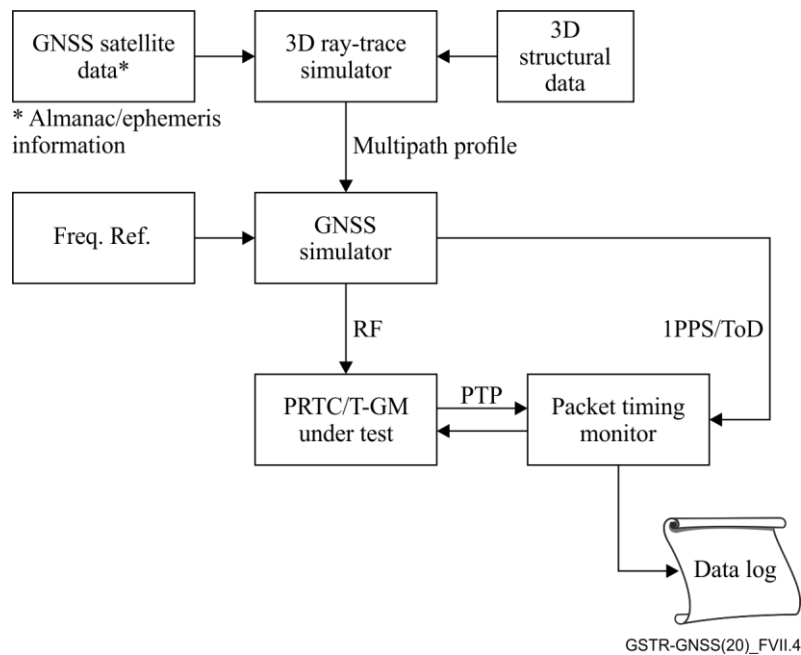


Figure VII.4 – Comparing time accuracy of a PRTC and T-GM against a GNSS simulator

For testing PRTCs, the GNSS simulator needs to have a frequency reference of better stability than the PRTC itself. The 1PPS output of the simulator is phase locked to the incoming frequency reference and aligned to the 1s boundary of the RF signal before impairment by the multipath model. The accuracy of the RF to 1PPS alignment should be defined by the GNSS simulator manufacturer and the best practice would be to have the simulator periodically calibrated to keep uncertainties at a minimum. The ToD associated with the 1PPS from the GNSS simulator is not required to do the time error measurement. It is used to verify the PTP second.