RECOMMENDATION ITU-R BO.1212*

Calculation of total interference between geostationary-satellite networks in the broadcasting-satellite service

(1995)

The ITU Radiocommunication Assembly,

considering

a) that successful implementation of satellite systems in the World Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service (Geneva, 1977) (WARC BS-77) and the First Session of the World Administrative Radio Conference on the Use of the Geostationary-Satellite Orbit and the Planning of the Space Services Utilizing It (WARC ORB-85) broadcasting-satellite service (BSS) plans is dependent upon accurate calculation of mutual interference between satellite networks;

b) that geostationary-satellite networks in the BSS operate in the same frequency bands;

c) that interference between networks in the BSS contributes to noise in the network;

d) that it is necessary to protect a network in the BSS from interference by other source networks;

e) that, due to increased orbit occupancy, the detailed estimation of mutual interference between satellite networks, requires more accurate values of polarization discrimination in order to take account of the use of different or identical polarizations by wanted and interfering systems,

recommends

1 that to calculate the total interference between two satellite networks considered, the method described in Annex 1 should be used.

ANNEX 1

Calculation of total interference

When evaluating the power produced at a given point by a single satellite (downlink) or at a given satellite location by an earth-station transmitter (uplink) the concept of an equivalent gain for each partial link may be employed.

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* Radiocommunication Study Group 6 made editorial amendments to this Recommendation in 2001 in accordance with Resolution ITU-R 44.

There are two antennas involved in each partial link, and these have both co-polar and cross-polar transmission and reception characteristics. In addition, atmospheric propagation effects, represented principally by co-polar attenuation and cross-polar discrimination, influence the net signal level.

The equivalent gain (as a power ratio) for one partial link can be represented by the following approximation:

$$G = G_1 \cos^2 \beta + G_2 \sin^2 \beta$$

$$G_1 = G_{tp}\, G_{rp}\, A + G_{tc}\, G_{rc}\, A + G_{tp}\, G_{rc}\, A\, X + G_{tc}\, G_{rp}\, A\, X \qquad (1)$$

$$G_2 = \left(\sqrt{G_{tp}\, G_{rc}\, A} + \sqrt{G_{tc}\, G_{rp}\, A}\right)^2 + G_{tp}\, G_{rp}\, A\, X + G_{tc}\, G_{rc}\, A\, X$$

where:

β: for linear polarization, is the relative alignment angle between the received signal polarization plane and the plane of polarization of the receive antenna;

for circular polarization, $\beta = 0°$ is assumed to correspond to co-polar transmission and reception and $\beta = 90°$ is assumed to correspond to mutually cross-polarized transmission and reception;

for cases of differing polarizations (e.g. linearly polarized wanted receive antenna and circularly polarized interfering transmissions, or *vice versa*), $\beta = 45°$

G_{tp}: co-polar gain characteristic of the transmit antenna expressed as a power ratio (Recommendation ITU-R BO.652)

G_{tc}: cross-polar gain characteristic of the transmit antenna expressed as a power ratio

G_{rp}: co-polar gain characteristic of the receive antenna expressed as a power ratio (Recommendation ITU-R BO.652)

G_{rc}: cross-polar gain characteristic of the receive antenna expressed as a power ratio

A: co-polar attenuation on the interfering partial link (as a power ratio ≤ 1)

X: cross-polar discrimination on the interfering partial link (as a power ratio ≤ 1)

$$X = 10^{-0.1\left[30 \log f - 40 \log (\cos \varepsilon_s) - 20 \log (-10 \log A)\right]} \qquad \text{for } 5° \le \varepsilon_s \le 60°$$

where:

f: frequency (GHz)

ε_s: satellite elevation angle as seen from the earth station (degrees).

For $\varepsilon_s > 60°$, use $\varepsilon_s = 60°$ in calculating the value of X.

(See Appendix 1 for derivation of the relative alignment angle, β.)

In the expression for G_1, power summation of the terms is assumed throughout. Near the main axis of the wanted transmission, a voltage addition of the first two terms may be more appropriate due to phase alignment while away from this axis random effects dictate power addition. However, since the second term is insignificant near this axis the assumption of power addition does not compromise the approximation. Atmospheric depolarization is a random effect thus the last two terms are power summed.

In the expression for G_2, voltage addition of the first two terms is assumed since, near axis, either term could be dominant and phase alignment of these terms would dictate voltage addition. Away from this main axis the third and fourth terms become the dominant contribution; thus, although a power addition of the first two terms is warranted, in this region as for the G_1 discussion, the validity of the assumed model is not unduly compromised by maintaining voltage addition in all regions. Since the transition from voltage addition near axis to power addition off-axis is nebulous, the above expressions, in view of the arguments presented, would appear to be a reasonable compromise between accuracy and simplicity.

Using the equivalent gain concept, the wanted carrier power, C, or the single-entry interfering power, I, on each partial link is simply given by:

$$C \text{ (or } I) = P_T - L_{FS} - L_{CA} + G \qquad \text{dBW} \tag{2}$$

where:

P_T: wanted (interfering) transmitting antenna power (dBW)

L_{FS}: free-space loss on the wanted (interfering) link (dB)

L_{CA}: clear-air absorption on the wanted (interfering) link (dB)

G: equivalent gain on the wanted (interfering) link (dB).

The aggregate interference power is obtained by adding the powers so calculated for all interferers. The ratio of the desired signal power to the aggregate interference power is the downlink aggregate carrier-to-interference ratio, C/I. The up-link aggregate interference power and C/I are obtained in a similar way, and the two aggregate values of C/I are then combined to obtain the total aggregate C/I.

If the ratio of the wanted carrier power to the power of an interfering signal, where both powers are calculated using equation (2), is to be evaluated for the worst case, such parameters as satellite station-keeping tolerances, satellite antenna pointing errors, and propagation conditions must be taken into account. The station-keeping and satellite transmit-antenna beam errors which should be included are those which result in the lowest receive level of the wanted signal and the highest receive level of the interfering satellite signal. When the interfering satellite is at a lower elevation angle than the wanted satellite, worst-case interference conditions usually occur during clear-sky operation. Conversely, if the interfering satellite is at a higher elevation angle, worst-case interference usually occurs during heavy rain conditions.

APPENDIX 1

TO ANNEX 1

Derivation of the relative alignment angle β for linear polarization

This Appendix defines the polarization angle of a linearly polarized radiowave and outlines the method for calculating polarization angles and relative alignment angles for both the downlink and feeder link interference cases. Calculation of relative alignment angles are necessary for determining the equivalent gain as defined by equation (1).

1 Definition of principle and cross-polarized components of a linearly polarized radiowave

In general, the polarization of a radiated electromagnetic wave in a given direction is defined to be the curve traced by the instantaneous electric field vector, at a fixed location and at a given frequency, in a plane perpendicular to the direction of propagation as observed along the direction of propagation. When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain. In practice, polarization of the radiated energy varies with the direction from the centre of the antenna, so that different parts of the pattern may have different polarizations. Polarization may be classified as linear, circular or elliptical. If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be linearly polarized. In the most general case, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized. Linear and circular polarizations are special cases of elliptical when the ellipse becomes a straight line or a circle, respectively. For the interference calculations, we are interested in the far-field polarization of the antenna where the E-field component in the direction of propagation is negligible so that the net electric field vector can be resolved into two (time-varying) orthogonal components that lie in a plane normal to the outward radial direction of propagation. In the case of linear polarization, the reference directions of these orthogonal components must first be defined before one can define a polarization angle. One of these reference directions is designated as the principle or main polarization component direction while the orthogonal reference direction is designated as the cross-polarization component direction. Surprisingly, there is no universally accepted definition for these reference directions. Some alternative definitions of principle and cross-polarization component directions are discussed in Arthur C. Ludwig's paper, "The Definition of Cross Polarization", in the IEEE Transactions on Antennas and Propagation, January 1973. In his paper, Ludwig derives expressions for the unit vectors for three different cross-polarization definitions in terms of a spherical antenna pattern coordinate system, which is the coordinate system usually adopted for antenna measurements. We briefly describe these three definitions below. In this Appendix the unit vector $\boldsymbol{u_p}$ represents the reference direction for the principle polarization component of the electric field vector while $\boldsymbol{u_c}$ represents the direction of the cross-polarized component. It is helpful to first review the transformation of vectors among rectangular, cylindrical and spherical coordinate systems.

1.1 Vector transformation among rectangular, cylindrical and spherical coordinate systems

Figure 1 shows the three coordinate systems and their associated unit vectors. The transformation matrix for transforming a vector **A** in rectangular components (A_x, A_y, A_z) to cylindrical components (A_p, A_φ, A_z) is:

$$M_{rc} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The transformation matrix for transforming a vector **A** in cylindrical components (A_p, A_φ, A_z) to spherical components (A_r, A_θ, A_φ) is given by:

$$M_{cs} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \tag{4}$$

The transformation matrix for transforming a vector **A** in rectangular components (A_x, A_y, A_z) to spherical components (A_r, A_θ, A_φ) is then:

$$M_{rs} = M_{cs}\,M_{rc} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \tag{5}$$

so that, in terms of components:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \tag{6}$$

Because the matrix is orthogonal, the transformation matrix for transforming from spherical (A_r, A_θ, A_φ) to rectangular (A_x, A_y, A_z) components is simply the transposed matrix:

$$M_{sr} = \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \tag{7}$$

so that:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} \tag{8}$$

The unit vectors $\boldsymbol{u_r}$, $\boldsymbol{u_\theta}$, and $\boldsymbol{u_\varphi}$ of the spherical coordinate system are in spherical coordinates

$$\boldsymbol{u_r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \boldsymbol{u_\theta} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \boldsymbol{u_\varphi} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

$$\boldsymbol{u_r} = \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix} \quad \boldsymbol{u_\theta} = \begin{bmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{bmatrix} \quad \boldsymbol{u_\varphi} = \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} \tag{10}$$

in rectangular coordinates.

1.2 Alternative definitions of principal polarization and cross-polarization reference directions

Ludwig describes three definitions for the cross polarization by deriving unit vector $\boldsymbol{i_{ref}}$ and $\boldsymbol{i_{cross}}$ (which we have renamed $\boldsymbol{u_p}$ and $\boldsymbol{u_c}$) such that the dot product of the electric field vector $\underline{\mathbf{E}}$ (t, θ, φ) along some direction (θ, φ) in the far-field antenna pattern with these unit vectors defines the principal and cross-polarization components, respectively. In the direction specified by the spherical coordinate angles (θ, φ), the principal and cross-polarization components of the electric field vector are therefore given by:

$$E_p(\theta, \varphi) = \underline{\mathbf{E}} \cdot \boldsymbol{u_p} \qquad E_c(\theta, \varphi) = \underline{\mathbf{E}} \cdot \boldsymbol{u_c} \tag{11}$$

(Note that, in general, $\underline{\mathbf{E}}$, $\boldsymbol{u_p}$ and $\boldsymbol{u_c}$ will themselves vary with θ and φ.)

Figure 2 illustrates the polarization patterns corresponding to the three definitions for the case in which the antenna is transmitting horizontal polarization along its main beam axis.

In the first definition, the reference unit vector $\boldsymbol{u_p}$ is simply taken to be one of the rectangular basis vectors of the antenna pattern coordinate system while $\boldsymbol{u_c}$ is another one of the basis unit vectors. For example, we can define:

$$\boldsymbol{u_p} = \boldsymbol{y_a} \qquad \boldsymbol{u_c} = \boldsymbol{x_a} \tag{12}$$

where $\boldsymbol{y_a}$ and $\boldsymbol{x_a}$ are unit vectors in the positive y and x directions.

From the transformation matrices above the spherical coordinate components of these vectors are given by:

$$\boldsymbol{u_p} = \boldsymbol{y_a} = \sin\theta \cdot \sin\varphi \cdot \boldsymbol{u_r} + \cos\theta \cdot \sin\varphi \cdot \boldsymbol{u_\theta} + \cos\varphi \cdot \boldsymbol{u_\varphi} \tag{13}$$

$$\boldsymbol{u_c} = \boldsymbol{x_a} = \sin\theta \cdot \cos\varphi \cdot \boldsymbol{u_r} + \cos\theta \cdot \cos\varphi \cdot \boldsymbol{u_\theta} - \sin\varphi \cdot \boldsymbol{u_\varphi} \tag{14}$$

Ludwig notes that this definition leads to inaccuracies, since in practice, the polarization of the radiated field does vary with direction from the centre of the antenna and that the far-field of the antenna is not planar, but tangent to a spherical surface. Ludwig’s second and third definitions of polarization therefore involve unit vectors which are tangent to a sphere. In his second definition, the principle polarization direction is chosen to be one of the spherical coordinate unit vectors while the cross-polarization direction is chosen to be one of the other spherical unit vectors. For example, we can choose:

$$\boldsymbol{u_p} = \boldsymbol{u}_\varphi \qquad \boldsymbol{u_c} = \boldsymbol{u}_\theta \tag{15}$$

In Ludwig's third definition, the principal and cross-polarization component directions are defined according to how one usually measures the polarization pattern of an antenna. The standard measurement method is described in Fig. 3. The probe polarization angle β (angle between $\boldsymbol{u}_\varphi$ and $\boldsymbol{u_p}$) is measured from $\boldsymbol{u}_\varphi$ towards $\boldsymbol{u}_\theta$. In the case where the transmitted field is linear horizontal polarization (i.e. in the +y direction) along the boresight ($\theta = 0°$), β turns out to be equal to φ. Therefore the principal and cross-polarization components in the direction (θ,φ) are given by:

$$E_p(t) = \underline{\mathbf{E}}(t) \cdot \boldsymbol{u_p} \qquad E_c(t) = \underline{\mathbf{E}}(t) \cdot \boldsymbol{u_c} \tag{16}$$

so that the electric field $\underline{\mathbf{E}}(t)$, in that direction can be expressed as:

$$\underline{\mathbf{E}}(t) = E_p(t) \cdot \boldsymbol{u_p} + E_c(t) \cdot \boldsymbol{u_c} = E_{pm} \cdot \cos(\omega t) \cdot \boldsymbol{u_p} + E_{cm} \cdot \cos(\omega t + \delta) \cdot \boldsymbol{u_c} \tag{17}$$

This is the general expression for an elliptically polarized wave. Note that in order for $\underline{\mathbf{E}}(t)$ to be linearly polarized the time phase δ between the two orthogonal linear components must be zero (or an integer multiple of π). The amplitudes of the components E_{pm} and E_{cm} however, need not be equal.

The principal and cross-polarization unit vectors, $\boldsymbol{u_p}$ and $\boldsymbol{u_c}$ can be expressed in terms of the spherical coordinate unit vectors, $\boldsymbol{u}_\theta$ and $\boldsymbol{u}_\varphi$, and the angle $\beta = \varphi$ (when the transmitted field is polarized in the +y direction at $\theta = 0°$) by:

$$\boldsymbol{u_p} = \sin\varphi \cdot \boldsymbol{u}_\theta + \cos\varphi \cdot \boldsymbol{u}_\varphi \tag{18}$$

$$\boldsymbol{u_c} = \cos\varphi \cdot \boldsymbol{u}_\theta - \sin\varphi \cdot \boldsymbol{u}_\varphi \tag{19}$$

Finally, by expressing $\boldsymbol{u}_\theta$ and $\boldsymbol{u}_\varphi$ in rectangular coordinates as shown above, $\boldsymbol{u_p}$ and $\boldsymbol{u_c}$ can be expressed in terms of the antenna coordinate system's rectangular unit vectors ($\boldsymbol{x_a}, \boldsymbol{y_a}, \boldsymbol{z_a}$) as:

$$\boldsymbol{u_p} = \sin\varphi \begin{bmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{bmatrix} + \cos\varphi \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\varphi\sin\varphi - \sin\varphi\cos\varphi \\ \cos\theta\sin(\varphi)^2 + \cos(\varphi)^2 \\ -\sin\theta\sin\varphi \end{bmatrix} \tag{20}$$

$$= (\cos\theta\cos\varphi\sin\varphi - \sin\varphi\cos\varphi) \cdot \boldsymbol{x_a} + (\cos\theta\sin(\varphi)^2 + \cos(\varphi)^2) \cdot \boldsymbol{y_a} - (\sin\theta\sin\varphi) \cdot \boldsymbol{z_a}$$

$$\boldsymbol{u_c} = \cos\varphi \begin{bmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{bmatrix} + \sin\varphi \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta\cos(\varphi)^2 + \sin(\varphi)^2 \\ \cos\theta\cos\varphi\sin\varphi - \sin\varphi\cos\varphi \\ -\sin\theta\cos\varphi \end{bmatrix} \tag{21}$$

$$= (\cos\theta\cos(\varphi)^2 + \sin(\varphi)^2) \cdot \boldsymbol{x_a} + (\cos\theta\cos\varphi\sin\varphi - \sin\varphi\cos\varphi) \cdot \boldsymbol{y_a} - (\sin\theta\cos\varphi) \cdot \boldsymbol{z_a}$$

These are the expressions for the principal and cross-polarization unit vectors for the case of the satellite antenna transmitting a horizontally linearly polarized field parallel to $\boldsymbol{y_a}$ (i.e. a polarization angle of $\gamma = 0°$). This means that the principal polarization direction $\boldsymbol{u_p}$ is in the direction of $\boldsymbol{y_a}$ (which lies in the equatorial plane) on the antenna boresight at $\theta = 0°$ (as can be seen from the above expression for $\boldsymbol{u_p}$ with $\theta = 0°$). Note that for off-axis angles, $\boldsymbol{u_p}$ is not parallel to $\boldsymbol{y_a}$ since their dot product is not equal to one. For the case when the satellite is not transmitting horizontal polarization, the principal and cross-polarization directions are, for a transmitted polarization angle γ, given by:

$$\begin{aligned} \boldsymbol{u_p} &= \sin(\varphi+\gamma)\cdot \boldsymbol{u_\theta} + \cos(\varphi+\gamma)\cdot \boldsymbol{u_\varphi} \\ \boldsymbol{u_c} &= \sin(\varphi+\gamma)\cdot \boldsymbol{u_\theta} - \sin(\varphi+\gamma)\cdot \boldsymbol{u_\varphi} \end{aligned} \tag{22}$$

The $\boldsymbol{x_a}, \boldsymbol{y_a}, \boldsymbol{z_a}$ components are thus:

$$\begin{aligned} \boldsymbol{u_p}(\gamma) &= \sin(\varphi+\gamma)\begin{bmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\varphi \end{bmatrix} + \cos(\varphi+\gamma)\begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta\cos\varphi\sin(\varphi+\gamma) - \sin\varphi\cos(\varphi+\gamma) \\ \cos\theta\sin\varphi\sin(\varphi+\gamma) + \cos\varphi\cos(\varphi+\gamma) \\ -\sin\theta\sin(\varphi+\gamma) \end{bmatrix} \end{aligned} \tag{23}$$

$$\begin{aligned} \boldsymbol{u_c}(\gamma) &= \cos(\varphi+\gamma)\begin{bmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{bmatrix} - \sin(\varphi+\gamma)\begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta\cos\varphi\cos(\varphi+\gamma) + \sin\varphi\sin(\varphi+\gamma) \\ \cos\theta\sin\varphi\cos(\varphi+\gamma) - \cos\varphi\sin(\varphi+\gamma) \\ -\sin\theta\cos(\varphi+\gamma) \end{bmatrix} \end{aligned} \tag{24}$$

This last definition for the polarization reference directions will be the one assumed in the analyses for the reasons cited in Ludwig's paper.

2 Coordinate system descriptions and transformation matrices

In performing polarization angle calculations, four types of Cartesian orthonormal coordinate systems will be considered. These are shown in Fig. 4. They are the boresight point coordinate system (denoted $\mathbf{R_b}$), the earth station coordinate system ($\mathbf{R_p}$), the satellite antenna coordinate system ($\mathbf{R_a}$), and the Earth-centred coordinate system ($\mathbf{R_g}$). For purposes of interference calculations, we can think of systems $\mathbf{R_b}$, $\mathbf{R_p}$, and $\mathbf{R_a}$ together as comprising the "wanted" satellite system. In a similar way, if an "interfering" satellite system is present, it will have its own satellite ($\mathbf{R_{a2}}$) earth station ($\mathbf{R_{p2}}$) and boresight point ($\mathbf{R_{b2}}$), so that the systems $\mathbf{R_{b2}}$, $\mathbf{R_{p2}}$, and $\mathbf{R_{a2}}$ together comprise the "interfering" satellite system. The Earth-centred system $\mathbf{R_g}$ serves as an intermediate system for transforming between any pair of previous systems. In this section, we therefore develop the transformation matrices for transforming a vector from the Earth-centred system $\mathbf{R_g}$ to each of these other systems. Determining these transformation matrices is fundamental to the calculations, since once they are obtained, computation of the polarization angles and relative alignment angles becomes a trivial matter. To aid in understanding the method, an example calculation is performed.

2.1 Symbol notation

Symbol definitions and arbitrarily chosen values for the example calculation are as follows:

GSO radius (earth radii):	$k = 6.61072$
Wanted earth station latitude (degrees):	$\psi_p = 20$
Wanted earth station longitude (degrees):	$\lambda_p = -80$
Wanted satellite boresight point latitude (degrees):	$\psi_b = 10$
Wanted satellite boresight point longitude (degrees):	$\lambda_b = -90$
Wanted satellite longitude (degrees):	$\lambda_a = -100$
Interfering earth station latitude (degrees):	$\psi_{p2} = 45$
Interfering earth station longitude (degrees):	$\lambda_{p2} = -115$
Interfering satellite boresight point latitude (degrees):	$\psi_{b2} = 35$
Interfering satellite boresight point longitude (degrees):	$\lambda_{b2} = -85$
Interfering satellite longitude (degrees):	$\lambda_{a2} = -110$

For notational clarity, unit vectors will be ***bolded in italics***, vectors with magnitudes other than one will be **<u>bolded and underlined</u>**, coordinate vectors which represent the coordinates of a point in 3-D space will be simply <u>underlined</u>, and transformation matrices for transforming between coordinate systems will be **CAPITALIZED AND BOLDED**. The unit of distance is assumed to be one earth radius (6 378.153 km).

NOTE – In order to render the symbolic notation clearer, specific characters have, exceptionally, been used and it is possible that, depending upon the computer hardware and software used, these same characters may be different to those which appear in the printed text.

2.2 The Earth-centred system $\mathbf{R_g}$

The Earth-centred system has its origin at the Earth's centre, +z-axis pointed north, +x-axis pointed toward the satellite, and +y-axis pointed 90° east of the +x-axis. Its unit vectors in $\mathbf{R_g}$ components are therefore simply:

$$\boldsymbol{x_g} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \boldsymbol{z_g} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \boldsymbol{y_g} = \boldsymbol{z_g} \times \boldsymbol{x_g} \qquad \boldsymbol{y_g} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{25}$$

where $\boldsymbol{z_g} \times \boldsymbol{x_g}$ denotes the vector cross product. Note that both $\boldsymbol{x_g}$ and $\boldsymbol{y_g}$ lie in the equatorial plane.

2.3 The wanted earth station system R_p

Recall that the local vertical at a given point on Earth is defined by the vector from the Earth's centre to the given Earth point. The unit vector along the local vertical at the earth station P therefore has Earth-centre components:

$$\boldsymbol{lv} = \begin{bmatrix} \cos\psi_p \cos(\lambda_p - \lambda_s) \\ \cos\psi_p \sin(\lambda_p - \lambda_s) \\ \sin\psi_p \end{bmatrix} \tag{26}$$

The Earth-centre coordinates of the Earth point P, boresight point B, and satellite S, are:

$$\underline{P} = \begin{bmatrix} \cos\psi_p \cos(\lambda_p - \lambda_s) \\ \cos\psi_p \sin(\lambda_p - \lambda_s) \\ \sin\psi_p \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} \cos\psi_b \cos(\lambda_b - \lambda_s) \\ \cos\psi_b \sin(\lambda_b - \lambda_s) \\ \sin\psi_b \end{bmatrix} \qquad \underline{S} = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix} \tag{27}$$

The Earth-centre components of the position vector $\underline{\mathbf{PS}}$ from the earth station P to the satellite S are then:

$$\underline{\mathbf{PS}} = \underline{\mathbf{S}} - \underline{\mathbf{P}} \qquad \underline{\mathrm{PS}} = \begin{pmatrix} 5.728 \\ -0.321 \\ -0.342 \end{pmatrix} \tag{28}$$

The earth-station coordinate system is defined to have its origin at the earth station with its +z-axis directed toward the satellite. The unit vector along its +z-axis is therefore:

$$\boldsymbol{z_p} = \frac{\underline{\mathbf{PS}}}{|\underline{\mathrm{PS}}|} \qquad \boldsymbol{z_p} = \begin{pmatrix} 0.997 \\ -0.056 \\ -0.06 \end{pmatrix} \tag{29}$$

Its x-axis unit vector is directed to the left of an observer who is located at the earth station and is facing the satellite (i.e. its direction is given by the vector cross-product of the local vertical $\boldsymbol{lv}$ with the $\boldsymbol{z_p}$ unit vector)

$$\boldsymbol{x_p} = \frac{\boldsymbol{lv} \times \boldsymbol{z_p}}{|\boldsymbol{lv} \times \boldsymbol{z_p}|} \qquad \boldsymbol{x_p} = \begin{pmatrix} 0 \\ 0.729 \\ -0.685 \end{pmatrix} \tag{30}$$

(Note that $\boldsymbol{x_p}$ cannot be determined by the above equation for the special case in which the earth point is located at the subsatellite point since $\boldsymbol{lv}$ and $\boldsymbol{z_p}$ are co-linear. For this special case, we just choose $\boldsymbol{x_p}$ to be equal to $\boldsymbol{y_g}$.)

Finally, the unit vector in the positive y-direction to complete the right-handed system is found by taking the cross product:

$$\boldsymbol{y_p} = \boldsymbol{z_p} \times \boldsymbol{x_p} \qquad \boldsymbol{y_p} = \begin{pmatrix} 0.082 \\ 0.683 \\ 0.726 \end{pmatrix} \tag{31}$$

We can verify that these unit vectors form a Cartesian right-handed system by checking their dot products (i.e. two vectors are perpendicular to each other only if their dot product is zero)

$$x_p \times y_p = 0 \qquad x_p \times z_p = 0 \qquad y_p \times z_p = 0 \tag{32}$$

It should also be noted that, in general, $\boldsymbol{y_p}$ is not in the same direction as the local vertical ***lv***. In fact, in this case, the angle between the two is found from the definition of the dot product to be:

$$a\cos(\boldsymbol{lv} \times y_p) = 57.325° \tag{33}$$

We can now determine the transformation matrix $\mathbf{M_p}$ for transforming a vector ***from*** the Earth-centred system $\mathbf{R_g}$ to the earth station system $\mathbf{R_p}$. The rows of the matrix are the x, y, and z components of the earth station system's unit vectors. Thus we have:

$$\mathbf{M_p} = \begin{bmatrix} x_{p1} & x_{p2} & x_{p3} \\ y_{p1} & y_{p2} & y_{p3} \\ z_{p1} & z_{p2} & z_{p3} \end{bmatrix} \qquad \mathbf{M_p} = \begin{pmatrix} 0 & 0.729 & -0.685 \\ 0.082 & 0.683 & 0.726 \\ 0.997 & -0.056 & -0.06 \end{pmatrix} \tag{34}$$

where x_{p2} for example, is the y-component of unit vector x_p.

Remember that a coordinate transformation ***does not change a vector's magnitude direction, or what it represents – it merely changes the basis of a vector***. Expressing a vector in coordinates of a particular system ***does not*** imply that the vector has its tail at the origin of that system. To express a *vector's components* in the earth-station system when it is specified in the Earth-centred system we use the matrix equation:

$$\underline{\mathbf{V_p}} = \mathbf{M_p} \cdot \underline{\mathbf{V_g}} \tag{35}$$

where $\underline{\mathbf{V_g}}$ is the vector in Earth-centred $\mathbf{R_g}$ components and $\underline{\mathbf{V_p}}$ is the vector in earth station $\mathbf{R_p}$ components. To perform the inverse transformation – from the $\mathbf{R_p}$ system to the $\mathbf{R_g}$ system – we need to find the inverse of matrix $\mathbf{M_p^T}$. Thus, the inverse transformation is simply:

$$\underline{\mathbf{V_g}} = \mathbf{M_p^T} \cdot \underline{\mathbf{V_p}} \tag{36}$$

Note that if we want to find the *coordinates of a point* $\underline{W'}$ (x′, y′, z′) in the earth station system when it is specified as $\underline{W}$ (x, y, z) in the Earth-centred system we use:

$$\underline{W'} = \mathbf{M_p} \cdot (\underline{W} - \underline{P}) \tag{37}$$

where P is the Earth-centred coordinates of the earth station found in equation (27).

2.4 The interfering earth station system $\mathbf{R_{p2}}$

We go through the same procedure as that above to find the transformation matrix for transforming from the Earth-centred system $\mathbf{R_g}$ to the interfering earth station system $\mathbf{R_{p2}}$.

The unit vector along the local $\boldsymbol{lv_2}$ at the interfering earth station P_2 has Earth-centre $\mathbf{R_g}$ components:

$$\boldsymbol{lv_2} = \begin{bmatrix} \cos\psi_{p2}\cos(\lambda_{p2}-\lambda_s) \\ \cos\psi_{p2}\sin(\lambda_{p2}-\lambda_s) \\ \sin\psi_{p2} \end{bmatrix} \qquad \boldsymbol{lv_2} = \begin{pmatrix} 0.683 \\ -0.183 \\ 0.707 \end{pmatrix} \tag{38}$$

(Note that we use λ_g above and not λ_{a2} since the reference x-axis for system $\mathbf{R_g}$ is towards the "wanted" satellite S.)

The Earth-centre coordinates of the interfering Earth point P_2, interfering satellite boresight point B_2, and interfering satellite S_2 are:

$$\underline{P_2} = \begin{bmatrix} \cos\psi_{p2}\cos(\lambda_{p2}-\lambda_s) \\ \cos\psi_{p2}\sin(\lambda_{p2}-\lambda_s) \\ \sin\psi_{p2} \end{bmatrix} \qquad \underline{B_2} = \begin{bmatrix} \cos\psi_{b2}\cos(\lambda_{b2}-\lambda_s) \\ \cos\psi_{b2}\sin(\lambda_{b2}-\lambda_s) \\ \sin\psi_{b2} \end{bmatrix} \tag{39}$$

$$\underline{S_2} = \begin{bmatrix} k\cos(\lambda_{s2}-\lambda_s) \\ k\sin(\lambda_{s2}-\lambda_s) \\ 0 \end{bmatrix}$$

The Earth-centre components of the position vector $\underline{\mathbf{P_2S_2}}$ from the earth station P_2 to the satellite S_2 are:

$$\underline{\mathbf{P_2S_2}} = \underline{\mathbf{S_2}} - \underline{\mathbf{P_2}} \qquad \underline{P_2S_2} = \begin{pmatrix} 5{,}827 \\ -0{,}965 \\ -0{,}707 \end{pmatrix} \qquad \left|\underline{P_2S_2}\right| = 5{,}949 \tag{40}$$

The interfering earth-station coordinate system is defined to have its origin at the earth station P_2 with its +z-axis directed toward the satellite S_2. The unit vector along its +z-axis is therefore:

$$\boldsymbol{z_{p2}} = \frac{\underline{\mathbf{P_2S_2}}}{|\underline{P_2S_2}|} \qquad \boldsymbol{z_{p2}} = \begin{pmatrix} 0.98 \\ -0.162 \\ 0.119 \end{pmatrix} \tag{41}$$

Its x-axis unit vector is directed to the left of an observer who is located at the earth station and is facing the satellite (i.e. its direction is given by the vector cross-product of the local vertical $\boldsymbol{lv_2}$ with the $\boldsymbol{z_{p2}}$ unit vector):

$$\boldsymbol{x_{p2}} = \frac{\boldsymbol{lv_2} \times \boldsymbol{z_{p2}}}{|\boldsymbol{lv_2} \times \boldsymbol{z_{p2}}|} \qquad \boldsymbol{x_{p2}} = \begin{pmatrix} 0.173 \\ 0.981 \\ 0.087 \end{pmatrix} \tag{42}$$

Finally, the unit vector in the positive y-direction to complete the right-handed system is found by taking the cross product:

$$\boldsymbol{y_{p2}} = \boldsymbol{z_{p2}} \times \boldsymbol{x_{p2}} \qquad \boldsymbol{y_{p2}} = \begin{pmatrix} 0.103 \\ -0.106 \\ 0.989 \end{pmatrix} \tag{43}$$

We can now determine the transformation matrix $\mathbf{M_{p2}}$ for transforming a vector *from* the Earth-centred system $\mathbf{R_g}$ *to* the interfering earth station system $\mathbf{R_{p2}}$:

$$\mathbf{M_{p2}} = \begin{bmatrix} x_{p2_1} & x_{p2_2} & x_{p2_3} \\ y_{p2_1} & y_{p2_2} & y_{p2_3} \\ z_{p2_1} & z_{p2_2} & z_{p2_3} \end{bmatrix} \qquad \mathbf{M_{p2}} = \begin{pmatrix} 0.173 & 0.981 & 0.087 \\ 0.103 & -0.106 & 0.989 \\ 0.98 & -0.162 & -0.119 \end{pmatrix} \tag{44}$$

2.5 The wanted satellite antenna coordinate system $\mathbf{R_a}$

The position vector from the boresight point B to the satellite S has Earth-centre components:

$$\underline{\mathbf{BS}} = \underline{\mathbf{S}} - \underline{\mathbf{B}} \qquad \underline{\mathrm{BS}} = \begin{pmatrix} 5.641 \\ -0.171 \\ -0.174 \end{pmatrix} \qquad |\underline{\mathrm{BS}}| = 5.646 \tag{45}$$

The unit vector along this position vector therefore has Earth-centre components:

$$z_b = \frac{\mathbf{BS}}{|\underline{\mathrm{BS}}|} \qquad z_b = \begin{pmatrix} 0.999 \\ -0.03 \\ -0.031 \end{pmatrix} \tag{46}$$

The satellite antenna coordinate system $\mathbf{R_a}$ has its origin *at the satellite*, +z-axis pointed along the antenna boresight towards point B, and +y-axis directed to the east *in the equatorial plane* (see Fig. 4). Thus the unit vector along the +z-axis is simply:

$$z_a = -z_b \qquad z_a = \begin{pmatrix} -0.999 \\ 0.03 \\ 0.031 \end{pmatrix} \tag{47}$$

The unit vector along the +y-axis, y_a, is now perpendicular to z_a and also perpendicular to z_g in order to lie in the equatorial plane. Thus, by using the vector cross-product, we have:

$$y_a = \frac{z_a \times z_g}{|z_a \times z_g|} \qquad y_a = \begin{pmatrix} 0.0303 \\ 0.99954 \\ 0 \end{pmatrix} \qquad |y_a| = 1 \tag{48}$$

And to complete the right-handed Cartesian system we have:

$$x_a = y_a \times z_a \qquad x_a = \begin{pmatrix} 0.031 \\ -9.32 \times 10^{-4} \\ 1 \end{pmatrix} \qquad |x_a| = 1 \tag{49}$$

As a check to see that we have a right-handed system and that y_a is in the equatorial plane we compute the dot products:

$$\begin{aligned} x_a \times y_a &= 0 \\ x_a \times z_a &= 0 \qquad y_a \times z_g \\ y_a \times z_a &= 0 \end{aligned} \tag{50}$$

Transformation from the Earth-centred system $\mathbf{R_g}$ to the wanted satellite antenna system $\mathbf{R_a}$ is then done using:

$$\mathbf{M_A} = \begin{bmatrix} x_{a1} & x_{a2} & x_{a3} \\ y_{a1} & y_{a2} & y_{a3} \\ z_{a1} & z_{a2} & z_{a3} \end{bmatrix} \qquad \mathbf{M_A} = \begin{pmatrix} 0.031 & 9.32 \times 10^{-4} & 1 \\ 0.03 & 1 & 0 \\ -0.999 & 0.03 & 0.031 \end{pmatrix} \tag{51}$$

2.6 The interfering satellite antenna coordinate system $\mathbf{R_{a2}}$

To derive the transformation matrix, we follow the same procedure as in § 2.5.

The position vector from the boresight point B_2 to the interfering satellite S_2 has Earth-centre components:

$$\underline{\mathbf{B_2S_2}} = \underline{\mathbf{S_2}} - \underline{\mathbf{B_2}} \qquad \underline{B_2S_2} = \begin{pmatrix} 5.719 \\ -1.36 \\ -0.574 \end{pmatrix} \qquad \left|\underline{B_2S_2}\right| = 5.906 \tag{52}$$

The unit vector along this position vector therefore has Earth-centre components:

$$z_{b2} = \frac{\underline{\mathbf{B_2S_2}}}{|\underline{B_2S_2}|} \qquad z_{b2} = \begin{pmatrix} 0.968 \\ -0.23 \\ -0.097 \end{pmatrix} \qquad |z_{b2}| = 1 \tag{53}$$

The interfering satellite antenna coordinate system $\mathbf{R_{a2}}$ has its origin *at the satellite* S_2, +z-axis pointed along the antenna boresight towards B_2, and +y-axis directed to the east *in the equatorial plane*. Thus the unit vector along the +z-axis is simply:

$$z_{a2} = -z_{b2} \qquad z_{a2} = \begin{pmatrix} -0.968 \\ 0.23 \\ 0.097 \end{pmatrix} \qquad |z_{a2}| = 1 \tag{54}$$

The unit vector along the +y-axis, y_{a2}, is now perpendicular to z_{a2} and also perpendicular to z_g in order to lie in the equatorial plane. Thus, by using the vector cross-product, we have:

$$y_{a2} = \frac{z_{a2} \times z_g}{|z_{a2} \times z_g|} \qquad y_{a2} = \begin{pmatrix} 0.231 \\ 0.973 \\ 0 \end{pmatrix} \qquad |y_{a2}| = 1 \tag{55}$$

And to complete the right-handed Cartesian system we have:

$$x_{a2} = y_{a2} \times z_{a2} \qquad x_{a2} = \begin{pmatrix} 0.094 \\ -0.022 \\ 0.995 \end{pmatrix} \qquad |x_{a2}| = 1 \tag{56}$$

Transformation from the Earth-centred system $\mathbf{R_g}$ to the interfering satellite antenna system $\mathbf{R_{a2}}$ is then done using:

$$\mathbf{M_{A2}} = \begin{bmatrix} x_{a2_1} & x_{a2_2} & x_{a2_3} \\ y_{a2_1} & y_{a2_2} & y_{a2_3} \\ z_{a2_1} & z_{a2_2} & z_{a2_3} \end{bmatrix} \qquad \mathbf{M_{A2}} = \begin{pmatrix} 0.094 & -0.022 & 0.995 \\ 0.231 & 0.973 & 0 \\ -0.968 & 0.23 & 0.097 \end{pmatrix} \tag{57}$$

We now have all the necessary transformation matrices.

3 Calculation of polarization angles and relative alignment angles

Using the above transformation matrices, we can now determine the relative alignment angles β_d and β_u, for the downlink and uplink interference cases, respectively.

3.1 Calculation of relative alignment angle β_d for the downlink case

In this case we need to determine the alignment angle between linearly polarized signals at the "wanted" earth station P being transmitted from the "wanted" satellite S and an interfering satellite S_2. We assume the wanted earth station P is pointed at its own satellite S. The problem can be broken down into three steps:

Step 1 calculate the polarization angle ε_{d1} of the wave received from the wanted satellite S by transforming the transmitted principle polarization vector $\boldsymbol{u_p}$ – defined in the wanted satellite's $\mathbf{R_a}$ antenna coordinate system – to coordinate system $\mathbf{R_p}$ of the wanted earth station;

Step 2 calculate the polarization angle ε_{d2} of the wave received from the interfering satellite S_2 by transforming polarization vector $\boldsymbol{u_{p2}}$ – defined in the interfering satellite's $\mathbf{R_{a2}}$ coordinate system – to coordinate system $\mathbf{R_p}$ of the wanted earth station; and

Step 3 take the difference between ε_{d1} and ε_{d2} to find the alignment angle β_d. Refer again to Fig. 4.

The polarization angle γ of a wave received from a satellite is specified in a plane which is normal to the antenna boresight axis (i.e. normal to $\boldsymbol{z_a}$). Within this plane it is measured positive counter-clockwise from the +y-axis defined by $\boldsymbol{y_a}$ when looking in the direction of $\boldsymbol{z_a}$. An angle $\gamma = 0°$, therefore represents a polarization vector which lies in the equatorial plane. Remember that the polarization orientation will vary with angular direction so that the reference polarization of the antenna is defined to be the polarization of the $\underline{\mathbf{E}}$-field on the boresight axis (i.e. at $\theta = 0°$ off-axis angle). At an angular position (θ,φ) in the far-field pattern, the principal (main) and cross-polarization components of the $\underline{\mathbf{E}}$-field are defined to lie along the orthogonal unit vectors $\boldsymbol{u_p}$ and $\boldsymbol{u_c}$ which are tangent to a sphere at the point (θ,φ). Therefore, in computing ε_{d1} and ε_{d2}, we need to determine $\boldsymbol{u_p}$ (and $\boldsymbol{u_{p2}}$ for the interfering satellite) in the angular direction of the receiving earth station P. Refer to the detailed diagram of the satellite antenna coordinate system shown in Fig. 4. The angular position of the earth station P in the satellite antenna system is defined by the angles θ_a and φ_a. The angle θ_a is the off-axis angle of $-\boldsymbol{z_p}$ which points toward the earth station P from the $\boldsymbol{z_a}$ axis (which is the satellite antenna boresight axis). The angle φ_a is the earth station orientation angle or azimuth angle. It is the angle measured in the plane normal to the boresight axis (i.e. the $\boldsymbol{x_a}$, $\boldsymbol{-y_a}$ plane) between the $\boldsymbol{x_a}$ axis and the *projection* of $\boldsymbol{-z_p}$ onto the $\boldsymbol{x_a}$, $\boldsymbol{-y_a}$ plane. φ_a is measured positive clockwise from the $\boldsymbol{x_a}$ axis when looking in the direction of $\boldsymbol{z_a}$. These angles are found by first transforming unit vector $\boldsymbol{-z_p}$ to the $\mathbf{R_a}$ system to get its $(\boldsymbol{x_a}, \boldsymbol{y_a}, \boldsymbol{z_a})$ components:

$$\begin{bmatrix} \boldsymbol{x_a} \\ \boldsymbol{y_a} \\ \boldsymbol{z_a} \end{bmatrix} = \mathbf{M_A} \cdot (-\boldsymbol{z_p}) \tag{58}$$

The off-axis and orientation angles are then:

$$\theta_a = a\cos \boldsymbol{z_a} \qquad \varphi_a = a\tan\left(\frac{\boldsymbol{y_a}}{\boldsymbol{x_a}}\right) \tag{59}$$

$$\theta_a = 2.212° \qquad \varphi_a = 41.747°$$

From equation (23), the principle polarization unit vector $\boldsymbol{u_p}$ in terms of $\boldsymbol{x_a}$, $\boldsymbol{y_a}$, $\boldsymbol{z_a}$ components for $\gamma = 0°$ is then:

$$\boldsymbol{u_p} = \sin(\varphi_a + \gamma)\begin{bmatrix} \cos\theta_a \cos\varphi_a \\ \cos\theta_a \sin\varphi_a \\ -\sin\theta_a \end{bmatrix} + \cos(\varphi_a + \gamma)\begin{bmatrix} -\sin\varphi_a \\ \cos\varphi_a \\ 0 \end{bmatrix} \tag{60}$$

$$\boldsymbol{u_p} = \begin{pmatrix} -3.7013 \times 10^{-4} \\ 0.9997 \\ -0.0257 \end{pmatrix} \qquad |\boldsymbol{u_p}| = 1$$

The principal polarization vector $\boldsymbol{u_p}$ above is expressed in antenna system $\mathbf{R_a}$ components. To determine the polarization angle of the principal component at the receiving earth station, we need to transform $\boldsymbol{u_p}$ to earth station $\mathbf{R_p}$ components. We do this by first transforming from antenna system $\mathbf{R_a}$ components to Earth-centre $\mathbf{R_g}$ components and then transforming from $\mathbf{R_g}$ components to the earth station $\mathbf{R_p}$ components. This is done through the matrix equation:

$$\boldsymbol{u_{pp}} = \mathbf{M_p} \cdot (\mathbf{M}_{\mathbf{A}}^{\mathbf{T}} \cdot \boldsymbol{u_p}) \qquad \boldsymbol{u_{pp}} = \begin{pmatrix} 0.728 \\ 0.685 \\ 0 \end{pmatrix} \qquad (\boldsymbol{u_p} \text{ in } \mathbf{R_p} \text{ components}) \tag{61}$$

The received polarization angle (measured from $\boldsymbol{x_p}$) is then from the x and y components of $\boldsymbol{u_{pp}}$:

$$\varepsilon_{d1} = a\tan\left(\frac{\boldsymbol{u_{pp2}}}{\boldsymbol{u_{pp1}}}\right) \qquad \varepsilon_{d1} = 43.248° \tag{62}$$

Now we follow a similar procedure to find ε_{d2}. To find the angular position of the wanted receiving earth station P with respect to the interfering satellite antenna system $\mathbf{R_{a2}}$, we find the unit vector which points from the interfering satellite S_2 to the earth station P.

The Earth-centre components of the position vector $\underline{\mathbf{PS_2}}$ from the earth station P to the satellite S_2 are:

$$\underline{\mathbf{PS_2}} = \underline{\mathbf{S_2}} - \underline{\mathbf{P}} \qquad \underline{\mathbf{PS_2}} = \begin{pmatrix} 5.627 \\ -1.469 \\ -0.342 \end{pmatrix} \qquad \left|\underline{\mathrm{PS_2}}\right| = 5.826 \tag{63}$$

The unit vector along this direction is therefore:

$$\boldsymbol{z_{p2}} = \frac{\underline{\mathbf{PS_2}}}{|\underline{\mathrm{PS_2}}|} \qquad \boldsymbol{z_{p2}} = \begin{pmatrix} 0.966 \\ -0.252 \\ -0.059 \end{pmatrix} \qquad \left|\boldsymbol{z_{p2}}\right| = 1 \tag{64}$$

The unit vector which points *from* the interfering satellite *to* the earth station is then simply $\boldsymbol{-z_{p2}}$. We now follow (58-62) above to compute the polarization angle ε_{d2} received from the interfering satellite.

$$\begin{bmatrix} \boldsymbol{x_{a2}} \\ \boldsymbol{y_{a2}} \\ \boldsymbol{z_{a2}} \end{bmatrix} = \mathbf{M_{A2}} \cdot (-\boldsymbol{z_{p2}}) \tag{65}$$

The off-axis and orientation angles are:

$$\theta_{a2} = a \cos \boldsymbol{z_{a2}} \qquad \varphi_{a2} = a \tan \left(\frac{\boldsymbol{y_{a2}}}{\boldsymbol{x_{a2}}} \right) \tag{66}$$

$$\theta_{a2} = 2.538^{\circ} \qquad \varphi_{a2} = 150.35^{\circ}$$

Then unit vector $\boldsymbol{u_{p2}}$ which defines the principle polarization direction in this part of the pattern in terms of $\mathbf{R_{a2}}$ components is, for $\gamma_2 = 0°$, given by:

$$\boldsymbol{u_{p2}} = \sin\left(\varphi_{a2} + \gamma_2\right) \begin{bmatrix} \cos\theta_{a2} \cos\varphi_{a2} \\ \cos\theta_{a2} \sin\varphi_{a2} \\ -\sin\theta_{a2} \end{bmatrix} + \cos\left(\varphi_{a2} + \gamma_2\right) \begin{bmatrix} -\sin\varphi_{a2} \\ \cos\varphi_{a2} \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} 4.219 \times 10^{-4} \\ 1 \\ -0.022 \end{pmatrix} \tag{67}$$

Again, the principal polarization vector $\boldsymbol{u_{p2}}$ above is expressed in antenna system $\mathbf{R_{a2}}$ components. To determine the polarization angle ε_{d2} at the receiving earth station, we need to transform $\boldsymbol{u_{p2}}$ to earth station $\mathbf{R_p}$ components. We do this first by transforming from antenna system $\mathbf{R_{a2}}$

components to Earth-centre $\mathbf{R_g}$ components and then transforming from $\mathbf{R_g}$ components to the earth station $\mathbf{R_p}$ components. This is done by the matrix equation:

$$\boldsymbol{u_{p2p}} = \mathbf{M_p} \cdot (\mathbf{M_{A2}^T} \cdot \boldsymbol{u_{p2}}) \qquad \boldsymbol{u_{p2p}} = \begin{pmatrix} 0.706 \\ 0.68 \\ 0.198 \end{pmatrix} \qquad (\boldsymbol{u_{p2}} \text{ in } \mathbf{R_p} \text{ components}) \tag{68}$$

The received polarization angle is then:

$$\varepsilon_{d2} = a \tan\left(\frac{\boldsymbol{u_{p2p_2}}}{\boldsymbol{u_{p2p_1}}}\right) \qquad \varepsilon_{d2} = 43.904° \tag{69}$$

The relative alignment angle β_d is then simply:

$$\beta_d = \left|\varepsilon_{d_1} - \varepsilon_{d_2}\right| \qquad \beta_d = 0.655° \tag{70}$$

3.2 Calculation of relative alignment angle β_u for the up link case

In this case we need to determine the alignment angle between linearly polarized signals at a "wanted" satellite S being transmitted from a "wanted" earth station P and an interfering earth station P_2. Note that the interfering earth station P_2 is assumed to be pointed at its own satellite, which is satellite S_2. The problem involves three steps:

Step 1: calculate the polarization angle ε_{u1} of the wave received from the "wanted" earth station P by transforming the transmitted polarization vector $\boldsymbol{u_p}$ – defined in the wanted earth station's $\mathbf{R_p}$ coordinate system – to antenna coordinate system $\mathbf{R_a}$ of the wanted satellite;

Step 2: calculate the polarization angle ε_{u2} of the wave received from the interfering earth station P_2 by transforming polarization vector $\boldsymbol{u_{p2}}$ – defined in the interfering earth station's $\mathbf{R_{p2}}$ coordinate system – to antenna coordinate system $\mathbf{R_a}$ of the wanted satellite; and

Step 3: take the difference between ε_{u1} and ε_{u2} to find the alignment angle β_u.

In determining the polarization vectors of the signals transmitted from the wanted and interfering earth stations to their respective satellites, it is assumed that they are matched (i.e. aligned) to the off-axis receive polarizations of their respective satellite antennas. By definition, the receive polarization of an antenna in a certain direction is the polarization of the signal transmitted by the antenna in that direction. Accordingly, to find the matching polarization vectors transmitted from the earth stations, it is first necessary to find the polarization vectors of the signals transmitted from the satellites in the direction of their respective earth stations. This is described below, first for the wanted system, then the interfering system.

We first determine the transmitted polarization vector $\boldsymbol{u_p}$ from the wanted earth station P to the wanted satellite S. As discussed above, it is assumed to be matched to the receiving polarization of the antenna at S in the direction of P. This, in turn, is just the polarization of the signal transmitted from S towards P, which has already been computed in equation (61) of the downlink calculations. Hence, the matching polarization vector transmitted from earth station P on the uplink to S is from (61):

$$\boldsymbol{u_p} = \boldsymbol{u_{pp}} \qquad \boldsymbol{u_p} = \begin{pmatrix} 0.728 \\ 0.685 \\ 0 \end{pmatrix} \qquad (\boldsymbol{u_p} \text{ in } \mathbf{R_p} \text{ components}) \tag{71}$$

As a check, we can transform this vector to antenna system $\mathbf{R_a}$ of the wanted satellite to obtain:

$$\boldsymbol{u_{pa}} = \mathbf{M_A} \; (\mathbf{M_P^T} \cdot \boldsymbol{u_p}) \qquad \boldsymbol{u_{pa}} = \begin{pmatrix} -3.7013 \times 10^{-4} \\ 0.9997 \\ -0.0257 \end{pmatrix} \qquad (\boldsymbol{u_p} \text{ in } \mathbf{R_a} \text{ components}) \tag{72}$$

Note that this vector matches that transmitted from the satellite on the downlink as shown in equation (60). Since the polarization angle in system $\mathbf{R_a}$ is defined to be measured positive counter-clockwise from the $+y_a$ axis when looking in the direction of $\boldsymbol{z_a}$, the received polarization angle ε_{u1} at the wanted satellite S is:

$$\varepsilon_{u1} = a \tan\left(\frac{\boldsymbol{u_{pa_1}}}{\boldsymbol{u_{pa_2}}}\right) \qquad \varepsilon_{u1} = -0.021^{\circ} \tag{73}$$

(Note that due to the way polarization angle is defined, the ratio is the x-component divided by the y-component.)

To find polarization vector $\boldsymbol{u_{p2}}$ transmitted from the interfering earth station P_2 in the direction of wanted satellite S it is first necessary to find the polarization vector that earth station P_2 transmits to its *own* satellite S_2. It is again assumed that this vector is matched to the receive polarization of S_2 in the direction of earth station P_2.

Recall that the Earth centre coordinates of earth station P_2 and satellite S_2 are:

$$\underline{P_2} = \begin{pmatrix} 0.683 \\ -0.183 \\ 0.707 \end{pmatrix} \qquad \underline{S_2} = \begin{pmatrix} 6.51 \\ -1.148 \\ 0 \end{pmatrix} \tag{74}$$

The position vector from the interfering earth station to the interfering satellite is then:

$$\underline{\mathbf{P_2S_2}} = \underline{\mathbf{S_2}} - \underline{\mathbf{P_2}} \qquad \underline{P_2S_2} = \begin{pmatrix} 5.827 \\ -0.965 \\ -0.707 \end{pmatrix} \tag{75}$$

and the unit vector from the interfering earth station to the interfering satellite is then:

$$\boldsymbol{z_{p2s2}} = \frac{\underline{\mathbf{P_2S_2}}}{\left|\underline{P_2S_2}\right|} \qquad \boldsymbol{z_{p2s2}} = \begin{pmatrix} 0.98 \\ -0.162 \\ -0.119 \end{pmatrix} \tag{76}$$

The unit vector *from* satellite S_2 *to* earth station P_2 then is simply:

$$-\boldsymbol{z}_{p2s2} = \begin{pmatrix} -0.98 \\ 0.162 \\ 0.119 \end{pmatrix} \tag{77}$$

Transforming this vector to the antenna coordinate system $\mathbf{R_{a2}}$ of interfering satellite S_2 we have:

$$\begin{bmatrix} x_{a2} \\ y_{a2} \\ z_{a2} \end{bmatrix} = \mathbf{M_{A2}} \cdot (-\boldsymbol{z}_{p2s2}) \qquad \begin{bmatrix} x_{a2} \\ y_{a2} \\ z_{a2} \end{bmatrix} = \begin{pmatrix} 0.022 \\ -0.069 \\ 0.997 \end{pmatrix} \tag{78}$$

The off-axis and orientation angles of the earth station P_2 as measured with respect to the antenna boresight of satellite S_2 (which is directed towards the boresight point B_2) are then:

$$\theta_{a2} = a \cos \boldsymbol{z}_{a2} \qquad \varphi_{a2} = a \tan \left(\frac{y_{a2}}{x_{a2}} \right)$$

$$\theta_{a2} = 4.145° \qquad \varphi_{a2} = -72.185° \tag{79}$$

From equation (23) the principal polarization unit vector $\boldsymbol{u}_{p2s2}$ that satellite S_2 transmits towards earth station P_2, assuming that S_2 is transmitting horizontal polarization (i.e. $\gamma = 0°$) on its antenna boresight axis is then:

$$\gamma = 0$$

$$\boldsymbol{u}_{p2s2} = \sin(\varphi_{a2} + \gamma) \begin{bmatrix} \cos \theta_{a2} \cos \varphi_{a2} \\ \cos \theta_{s2} \sin \varphi_{a2} \\ -\sin \theta_{a2} \end{bmatrix} + \cos(\varphi_{a2} + \gamma) \begin{bmatrix} -\sin \varphi_{a2} \\ \cos \varphi_{a2} \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} 7.6183 \times 10^{-4} \\ 0.9976 \\ 0.0688 \end{pmatrix} \tag{80}$$

The principal polarization vector above is expressed in antenna system $\mathbf{R_{a2}}$ components. To determine the polarization angle of the principal component at the receiving earth station P_2, we need to transform it to earth station $\mathbf{R_{p2}}$ components. We do this by first transforming from antenna system $\mathbf{R_{a2}}$ components to Earth-centre $\mathbf{R_g}$ components and then transforming from $\mathbf{R_g}$ components to the earth station $\mathbf{R_{p2}}$ components. This is done through the matrix equation:

$$\boldsymbol{u}_{pp} = \mathbf{M_{p2}} \cdot (\mathbf{M}^{\mathbf{T}}_{\mathbf{A2}} \cdot \boldsymbol{u}_{p2s2}) \qquad \boldsymbol{u}_{pp} = \begin{pmatrix} 0.997 \\ -0.08 \\ 0 \end{pmatrix} \qquad (\boldsymbol{u}_p \text{ in } \mathbf{R_{p2}} \text{ components}) \tag{81}$$

We now have the ($\boldsymbol{x_{p2}}$, $\boldsymbol{y_{p2}}$, $\boldsymbol{z_{p2}}$) components of the polarization vector received at earth station P_2 from its satellite S_2 from which we can find the polarization angle. At this point, it is important to keep in mind the definition of polarization angle and the reference directions that are used to define it. Recall that in the satellite antenna coordinate systems, polarization angle is the angle between the +y-axis (which lies in the equatorial plane) and the *projection* of the polarization vector on the x-y plane (which is normal to the antenna boresight) measured positive CCW (counter clockwise) while looking along the +z-axis (which is the antenna boresight axis). Hence, in the satellite antenna coordinate systems, a polarization vector which is parallel to the equatorial plane has a polarization angle of 0° while one that lies along the +x-axis has a polarization angle of 90°. To be consistent, we will define the polarization angle in the earth station coordinate systems *to be the same*, even though their axes are oriented differently. Hence, polarization vectors that are oriented along the local vertical will have polarization angles close to 0° while those that are oriented along the local horizontal will have polarization angles close to 90°. The received polarization angle (measured + CCW from the y_{p2} axis while looking along the z_{p2} axis) is then from the x and y components of $\boldsymbol{u_{pp}}$,

$$\varepsilon = a\tan\left(\frac{\boldsymbol{u_{pp_1}}}{\boldsymbol{u_{pp_2}}}\right) \qquad \varepsilon = 94.587° \tag{82}$$

(Note that the ratio is the x-component over the y-component and that the proper quadrant is accounted for in the arctangent operation.)

Assuming the signal transmitted from earth station P_2 is polarization matched to the receive polarization of satellite S_2, this is therefore also the polarization angle of the wave transmitted on the uplink towards satellite S_2. What we need, however, is the polarization vector of the signal transmitted in the direction of the wanted satellite S. To find this vector, it is necessary to find the angular position of S in the system $\mathbf{R_{p2}}$ of earth station P_2. Given the Earth-centre coordinates of the interfering earth station $\underline{P_2}$ in (39) and wanted satellite S in (27), the position vector $\underline{\mathbf{P_2S}}$ *from* the interfering earth station *to* the wanted satellite has Earth-centre $\mathbf{R_g}$ components:

$$\underline{\mathbf{P_2S}} = \underline{\mathbf{S}} - \underline{\mathbf{P_2}} \qquad \underline{P_2S} = \begin{pmatrix} 5.928 \\ 0.183 \\ -0.707 \end{pmatrix} \qquad \left|\underline{P_2S}\right| = 5.973 \tag{83}$$

The unit vector along this position vector is then simply:

$$\boldsymbol{z_{p2s}} = \frac{\underline{\mathbf{P_2S}}}{\underline{|P_2S|}} \qquad \boldsymbol{z_{p2s}} = \begin{pmatrix} 0.992 \\ 0.031 \\ -0.118 \end{pmatrix} \qquad \left|\boldsymbol{z_{p2s}}\right| = 1 \tag{84}$$

Now, we transform from $\mathbf{R_g}$ components to $\mathbf{R_{p2}}$ components using matrix $\mathbf{M_{p2}}$:

$$\begin{bmatrix} x_{p2s} \\ y_{p2s} \\ z_{p2s} \end{bmatrix} = \mathbf{M_{p2}} \cdot (-\mathbf{z}_{p2s}) \qquad \begin{bmatrix} x_{p2s} \\ y_{p2s} \\ z_{p2s} \end{bmatrix} = \begin{pmatrix} 0.191 \\ -0.019 \\ 0.981 \end{pmatrix} \tag{85}$$

The off-axis and orientation angles of S with respect to the $\mathbf{R_{p2}}$ system are then:

$$\theta_{p2s} = a\cos z_{p2s} \qquad \varphi_{p2s} = a\tan\left(\frac{y_{p2s}}{x_{p2s}}\right)$$

$$\theta_{p2s} = 11.091° \qquad {}_{p2s} = -5.541° \tag{86}$$

We now again use equation (23) (Ludwig's third definition) to find the transmitted polarization vector in the angular direction ($\theta_{p2s}\ \varphi_{p2s}$) of the wanted satellite S, given the polarization angle on the interfering earth station's antenna boresight (which is the angle ε computed above). Hence, the polarization vector $\boldsymbol{u_{p2}}$ is:

$$\gamma_2 = \varepsilon \qquad \gamma_2 = 94.587°$$

$$\boldsymbol{u_{p2}} = \sin(\varphi_{p2s} + \gamma_2) \begin{bmatrix} \cos\theta_{p2s}\cos\varphi_{p2s} \\ \cos\theta_{p2s}\sin\varphi_{p2s} \\ -\sin\theta_{p2s} \end{bmatrix} + \cos(\varphi_{p2s} + \gamma_2) \begin{bmatrix} -\sin\varphi_{p2s} \\ \cos\varphi_{p2s} \\ 0 \end{bmatrix} \tag{87}$$

$$= \begin{pmatrix} 0.978 \\ -0.078 \\ -0.192 \end{pmatrix}$$

Finally, to find the received polarization angle at the satellite S of the wave transmitted from the interfering earth station P_2, we transform vector $\boldsymbol{u_{p2}}$ from system $\mathbf{R_{p2}}$ to system $\mathbf{R_a}$ of the wanted satellite using the matrix equation:

$$\boldsymbol{u_{p2a}} = \mathbf{M_A}(\mathbf{M_{p2}^T} \cdot \boldsymbol{u_{p2}}) \qquad \boldsymbol{u_{p2a}} = \begin{pmatrix} 0.029 \\ 0.998 \\ 0.058 \end{pmatrix} \qquad (\boldsymbol{u_{p2}} \text{ in } \mathbf{R_a} \text{ components}) \tag{88}$$

The received polarization angle ε_{u2} is then:

$$\varepsilon_{u2} = a\tan\left(\frac{u_{p2a_1}}{u_{p2a_2}}\right) \qquad \varepsilon_{u2} = 1.647° \tag{89}$$

Finally, the alignment angle β_u between the linearly polarized signal received from the wanted earth station and the linearly polarized signal received from the interfering earth station is:

$$\beta_u = |\varepsilon_{u1} - \varepsilon_{u2}| \qquad \beta_u = 1.668° \tag{90}$$

FIGURE 1

Rectangular, cylindrical and spherical coordinate systems

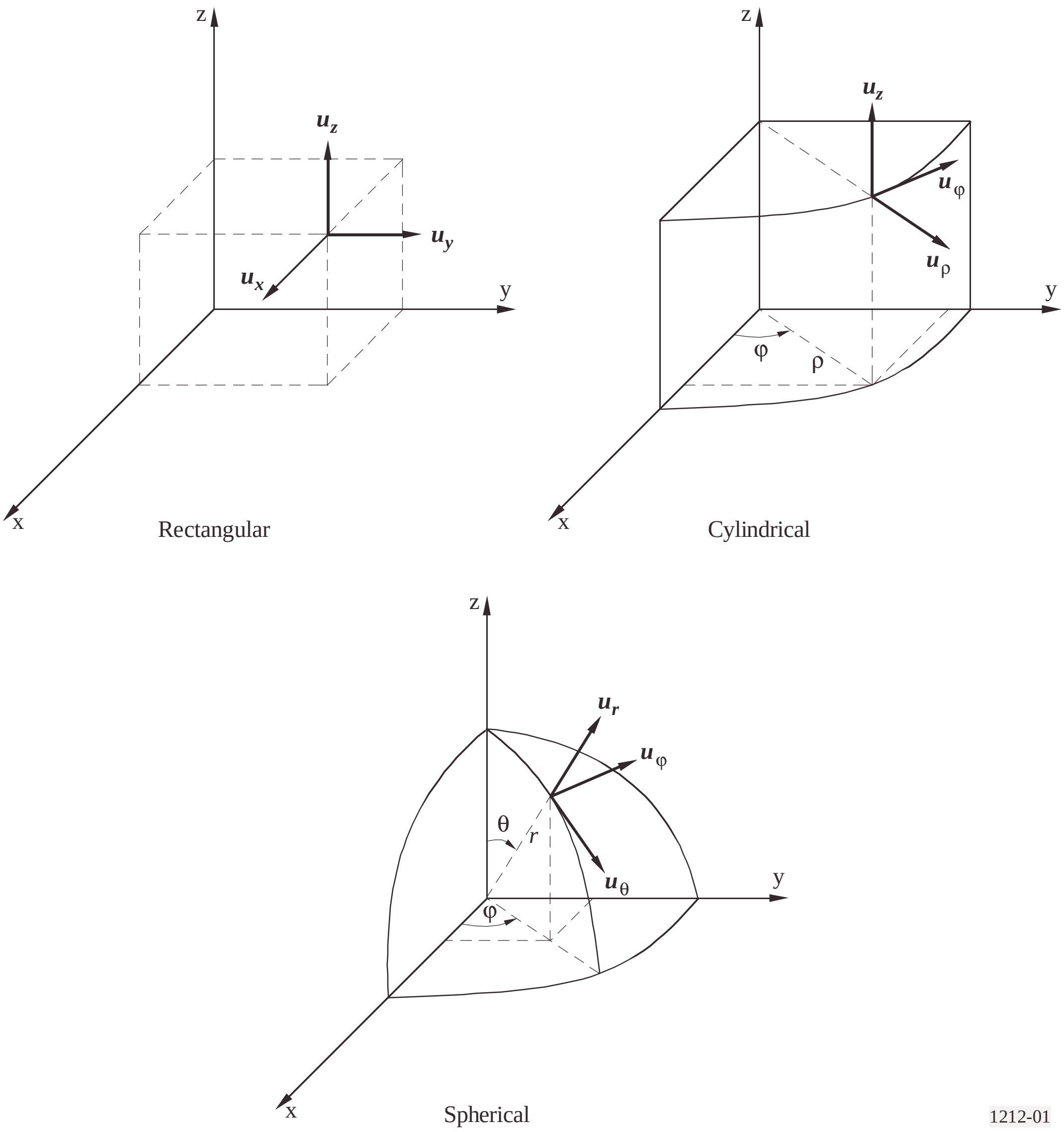


FIGURE 2

Alternate definitions of polarization

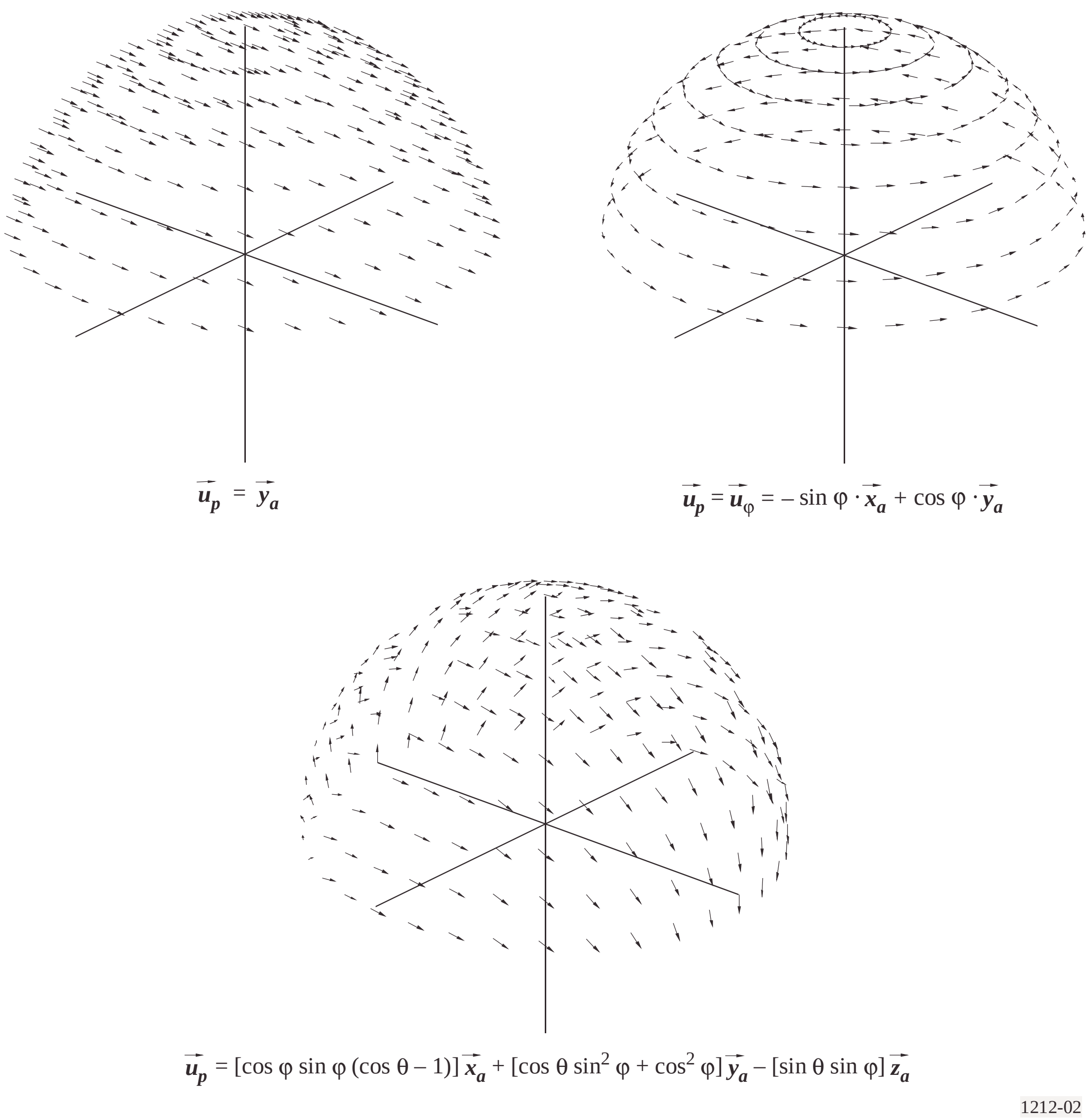
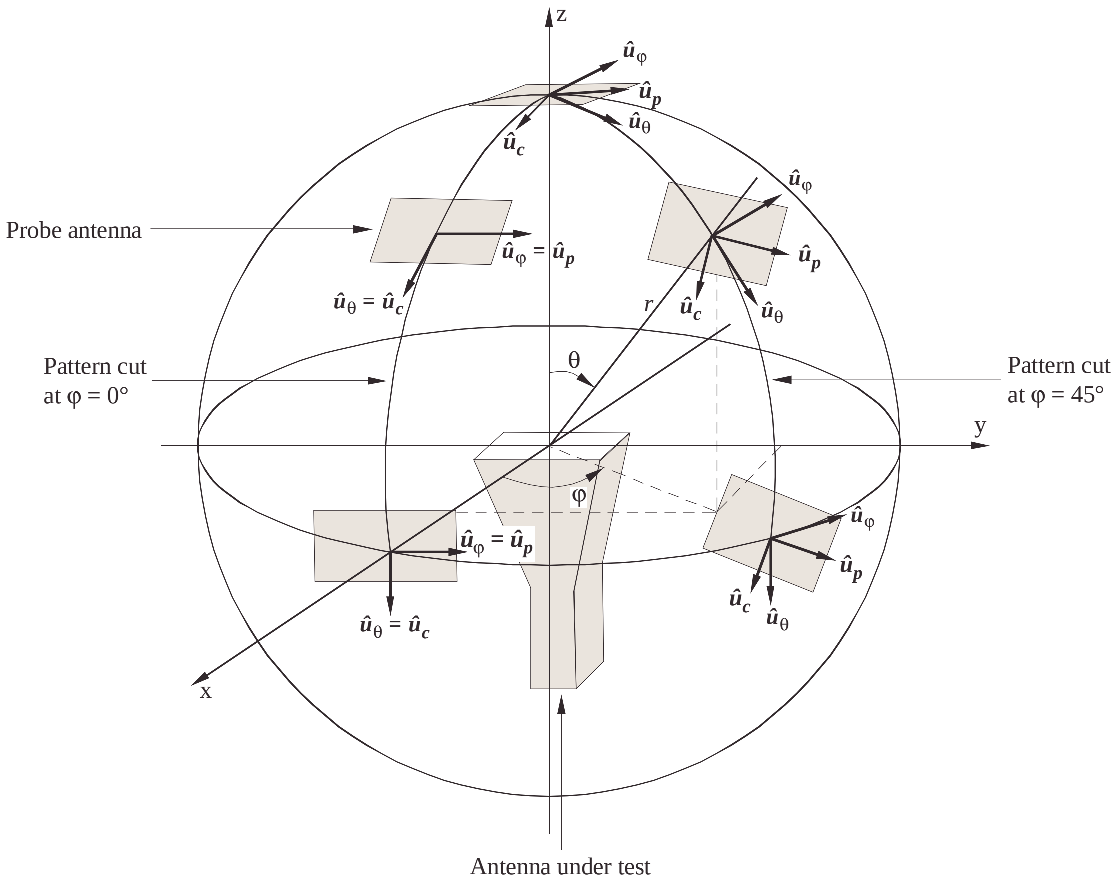


FIGURE 3

Definition of principal and cross-polarization from the antenna pattern measurement method



$\hat{\boldsymbol{u}}_\theta$ and $\hat{\boldsymbol{u}}_\varphi$ are spherical coordinate unit vectors
$\hat{\boldsymbol{u}}_p$ defines the principal polarization direction at a point (θ, φ)
$\hat{\boldsymbol{u}}_c$ defines the cross-polarization direction at a point (θ, φ)

The principal and cross-polarization directions as defined by the antenna pattern measurement method. The antenna under test is mounted at the origin of a spherical coordinate system. A probe antenna which is linearly polarized is used to determine the polarization pattern of the test antenna by making pattern cuts at various azimuth angles φ. Each pattern cut begins at θ = 0° (on the z-axis) where the probe is rotated about its axis in order to align its polarization with that of the test antenna. The orientation of the polarization at θ = 0° defines the basic polarization direction of the test antenna. For a given φ, a pattern cut is then taken by varying θ by moving the probe along a great circle arc as shown. The probe remains fixed about its axis so it retains the same orientation with respect to the unit vectors $\hat{\boldsymbol{u}}_\theta$ and $\hat{\boldsymbol{u}}_\varphi$ and the same polarization angle β. The orthogonal unit vectors $\hat{\boldsymbol{u}}_p$ and $\hat{\boldsymbol{u}}_c$ (which are also tangent to the sphere at a point (θ, φ)) are then defined to be the principal and cross-polarization directions, respectively.

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FIGURE 4

Illustration of the various coordinate systems