RECOMMENDATION ITU-R F.1336-2*

Reference radiation patterns of omnidirectional, sectoral and other antennas in point-to-multipoint systems for use in sharing studies in the frequency range from 1 GHz to about 70 GHz

(Question ITU-R 202/9)

(1997-2000-2007)

Scope

This Recommendation gives reference models of the peak and average antenna patterns of omnidirectional, sectoral and directional antennas in point-to-multipoint systems to be used in sharing studies in the frequency range 1 GHz to about 70 GHz.

The ITU Radiocommunication Assembly,

considering

a) that, for coordination studies and for the assessment of mutual interference between point-to-multipoint (P-MP) fixed wireless systems (FWSs) and between stations of such systems and stations of space radiocommunication services sharing the same frequency band, it may be necessary to use reference radiation patterns for FWS antennas;

b) that, depending on the sharing scenario, it may be appropriate to consider the peak envelope or average sidelobe patterns in the sharing studies;

c) that it may be appropriate to use the antenna radiation pattern representing average side-lobe levels in the following cases:

– to predict the aggregate interference to a geostationary or non-geostationary satellite from numerous fixed wireless stations;

– to predict the aggregate interference to a fixed wireless station from many geostationary satellites;

– to predict interference to a fixed wireless station from one or more non-geostationary-satellites under continuously varying angles;

– in any other cases where the use of the radiation pattern representing average side-lobe levels is appropriate;

d) that reference radiation patterns may be required in situations where information concerning the actual radiation pattern is not available;

e) that, at large angles, the likelihood of local ground reflections must be considered;

* This Recommendation should be brought to the attention of Radiocommunication Study Groups 4 (WP 4A), 6 (WP 6S), 7 (WP 7B), 8 (WP 8D) and WP 4-9S.

f) that the use of antennas with the best available radiation patterns will lead to the most efficient use of the radio-frequency spectrum,

noting

a) that Recommendations ITU-R F.699 and ITU-R F.1245 give the peak and average reference antenna patterns respectively to be used in coordination studies and interference assessment in cases not referred to in *recommends* 1 to 4 below,

recommends

1 that, in the absence of particular information concerning the radiation pattern of the P-MP FWS antenna involved (see Note 1), the reference radiation pattern as stated below should be used for:

1.1 interference assessment between line-of-sight (LoS) P-MP FWSs;

1.2 coordination studies and interference assessment between P-MP LoS FWSs and other stations of services sharing the same frequency band;

2 that, in the frequency range from 1 GHz to about 70 GHz, the following reference radiation patterns should be used in cases involving stations that use omnidirectional (in azimuth) antennas:

2.1 in the case of peak side-lobe patterns referred to in *considering* b), the following equations should be used for elevation angles that range from 0° to 90° (see Annex 1):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\theta_3}\right)^2 & \text{for} \quad 0 \le \theta < \theta_4 \\ G_0 - 12 + 10\log(k+1) & \text{for} \quad \theta_4 \le \theta < \theta_3 \\ G_0 - 12 + 10\log\left[\left(\dfrac{|\theta|}{\theta_3}\right)^{-1.5} + k\right] & \text{for} \quad \theta_3 \le \theta \le 90° \end{cases} \tag{1a}$$

with:

$$\theta_3 = 107.6 \times 10^{-0.1 G_0} \tag{1b}$$

$$\theta_4 = \theta_3 \sqrt{1 - \frac{1}{1.2}\log(k+1)} \tag{1c}$$

where:

- $G(\theta)$: gain relative to an isotropic antenna (dBi)
- G_0: the maximum gain in or near the horizontal plane (dBi)
- θ: absolute value of the elevation angle relative to the angle of maximum gain (degrees)
- θ_3: the 3 dB beamwidth in the vertical plane (degrees)
- k: parameter which accounts for increased side-lobe levels above what would be expected for an antenna with improved side-lobe performance (see *recommends* 2.3 and 2.4);

2.2 in the case of average side-lobe patterns referred to in *considering* c), the following equations should be used for elevation angles that range from 0° to 90° (see Annex 1 and Annex 5):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\theta_3}\right)^2 & \text{for} \quad 0 \le \theta < \theta_3 \\ G_0 - 15 + 10\,\log(k+1) & \text{for} \quad \theta_3 \le \theta < \theta_5 \\ G_0 - 15 + 10\,\log\left[\left(\dfrac{|\theta|}{\theta_3}\right)^{-1.5} + k\right] & \text{for} \quad \theta_5 \le \theta \le 90° \end{cases} \tag{1d}$$

with:

$$\theta_5 = \theta_3 \sqrt{1.25 - \frac{1}{1.2}\log(k+1)}$$

where θ, θ_3, G_0 and k are defined and expressed in *recommends* 2.1;

2.3 in cases involving typical antennas operating in the 1-3 GHz range, the parameter k should be 0.7;

2.4 in cases involving antennas with improved side-lobe performance in the 1-3 GHz range, and for all antennas operating in the 3-70 GHz range, the parameter k should be 0;

3 that, in the frequency range from 1 GHz to about 70 GHz, the following reference radiation patterns should be used in cases involving stations that use sectoral antennas with a 3 dB beamwidth in the azimuthal plane less than about 120° (see Annex 4 and Note 4);

3.1 in the case of peak side-lobe patterns referred to in *considering* b), the following equations should be used for elevation angles that range from 0° to 90° and for azimuth angles that range from –180° to 180° (see Note 2):

$$G(\varphi,\theta) = G_{ref}(x) \tag{2a1}$$

$$\alpha = \arctan\left(\frac{\tan\theta}{\sin\varphi}\right) \tag{2a2}$$

$$\psi_\alpha = \frac{1}{\sqrt{\left(\dfrac{\cos\alpha}{\varphi_3}\right)^2 + \left(\dfrac{\sin\alpha}{\theta_3}\right)^2}} \tag{2a3}$$

$$= \varphi_3 \cdot \theta_3 \sqrt{\frac{(\sin\theta)^2 + (\sin\varphi \cdot \cos\theta)^2}{(\varphi_3 \cdot \sin\theta)^2 + (\theta_3 \cdot \sin\varphi \cdot \cos\theta)^2}} \quad \text{degrees}$$

$$\psi = \arccos(\cos\varphi \cdot \cos\theta) \quad \text{degrees} \tag{2a4}$$

$$x = \psi / \psi_\alpha \tag{2a5}$$

where:

- φ : azimuth angle relative to the angle of maximum gain (degrees)
- φ_3: the 3 dB beamwidth in the azimuth plane (degrees) (generally equal to the sectoral beamwidth).

Other variables and parameters are as defined in *recommends* 2.1;

3.1.1 in the frequency range from 1 GHz to about 6 GHz (see Annex 6)

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < x_k \qquad (2b)$$

$$G_{ref}(x) = G_0 - 12 + 10\log(x^{-1.5} + k) \quad \text{for} \quad x_k \leq x < 4$$

$$G_{ref}(x) = G_0 - \lambda_k - 15\log(x) \quad \text{for} \quad x \geq 4$$

with $\lambda_k = 12 - 10\log(1 + 8k)$ and $x_k = \sqrt{1 - 0.36k}$;

3.1.1.1 in cases involving typical antennas the parameter k should be 0.7 (therefore, $\lambda_{k=0.7} = 3.8$ and $x_{k=0.7} = 0.86$);

3.1.1.2 in cases involving antennas with improved side-lobe performance the parameter k should be 0 (therefore, $\lambda_{k=0} = 12$ and $x_{k=0} = 1$);

3.1.2 in the frequency range from 6 GHz to about 70 GHz:

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < 1 \qquad (2c)$$

$$G_{ref}(x) = G_0 - 12 - 15\log(x) \quad \text{for} \quad 1 \leq x$$

3.2 in the case of average side-lobe patterns referred to in *considering* c), for use in a statistical interference assessment, the following equations should be used for elevation angles that range from 0° to 90° and for azimuth angles that range from –180° to 180° (see Annex 5 and Note 2):

$$G(\varphi, \theta) = G_{ref}(x)$$

3.2.1 in the frequency range from 1 GHz to about 6 GHz (see Annex 6):

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < x_k \qquad (2d)$$

$$G_{ref}(x) = G_0 - 15 + 10\log(x^{-1.5} + k) \quad \text{for} \quad x_k \leq x < 4$$

$$G_{ref}(x) = G_0 - \lambda_k - 3 - 15\log(x) \quad \text{for} \quad x \geq 4$$

with $\lambda_k = 12 - 10\log(1 + 8k)$ and $x_k = \sqrt{1.25 - 0.36k}$;

3.2.1.1 in cases involving typical antennas the parameter k should be 0.2 (therefore, $\lambda_{k=0.2} = 7.85$ and $x_{k=0.2} = 1.08$);

3.2.1.2 in cases involving antennas with improved side-lobe performance the parameter k should be 0 (therefore, $\lambda_{k=0} = 12$ and $x_{k=0} = 1.118$);

3.2.2 in the frequency range from 6 GHz to about 70 GHz:

$$G_{ref}(x) = G_0 - 12x^2 \quad \text{for} \quad 0 \leq x < 1.152 \qquad (2e)$$

$$G_{ref}(x) = G_0 - 15 - 15\log(x) \quad \text{for} \quad 1.152 \leq x$$

3.3 in cases involving sectoral antennas with a 3 dB beamwidth in the azimuthal plane less than about 120°, the relationship between the maximum gain and the 3 dB beamwidth in both the azimuthal plane and the elevation plane, on a provisional basis, is (see Annex 3 and Note 4):

$$\theta_3 = \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_3} \tag{3}$$

where all parameters are as defined under *recommends* 3.1;

4 that, in the frequency range from 1 GHz to about 3 GHz, the following reference radiation patterns should be used in cases involving stations that use low-gain antennas with circular symmetry about the 3 dB beamwidth and with a main lobe antenna gain less than about 20 dBi:

4.1 the following equations should be used in the case of peak side-lobe patterns referred to in *considering* b) (see Annex 2 and Note 3):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\varphi_3}\right)^2 & \text{for } 0 \leq \theta < 1.08\,\varphi_3 \\ G_0 - 14 & \text{for } 1.08\,\varphi_3 \leq \theta < \varphi_1 \\ G_0 - 14 - 32\,\log\left(\dfrac{\theta}{\varphi_1}\right) & \text{for } \varphi_1 \leq \theta < \varphi_2 \\ -8 & \text{for } \varphi_2 \leq \theta \leq 180° \end{cases} \tag{4}$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: the main lobe antenna gain (dBi)

θ: off-axis angle (degrees)

φ_3: the 3 dB beamwidth of the low-gain antenna (degrees)

$= \sqrt{27\,000 \times 10^{-0.1\,G_0}}$ (degrees)

$\varphi_1 = 1.9\,\varphi_3$ (degrees)

$\varphi_2 = \varphi_1 \times 10^{(G_0 - 6)/32}$ (degrees);

4.2 in the case of average side-lobe patterns referred to in *considering* c), the antenna pattern given in Recommendation ITU-R F.1245 should be used;

5 that the following Notes should be regarded as part of this Recommendation:

NOTE 1 – It is essential that every effort be made to utilize the actual antenna pattern in coordination studies and interference assessment.

NOTE 2 – To evaluate the gain for all elevation angles, θ' from 0 to 180°, in a vertical plane, the value of x for elevation angles beyond 90° must be determined by using the complementary value of the elevation angle (180° – θ') at the supplementary value of the azimuth, i.e. 180 ± φ.

NOTE 3 – The different values of parameter k in *recommends* 3.1.1.1 and 3.2.1.1 are derived taking into account peak envelopes and average side-lobe levels of a number of typical measured antenna patterns in the 1 to 6 GHz frequency range.

NOTE 4 – In a case involving an antenna whose main beam width is different that this calculated with equation (3), it is recommended to use θ_3 as an input parameter.

NOTE 5 – As discussed in Annex 3, an exponential factor has been replaced by unity. As a result, the theoretical error introduced by this approximation will be less than 6% for 3 dB beamwidths in the elevation plane less than 45°.

NOTE 6 – The reference radiation pattern given in *recommends* 4.1 primarily applies in situations where the main lobe antenna gain is less than or equal to 20 dBi and the use of Recommendation ITU-R F.699 produces inadequate results. Further study is required to establish the full range of frequencies and gain over which the equations are valid.

NOTE 7 – Measured results of a specially designed sectoral antenna for use around 20 GHz indicate the possibility of compliance with a more restrictive reference side-lobe radiation pattern. Further studies are required to develop such an optimized pattern.

Annex 1

Reference radiation pattern for omnidirectional antennas as used in P-MP radio-relay systems

1 Introduction

An omnidirectional antenna is frequently used for transmitting and receiving signals at central stations of P-MP radio-relay systems. Studies involving sharing between these types of radio-relay systems and space service systems in the 2 GHz bands have used the reference radiation pattern described here.

2 Analysis

The reference radiation pattern is based on the following assumptions concerning the omnidirectional antenna:

– that the antenna is an *n*-element linear array radiating in the broadside mode;

– the elements of the array are assumed to be dipoles;

– the array elements are spaced 3λ/4.

The 3 dB beamwidth θ_3 of the array in the elevation plane is related to the directivity D by (see Annex 3 for the definition of D):

$$D = 10 \log\left[191.0\sqrt{0.818 + 1/\theta_3} - 172.4\right] \quad \text{dBi} \tag{5a}$$

Equation (5a) may be solved for θ_3 when the directivity is known:

$$\theta_3 = \frac{1}{\alpha^2 - 0.818} \tag{5b}$$

$$\alpha = \frac{10^{0.1D} + 172.4}{191.0} \tag{5c}$$

The relationship between the 3 dB beamwidth in the elevation plane and the directivity was derived on the assumption that the radiation pattern in the elevation plane was adequately approximated by:

$$f(\theta) = \cos^{m}(\theta)$$

where m is an arbitrary parameter used to relate the 3 dB beamwidth and the radiation pattern in the elevation plane. Using this approximation, the directivity was obtained by integrating the pattern over the elevation and azimuth planes.

The intensity of the far-field of a linear array is given by:

$$E_T(\theta) = E_e(\theta) \cdot AF(\theta) \quad (6)$$

where:

$E_T(\theta)$: total E-field at an angle of θ normal to the axis of the array

$E_e(\theta)$: E-field at an angle of θ normal to the axis of the array caused by a single array element

$AF(\theta)$: array factor at an angle θ normal to the axis of the array.

The normalized E-field of a dipole element is:

$$E_e(\theta) = \cos(\theta) \quad (7)$$

The array factor is:

$$AF_N = \frac{1}{N}\left[\frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] \quad (8)$$

where:

N: number of elements in the array

$\frac{\psi}{2} = \frac{1}{2}\left[2\pi\frac{d}{\lambda}\sin\theta\right]$

d: spacing of the radiators

λ: wavelength.

The following procedure has been used to estimate the number of elements N in the array. It is assumed that the maximum gain of the array is identical to the directivity of the array.

– Given the maximum gain of the omnidirectional antenna in the elevation plane, compute the 3 dB beamwidth, θ_3, using equations (5b) and (5c);

– Ignore the small reduction in off-axis gain caused by the dipole element, and note that the array factor, AF_N, evaluates to 0.707 (− 3 dB) when $N\frac{\psi}{2} = 1.396$; and

– N is then determined as the integer value of:

$$N = \left\lfloor \frac{2 \times 1.3916}{2\pi \frac{d}{\lambda} \sin\left(\frac{\theta_3}{2}\right)} \right\rfloor \tag{9}$$

where $|x|$ means the maximum integer value not exceeding x.

The normalized off-axis discrimination ΔD is given by:

$$\Delta D = 20 \log \left[\left| AF_N \times \cos(\theta) \right| \right] \qquad \text{dB} \tag{10}$$

Equation (10) has been evaluated as a function of the off-axis angle (i.e., the elevation angle) for several values of maximum gain. For values in the range of 8 dBi to 13 dBi, it has been found that the envelope of the radiation pattern in the elevation plane may be adequately approximated by the following equations:

$$G(\theta) = \max\left[G_1(\theta), G_2(\theta)\right] \tag{11a}$$

$$G_1(\theta) = G_0 - 12\left(\frac{\theta}{\theta_3}\right)^2 \qquad \text{dBi} \tag{11b}$$

$$G_2(\theta) = G_0 - 12 + 10 \log\left[\left(\max\left\{\frac{|\theta|}{\theta_3}, 1\right\}\right)^{-1.5} + k\right] \qquad \text{dBi} \tag{11c}$$

k is a parameter which accounts for increased side-lobe levels above what would be expected for an antenna with improved side-lobe performance.

Figures 1 to 4 compare the reference radiation envelopes with the theoretical antenna patterns generated from equation (11), for gains from 8 dBi to 13 dBi, using a factor of $k = 0$. Figures 5 to 8 compare the reference radiation envelopes with actual measured antenna patterns using a factor of $k = 0$. In Figs. 7 and 8, it can be seen that the side lobes are about 15 dB or more below the level of the main lobe, allowing for a small percentage of side-lobe peaks which might exceed this value. However practical factors such as the use of electrical downtilt, pattern degradations at band-edges and production variations would further increase the side lobes to about 10 dB below the main lobe in actual field installations. The k factor, mentioned above, in equation (11), is intended to characterize this variation in side-lobe levels. Figures 9 and 10 provide a comparison of a 10 dBi and a 13 dBi gain antenna, at 2.4 GHz, with the reference radiation pattern envelope, using $k = 0.5$. A factor of $k = 0.5$ represents side-lobe levels about 15 dB below the main-lobe peak. However, to account for increases in side-lobe levels which may be found in field installations, for typical antennas a factor of $k = 0.7$ should be used, representing side-lobe levels about 13.5 dB below the level of the main lobe. Finally, Figs. 11 and 12 illustrate the effect on elevation patterns of using various values of k.

FIGURE 1

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 10 dBi, k = 0

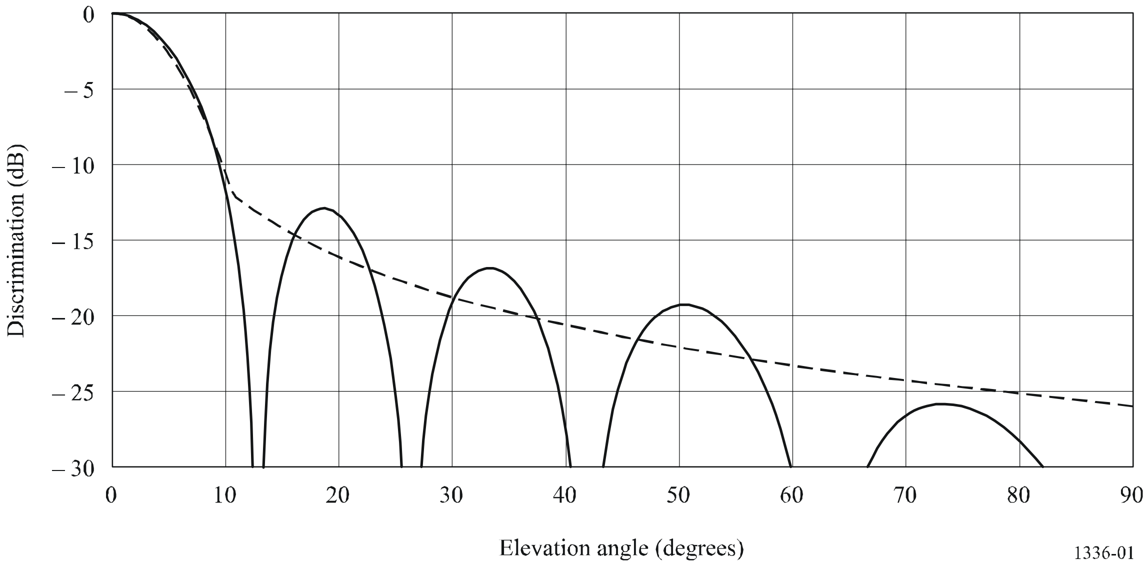


FIGURE 2

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 11 dBi, k = 0

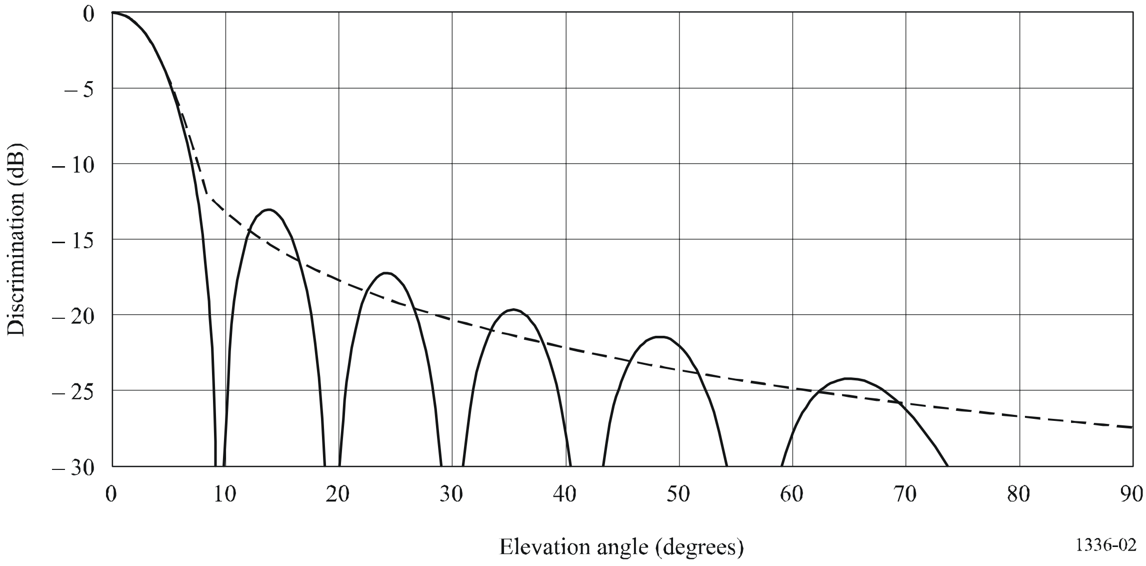


FIGURE 3

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 12 dBi, *k* = 0

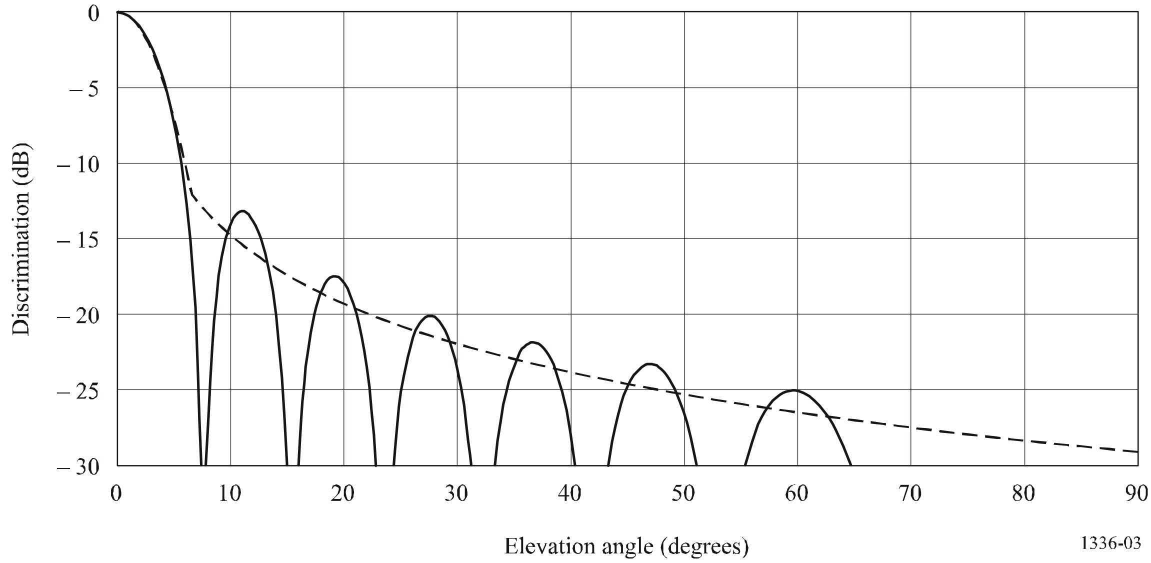


FIGURE 4

Normalized radiation pattern of a linear array of dipole elements compared with the approximate envelope of the radiation pattern

G_0 = 13 dBi, *k* = 0

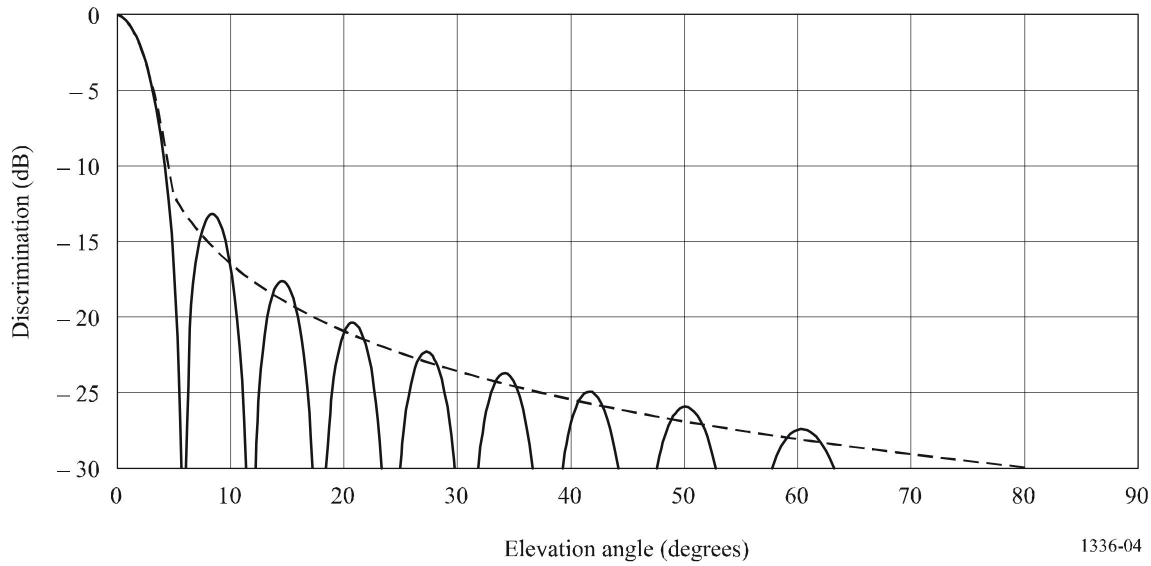


FIGURE 5

Comparison of measured pattern and reference radiation pattern envelope for an omnidirectional antenna with 11 dBi gain and operating in the band 928-944 MHz, $k = 0$

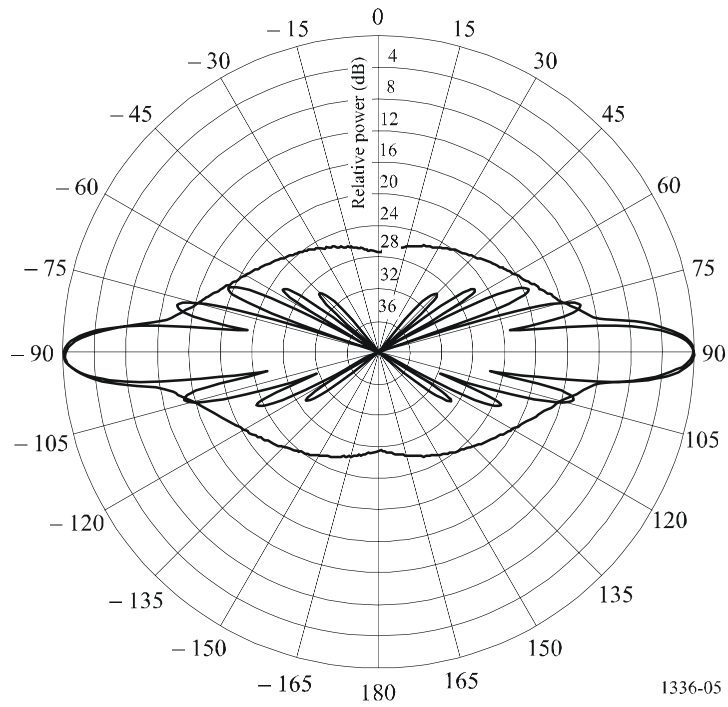


FIGURE 6

Comparison of measured pattern and the reference radiation pattern envelope for an omnidirectional antenna with 8 dBi gain and operating in the band 1 850-1 990 MHz, $k = 0$

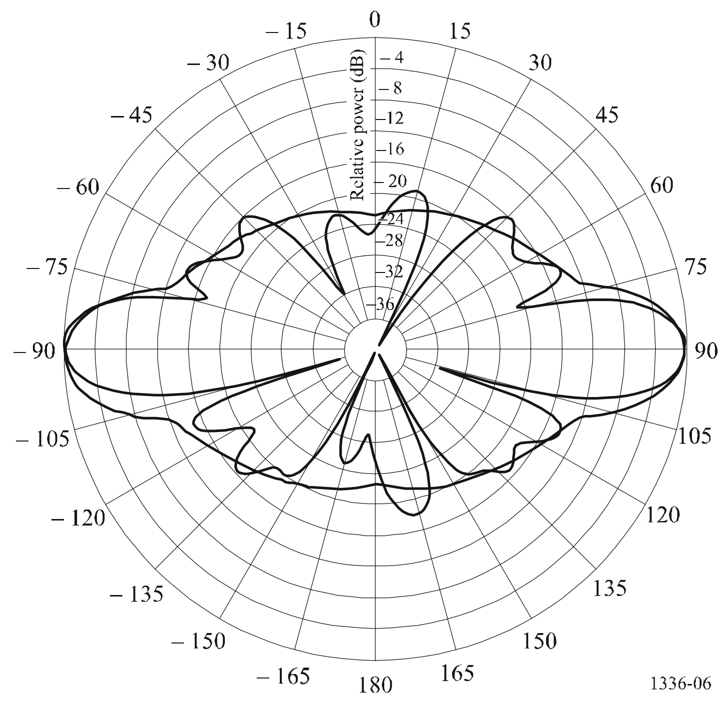


FIGURE 7

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0 for an omnidirectional antenna with 10 dBi gain and operating in the 1.4 GHz band

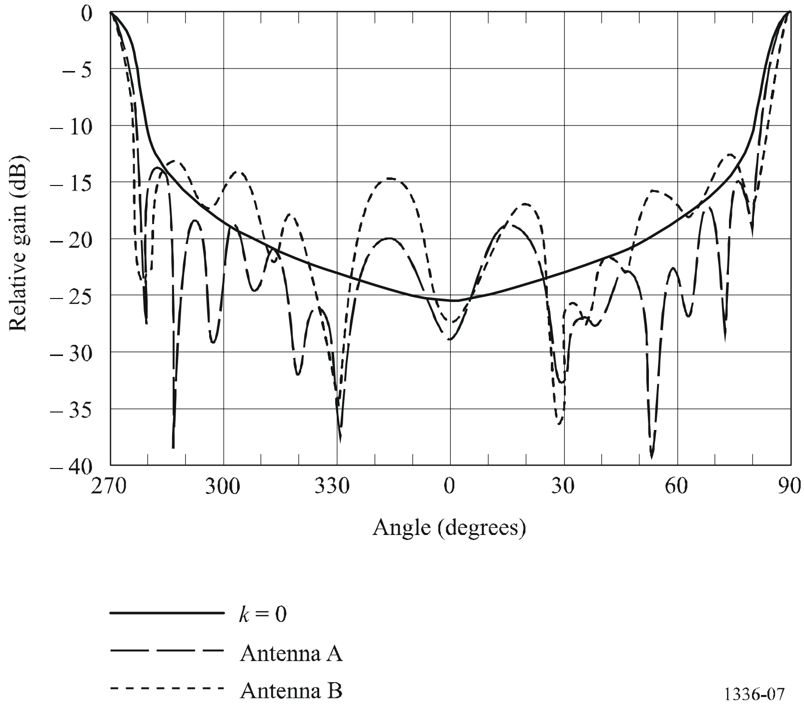


FIGURE 8

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0 for an omnidirectional antenna with 13 dBi gain and operating in the 1.4 GHz band

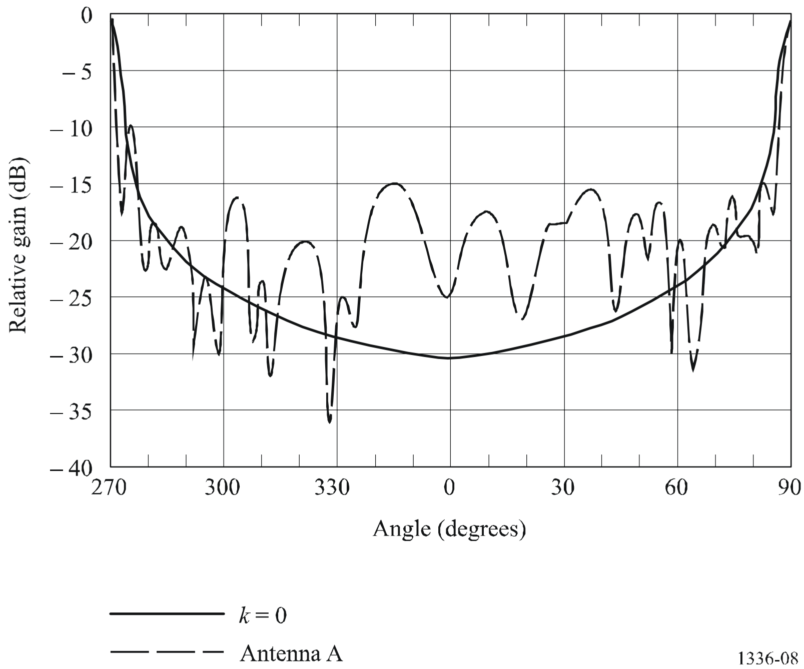


FIGURE 9

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0.5 for an omnidirectional antenna with 10 dBi gain and operating in the 2.4 GHz band

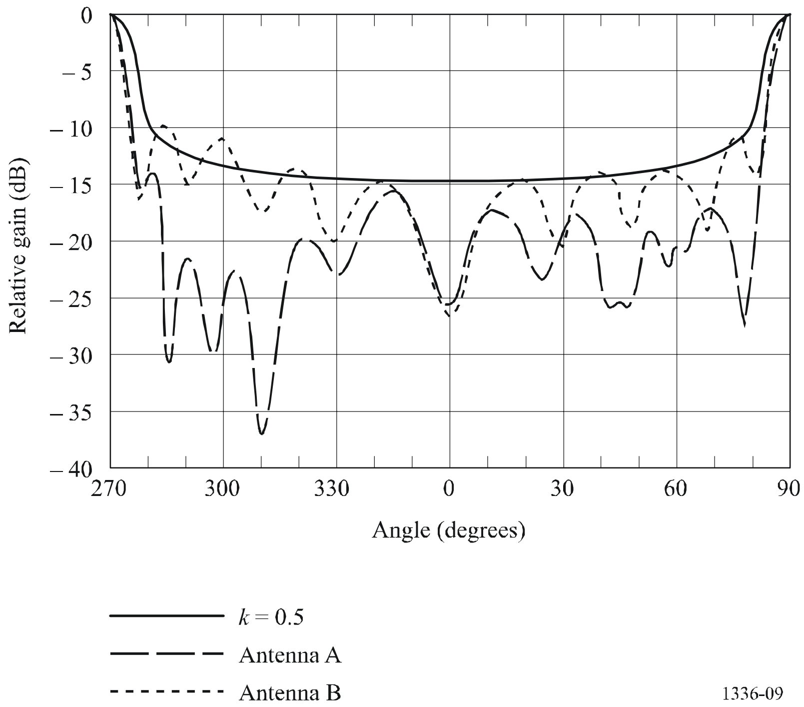


FIGURE 10

Comparison of measured pattern and the reference radiation pattern envelope with *k* = 0.5 for an omnidirectional antenna with 13 dBi gain and operating in the 2.4 GHz band

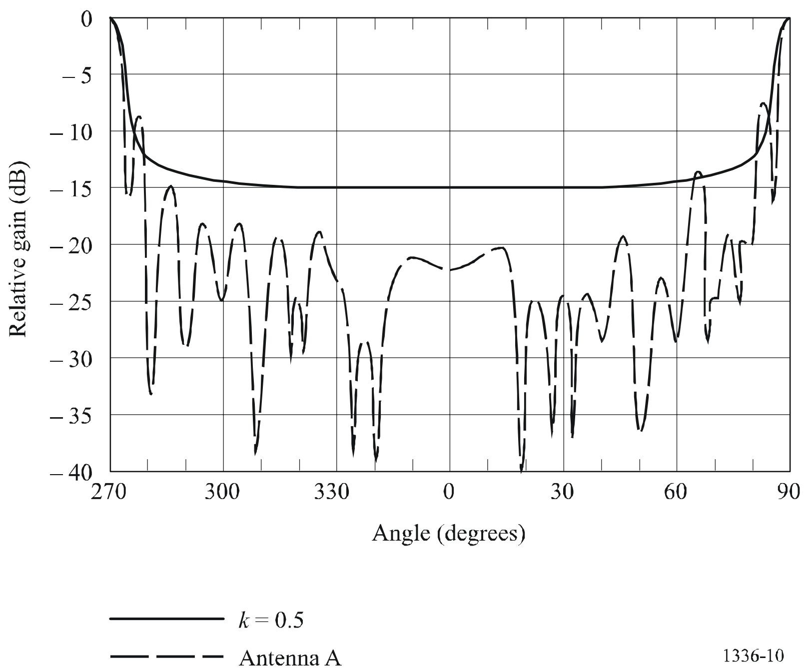


FIGURE 11

Reference radiation pattern envelopes for various values of *k* for an omnidirectional antenna with 10 dBi gain

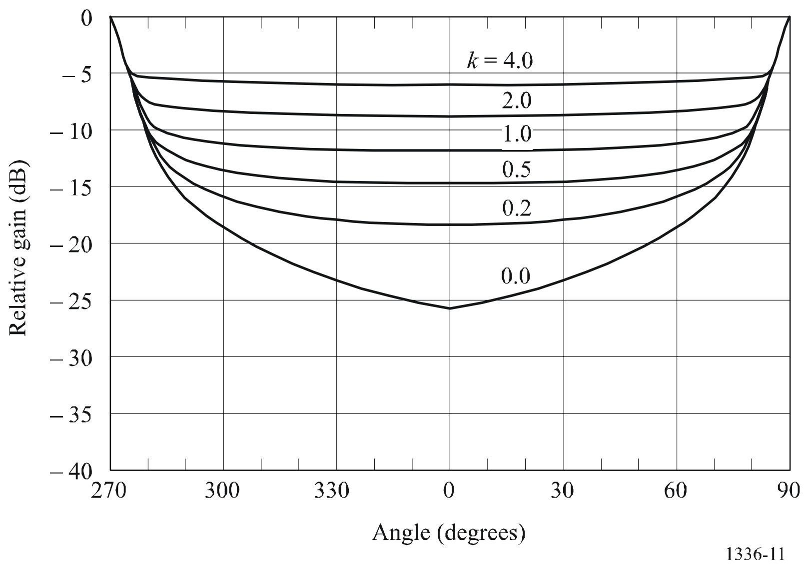
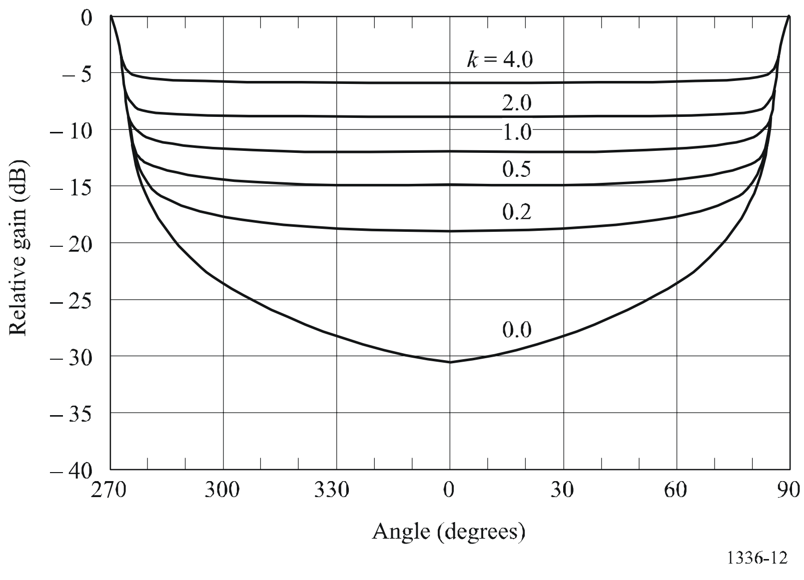


FIGURE 12

Reference radiation pattern envelopes for various values of *k* for an omnidirectional antenna with 13 dBi gain



3 Summary, conclusions and further analyses

A reference radiation pattern has been presented for omnidirectional antennas exhibiting a gain between 8 dBi and 13 dBi. The reference radiation pattern has been derived on the basis of theoretical considerations of the radiation pattern of a collinear array of dipoles. The proposed pattern has been shown to adequately represent the theoretical patterns and measured patterns over the range from 8 dBi to 13 dBi. Further work is required to determine the range of gain over which the reference radiation pattern is appropriate especially with regard to antennas operating in frequency bands above 3 GHz.

Annex 2

Reference radiation pattern for low-gain circularly symmetric subscriber antennas as used in P-MP radio-relay systems in the 1-3 GHz range

1 Introduction

An antenna with relatively low gain is frequently used for transmitting and receiving signals at the out-stations or in sectors of central stations of P-MP radio-relay systems. These antennas may exhibit a gain of the order of 20 dBi or less. It has been found that using the reference radiation pattern given in Recommendation ITU-R F.699 for these relatively low-gain antennas will result in an overestimate of the gain for relatively large off-axis angles. As a consequence, the amount of interference caused to other systems and the amount of interference received from other systems at relatively large off-axis angles will likely be substantially overestimated if the pattern of Recommendation ITU-R F.699 is used.

2 Analysis

The reference radiation pattern for a subscriber antenna is based on the following assumptions:

– that the directivity of the antenna is less than about 20 dBi;

– that the antenna pattern exhibits circularly symmetric about the main lobe;

– that the main-lobe gain is equal to the directivity.

The proposed reference radiation pattern is given by:

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\varphi_3}\right)^2 & \text{for } 0 \leq \theta < 1.08\,\varphi_3 \quad (12) \\ G_0 - 14 & \text{for } 1.08\,\varphi_3 \leq \theta < \varphi_1 \quad (13) \\ G_0 - 14 - 32\log\left(\dfrac{\theta}{\varphi_1}\right) & \text{for } \varphi_1 \leq \theta < \varphi_2 \quad (14) \\ -8 & \text{for } \varphi_2 \leq \theta \leq 180° \quad (15) \end{cases}$$

where:

$G(\theta)$: gain relative to an isotropic antenna (dBi)

G_0: maximum on-axis gain (dBi)

θ: off-axis angle (degrees)

φ_3: the 3 dB beamwidth (degrees)

$= \sqrt{27\,000 \times 10^{-0.1\,G_0}}$ degrees

$\varphi_1 = 1.9\,\varphi_3$ degrees

$\varphi_2 = \varphi_1 \times 10^{(G_0 - 6)/32}$ degrees.

3 Summary and conclusions

A reference radiation pattern has been presented for low-gain subscriber antennas exhibiting a gain of less than or equal to 20 dBi. The reference radiation pattern has been derived on the basis of limited data on the radiation patterns of flat plate array antennas considered for use in a local access P-MP system operating in the 2 GHz bands. The proposed pattern has been shown to more accurately represent the actual pattern than the pattern given in Recommendation ITU-R F.699. Further work is required to determine the range of gain over which the reference radiation pattern is appropriate and to compare the reference radiation pattern to measured patterns.

Annex 3

Relationship between gain and beamwidth for omnidirectional and sectoral antennas

1 Introduction

The purpose of this Annex is to derive the relationship between the gain of omnidirectional and sectoral antennas and their beamwidth in the azimuthal and elevation planes. Section 2 is an analysis of the directivity of omnidirectional and sectoral antennas assuming two different radiation intensity functions in the azimuthal plane. For both cases, the radiation intensity in the elevation plane was assumed to be an exponential function. Section 3 provides a comparison between the gain-beamwidth results obtained using the methods of Section 2 and results contained in the previous version of this Recommendation for omnidirectional antennas. Section 4 summarizes the results, proposes a provisional equation for gain-beamwidth for omnidirectional and sectoral antennas, and suggests areas for further study.

2 Analysis

The far-field pattern of the sectoral antenna in the elevation plane is assumed to conform to an exponential function, whereas the far-field pattern in the azimuth plane is assumed to conform to either a rectangular function or an exponential function. With these assumptions, the directivity, *D*, of the sectoral antenna may be derived from the following formulation in (spherical coordinates):

$$D = \frac{U_M}{U_0} \tag{16}$$

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} F(\varphi)\, F(\theta) \cos(\theta)\, \mathrm{d}\theta\, \mathrm{d}\varphi \tag{17}$$

where:

U_M: maximum radiation intensity

U_0: radiation intensity of an isotropic source

φ: angle in the azimuthal plane

θ: angle in the elevation plane

$F(\varphi)$: radiation intensity in the azimuthal plane

$F(\theta)$: radiation intensity in the elevation plane.

The directivity of omnidirectional and sector antennas is evaluated in the following sub-sections assuming the radiation intensity in the azimuthal plane is either a rectangular function or an exponential function.

2.1 Rectangular sectoral radiation intensity

Rectangular sectoral radiation intensity function, $F(\varphi)$, is assumed to be:

$$F(\varphi) = U\left(\frac{\varphi_s}{2} - |\varphi|\right) \tag{18}$$

where:

φ_s: beamwidth of the sector,

$$U(x) = 1 \quad \text{for} \quad x \geq 0$$
$$U(x) = 0 \quad \text{for} \quad x < 0 \tag{19}$$

For either rectangular or exponential sectoral radiation intensity functions, it is assumed that the radiation intensity in the elevation plane is given by:

$$F(\theta) = \mathrm{e}^{-a^2\theta^2} \tag{20}$$

where:

$$a^2 = -\ln(0.5) \times \left(\frac{2}{\theta_3}\right)^2 = \frac{2.773}{\theta_3^2} \tag{21}$$

θ_3: 3 dB beamwidth of the antenna in the elevation plane (degrees).

Substituting equations (18) and (20) into equation (17) results in:

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} U\left(\frac{\varphi_s}{2} - |\varphi|\right) \mathrm{d}\varphi \int_{-\pi/2}^{\pi/2} \mathrm{e}^{-a^2\theta^2} \cos(\theta)\, \mathrm{d}\theta \tag{22}$$

This double integral may be solved as the product of two independent integrals. The integral over φ is evaluated in a straightforward way. However, evaluating the integral over θ is somewhat more difficult. The integral over θ could be evaluated numerically with the results either tabulated or a polynomial fitted to the data. However, it is noted that if the limits of integration are changed to $\pm\infty$, the integral over θ is given in closed-form by:

$$\int_{-\pi/2}^{\pi/2} \mathrm{e}^{-a^2\theta^2} \cos(\theta)\, \mathrm{d}\theta \approx \int_{-\infty}^{\infty} \mathrm{e}^{-a^2\theta^2} \cos(\theta)\, \mathrm{d}\theta = \frac{1}{a}\sqrt{\pi}\, \mathrm{e}^{-1/4a^2} \tag{23}$$

This is a rather simple and flexible formulation that, depending on its accuracy, could be quite useful in evaluating the directivity of sector antennas as well as omnidirectional antennas.

The accuracy with which the infinite integral approximates the finite integral has been evaluated. The finite integral, i.e., the integral on the left-hand side of equation (23), has been evaluated for several values of 3 dB beamwidth using the 24 point Gaussian Quadrature method and compared with the value obtained using the formula corresponding to the infinite integral on the right-hand side of equation (23). (Actually, because of its symmetry, the finite integral has been numerically evaluated over the range 0 to π/2 and the result doubled.) The results for a range of example values of the 3 dB beamwidth in the elevation plane are shown in Table 1. The Table shows that for a 3 dB beamwidth of 45°, the difference between the values produced by the finite integral and the infinite integral approximation is less than 0.03%. At 25° and below, the error is essentially zero. Equation (22) is now readily evaluated:

$$U_0 = \frac{\varphi_s \theta_3}{4\pi} \sqrt{\frac{\pi}{2.773}} \times e^{\frac{\theta_3^2}{11.09}} \qquad (24)$$

TABLE 1

Relative accuracy of the infinite integral in equation (23) in the evaluation of the average radiation intensity

3 dB beamwidth in the elevation plane (degrees)	Finite integral	Infinite integral	Relative error (%)
45	1.116449558	1.116116449	0.0298
25	0.67747088	0.67747088	0.0000
20	0.549744213	0.549744213	0.0000
15	0.416896869	0.416896869	0.0000
10	0.280137168	0.280137168	0.0000
5	0.140734555	0.140734558	0.0000

From equations (18) and (20), $U_M = 1$. Substituting these values and equation (24) into equation (16) yields the directivity of a sector antenna given the beamwidth in the elevation and azimuthal planes:

$$D = \frac{11.805}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{11.09}} \qquad (25)$$

where the angles are given in radians. When the angles are expressed in degrees, equation (25) becomes:

$$D = \frac{38\,750}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{36\,400}} \qquad (26)$$

Note that for an omnidirectional antenna, equation (26) reduces to:

$$D = \frac{107.64}{\theta_3} e^{\frac{\theta_3^2}{36\,400}} \qquad (27a)$$

If it is assumed that the radiation efficiency is 100% and that the antenna losses are negligible, then the gain and the directivity of the omnidirectional antenna are identical. Additionally, for omnidirectional antennas with a 3 dB beamwidth less than about 45°, the relationship between the gain and the 3 dB beamwidth in the elevation plane may be simplified by setting the exponential factor equal to unity. The resulting error is less than 6%.

$$G_0 \approx \frac{107.64}{\theta_3} \tag{27b}$$

2.2 Exponential sectoral radiation intensity

The second case considered for the sectoral radiation intensity is that of an exponential function. Specifically:

$$F(\varphi) = \mathrm{e}^{-b^2 \varphi^2} \tag{28}$$

where:

$$b^2 = -\ln(0.5) \times \left(\frac{2}{\varphi_s} \right)^2 \tag{29}$$

and φ_s is the 3 dB beamwidth of the sector.

Substituting equations (20) and (28) into equation (14), changing the limits of integration so that the finite integrals become infinite integrals, integrating and then substituting the result into equation (16) yields the following approximation:

$$D = \frac{11.09}{\varphi_s \theta_3} \mathrm{e}^{\frac{\theta_3^2}{11.09}} \tag{30}$$

where the angles are as defined previously and are expressed in radians. Converting the angles to degrees transforms equation (30) into:

$$D = \frac{36\,400}{\varphi_s \theta_3} \mathrm{e}^{\frac{\theta_3^2}{36\,400}} \tag{31}$$

Comparing equations (26) and (31), it is seen that the difference between the directivity computed using either of the equations is less than 0.3 dB.

The results given by equation (31) should be compared to a number of measured patterns to determine the inherent effect of the radiation efficiency of the antenna and other losses on the coefficient. At this time, only two sets of measurements are available for sectoral antennas designed to operate in the 25.25 GHz to 29.5 GHz band. Measured patterns in the azimuthal and elevation planes are given, respectively, in Figs. 13 and 14 for one set of antennas and Figs. 15 and 16, respectively, for the second set. From Figs. 13 and 14, the 3 dB beamwidth in the azimuthal plane is 90° and the 3 dB beamwidth in the elevation plane is 2.5°. From equation (31), the directivity is 22.1 dB. This is to be compared with a measured gain of 20.5-21.4 dBi for the antenna over the range 25.5-29.5 GHz. Assuming the gain G_0 of the antenna in the band around 28 GHz is 0.7 dB less than its directivity, and the exponential factor is replaced by unity which introduces an increasing error with increasing beamwidth. The error reaches 6% at 45°. A larger beamwidth leads to a larger error. Based on these considerations, the semi-empirical relationship between the gain and the beamwidth of a sectoral antenna is given by:

$$G_0 \approx \frac{31\,000}{\varphi_s \theta_3} \tag{32a}$$

Similarly, from Figs. 15 and 16, the semi-empirical relationship between the gain and the beamwidth of that sectoral antenna is:

$$G_0 \approx \frac{34\,000}{\varphi_s \theta_3} \tag{32b}$$

FIGURE 13

Measured pattern in the azimutal plane of a 90° sector antenna. Pattern measured over the band 27.5 GHz to 29.5 GHz. The hand drawn cross marks on the left side of the Figure correspond to values obtained from equation (28) (when expressed in dB) for an assumed 3 dB beamwidth of 90° in the azimuthal plane

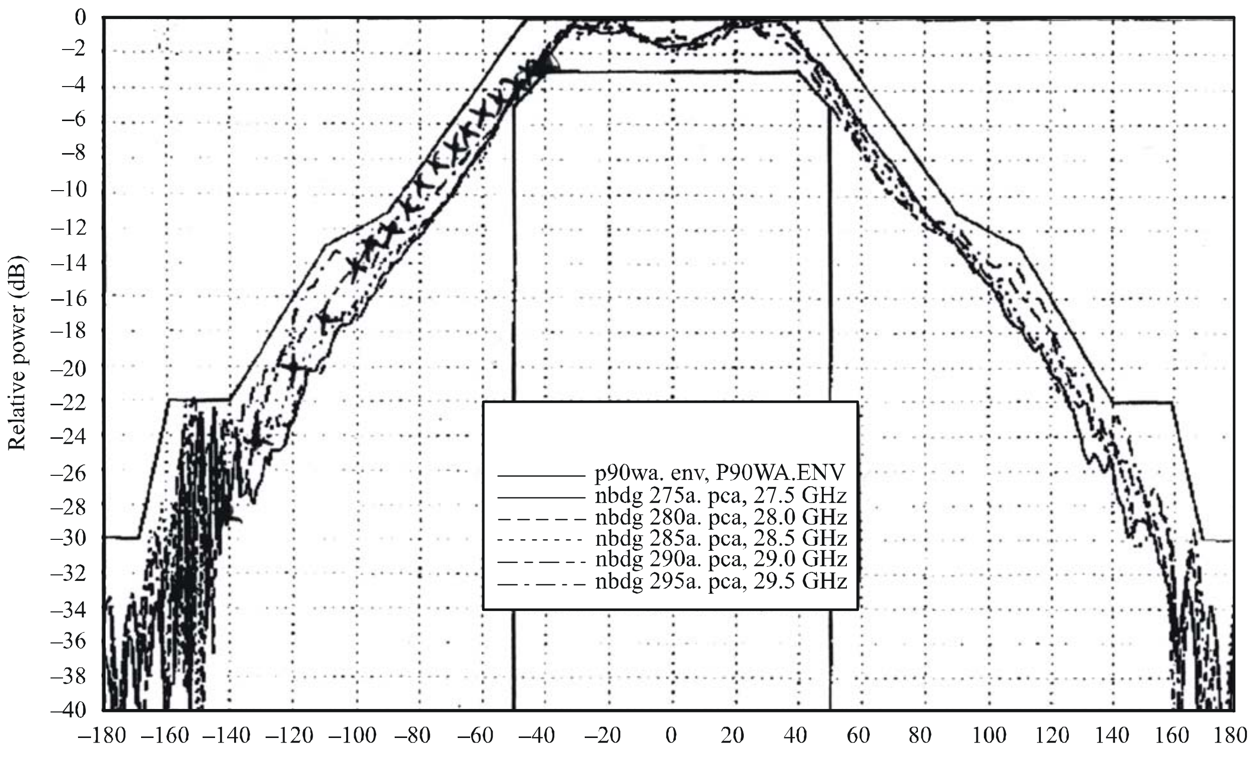


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FIGURE 14

Measured pattern in the azimutal plane of a 90° sector antenna.
Pattern measured over the band 27.5 GHz to 29.5 GHz

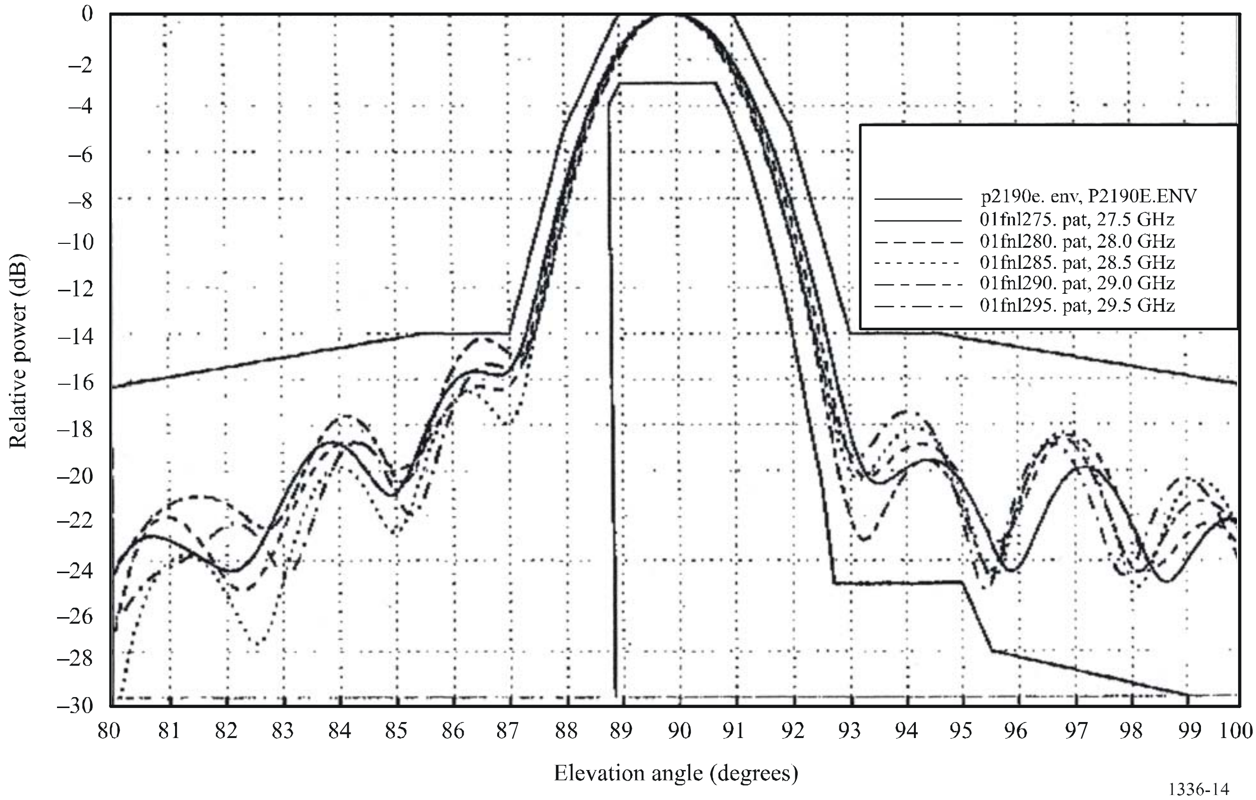


FIGURE 15

Azimuth pattern of typical 90° sectoral antenna (V-polarization)
15 dBi half-value angle: 90° (horn type antenna at 26 GHz)

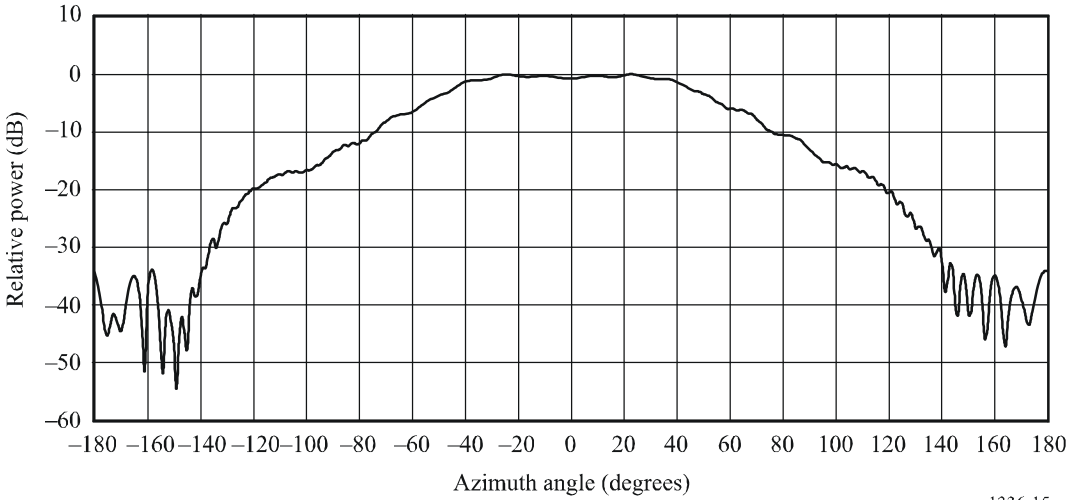
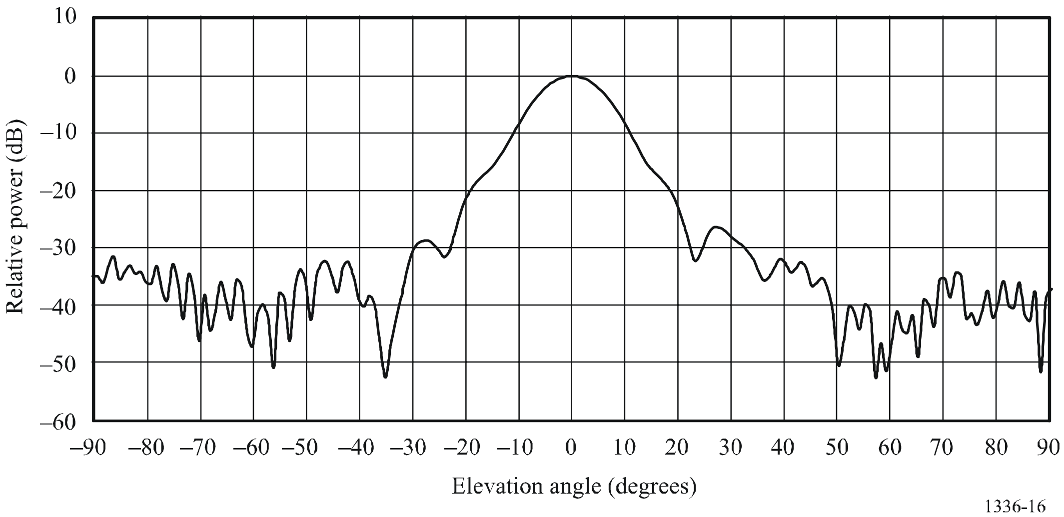


FIGURE 16

Elevation pattern of typical 90° sectoral antenna (V-polarization)
15 dBi half-value angle: 12° (horn type antenna at 26 GHz)



3 Comparison with previous results for omnidirectional antennas

The purpose of this section is to compare the results obtained for an omnidirectional antenna given by equation (27) with previous results reported in and summarized in Annex 1 of this Recommendation.

The radiation intensity in the elevation plane used in for an omnidirectional antenna was of the form:

$$F(\theta) = \cos^{2N} \theta \tag{33}$$

Substituting equation (33) into equation (17), and assuming that $F(\varphi) = 1$, yields:

$$U_0 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} \cos^{2N} (\theta) \cos(\theta) \, d\theta \, d\varphi \tag{34}$$

This double integral evaluates to:

$$U_0 = \frac{(2N)!!}{(2N+1)!!} \tag{35}$$

where $(2N)!!$ is the double factorial defined as $(2 \cdot 4 \cdot 6...(2N))$, and $(2N + 1)!!$ is also a double factorial defined as $(1 \cdot 3 \cdot 5...(2N + 1))$.

Thus, the directivity becomes:

$$D = \frac{(2N+1)!!}{(2N)!!} \tag{36}$$

The 3 dB beamwidth in the elevation plane is given by:

$$\theta_3 = 2 \cos^{-1}\left(0.5^{1/2N}\right) \tag{37}$$

A comparison between the directivity computed using the assumptions and methods embodied in equation (27) and those used in the derivation of equations (36) and (37) is given in Table 2. It is shown that results obtained using equation (27) compare favourably with the results using equations (36) and (37). In all cases equation (27) slightly underestimates the directivity obtained using equations (36) and (37). The relative error (%) of the estimates, when expressed in dB, is greatest for a 3 dB beamwidth in the elevation plane of 65°, amounting to –2.27%. The error (dB) for this case, expressed in dB, is –0.062 dB. For 3 dB beamwidth angles less than 65°, the relative error (%) and the error (dB), are monotonically decreasing functions as the 3 dB beamwidth decreases. For a 16° 3 dB beamwidth, the relative error (%) is about –0.01% and the error (dB) is less than about –0.0085 dB. An evaluation similar to that shown in Table 2 for values of 2*N* up to 10 000 (corresponds to a 3 dB beamwidth of 1.35° and a directivity of 19.02 dB) confirms that the results of the two approaches converge.

TABLE 2

Comparison of the directivity of omnidirectional antennas computed using equation (27a) with the directivity computed using equations (36) and (37)

2*N*	θ_3 (degrees) (equation (37))	Directivity (dB) (equation (36))	Directivity (dB) (equation (27a))	Relative error (%)	Error (dB)
2	90.0000	1.7609	1.7437	–0.98	–0.0172
4	65.5302	2.7300	2.6677	–2.28	–0.0623
6	54.0272	3.3995	3.3419	–1.69	–0.0576
8	47.0161	3.9110	3.8610	–1.28	–0.0500
10	42.1747	4.3249	4.2814	–1.01	–0.0435
12	38.5746	4.6726	4.6343	–0.82	–0.0383
14	35.7624	4.9722	4.9381	–0.69	–0.0341
16	33.4873	5.2355	5.2047	–0.59	–0.0307
18	31.5975	5.4703	5.4423	–0.51	–0.0280
20	29.9953	5.6822	5.6565	–0.45	–0.0256
22	28.6145	5.8752	5.8516	–0.40	–0.0237
24	27.4083	6.0525	6.0305	–0.36	–0.0220
26	26.3428	6.2164	6.1959	–0.33	–0.0205
28	25.3927	6.3688	6.3496	–0.30	–0.0192
30	24.5384	6.5112	6.4931	–0.28	–0.0181
32	23.7649	6.6449	6.6278	–0.26	–0.0171
34	23.0603	6.7708	6.7545	–0.24	–0.0162
36	22.4148	6.8897	6.8743	–0.22	–0.0154
38	21.8206	7.0026	6.9879	–0.21	–0.0147
40	21.2714	7.1098	7.0958	–0.20	–0.0140
42	20.7616	7.2120	7.1986	–0.19	–0.0134
44	20.2868	7.3096	7.2967	–0.18	–0.0129
46	19.8431	7.4030	7.3906	–0.17	–0.0124
48	19.4274	7.4925	7.4806	–0.16	–0.0119

TABLE 2 (*end*)

2*N*	**θ_3 (degrees) (equation (37))**	**Directivity (dB) (equation (36))**	**Directivity (dB) (equation (27a))**	**Relative error (%)**	**Error (dB)**
50	19.0367	7.5785	7.5671	–0.15	–0.0115
52	18.6687	7.6613	7.6502	–0.14	–0.0111
54	18.3212	7.7410	7.7302	–0.14	–0.0107
56	17.9924	7.8178	7.8075	–0.13	–0.0104
58	17.6808	7.8921	7.8820	–0.13	–0.0100
60	17.3847	7.9638	7.9541	–0.12	–0.0097
62	17.1031	8.0333	8.0239	–0.12	–0.0094
64	16.8347	8.1007	8.0915	–0.11	–0.0092
66	16.5786	8.1660	8.1571	–0.11	–0.0089
68	16.3338	8.2294	8.2207	–0.11	–0.0087
70	16.0996	8.2910	8.2825	–0.10	–0.0085
72	15.8751	8.3509	8.3426	–0.10	–0.0083
74	15.6598	8.4092	8.4011	–0.10	–0.0081

4 Summary and conclusions

Equations have been developed that permit easy calculation of the directivity and the relationship between the beamwidth and gain of omnidirectional and sectoral antennas as used in P-MP radio-relay systems. It is proposed to use the following equations to determine the directivity of sectoral antennas:

$$D = \frac{k}{\varphi_s \theta_3} e^{\frac{\theta_3^2}{36\,400}} \tag{38}$$

where:

$$k = 38\,750 \quad \text{for} \quad \varphi_s > 120° \qquad k = 36\,400 \quad \text{for} \quad \varphi_s \leq 120° \tag{39}$$

and φ_s = 3 dB beamwidth of the sectoral antenna in the azimuthal plane (degrees) for an assumed exponential radiation intensity in azimuth and θ_3 is the 3 dB beamwidth of the sectoral antenna in the elevation plane (degrees).

For omnidirectional antennas, it is proposed to use the following simplified equation to determine the 3 dB beamwidth in the elevation plane given the gain in dBi (see equation (27b)):

$$\theta_3 \approx 107.6 \times 10^{-0.1\,G_0}$$

It is proposed to use, on a provisional basis, the following semi-empirical equation relating the gain of a sectoral antenna (dBi) to the 3 dB beamwidths in the elevation plane and the azimuthal plane, where the sector is on the order of 120° or less and the 3 dB beamwidth in the elevation plane is less than about 45° (see equation (32a)):

$$\theta_3 \approx \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_s}$$

Further study is required to determine how to handle the transition region implicit in equation (39), and to determine the accuracy of these approximations as they apply to measured patterns of sectoral and omnidirectional antennas designed for use in P-MP radio-relay systems for bands in the range from 1 GHz to about 70 GHz.

Annex 4

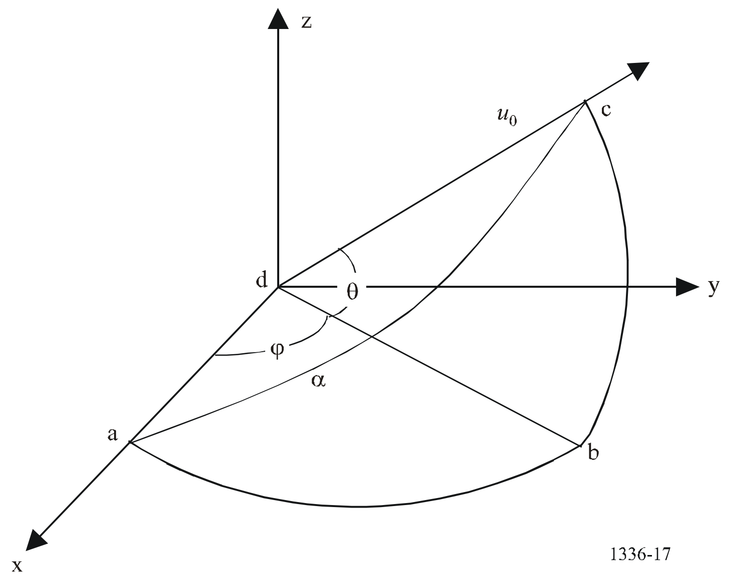
Procedure for determining the gain of a sectoral antenna at an arbitrary off-axis angle specified by an azimuth angle and an elevation angle referenced to the boresight of the antenna

1 Analysis

The basic geometry for determining the gain of a sectoral antenna at an arbitrary off-axis angle is shown in Fig. 17. It is assumed that the antenna is located at the centre of the spherical coordinate system; the direction of maximum radiation is along the x-axis; the x-y plane is the local horizontal plane; the elevation plane contains the z-axis; and, u_0 is a unit vector whose direction is used to determine the gain of the sectoral antenna.

FIGURE 17

Determining the off-boresight angle given the azimuth and elevation angle of interest



The two fundamental assumptions regarding this procedure are that:

– the –3 dB gain contour of the far-field pattern when plotted in two-dimensions as a function of the azimuth and elevation angles will be an ellipse as shown in Fig. 2; and

– the gain of the sectoral antenna at an arbitrary off-axis angle is a function of the 3 dB beamwidth and the beamwidth of the antenna when measured in the plane containing the x-axis and the unit vector u_0 (see Fig. 1).

Given the 3 dB beamwidth (degrees) of the sectoral antenna in the azimuth and elevation planes, φ_3 and θ_3, the numerical value of the boresight gain is given, on a provisional basis, by (see *recommends* 3.3 and equation (32a)).

$$G_0 = \frac{31\,000}{\varphi_3 \theta_3} \tag{40}$$

The first step in evaluating the gain of the sectoral antenna at an arbitrary off-axis angle, φ and θ, is to determine the value of α. Referring to Fig. 1 and recognizing that abc is a right-spherical triangle, α is given by:

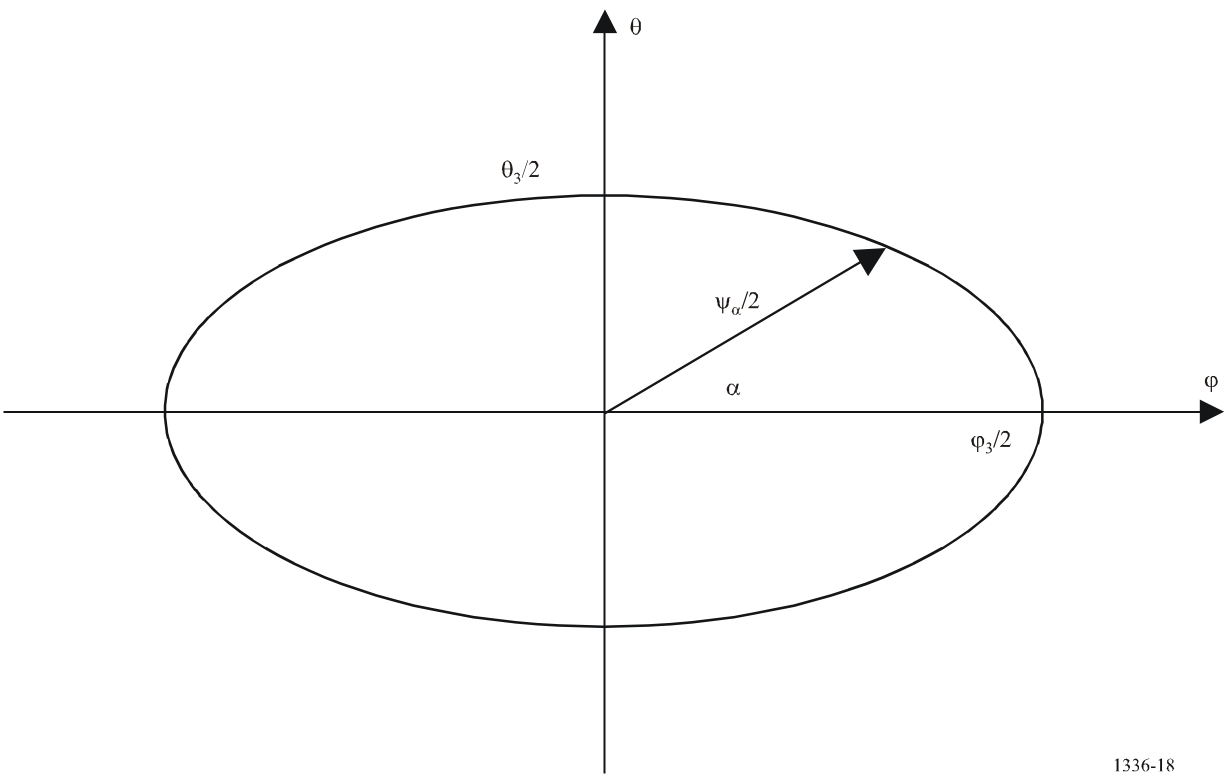
$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{\sin\varphi}\right) \tag{41a}$$

and the off-axis angle in the plane adc is given by:

$$\psi_\alpha = \cos^{-1}(\cos\varphi \cos\theta) \tag{41b}$$

FIGURE 18

Determination of the 3 dB beamwidth of an elliptical beam at an arbitrary inclination angle α



Given that the beam is elliptical, the 3 dB beamwidth of the sectoral antenna in the plane adc is determined from:

$$\frac{1}{\psi_\alpha^2} = \left(\frac{\cos\alpha}{\varphi_3}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2 \tag{42a}$$

or

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\alpha}{\varphi_3}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2}} \tag{42b}$$

The gain of the sectoral antenna at this arbitrary off-axis angle may be determined, on a provisional basis, using the reference radiation pattern given in *recommends* 3.1 and 3.2 of this Recommendation.

2 Conclusion

A procedure has been given to evaluate the gain of a sectoral antenna at an arbitrary off-axis angle. Further study is required to demonstrate the range over which this procedure is valid for sectoral antennas. Administrations are requested to submit measured patterns of sectoral antennas in order that this determination may be made.

Annex 5

Mathematical model of generic radiation patterns of omnidirectional and sectoral antennas for P-MP FWSs for use in statistical interference assessment

1 Introduction

The main text of this Recommendation (in *recommends* 2.2 and 3.2) gives reference radiation patterns, representing average side-lobe levels for both omnidirectional (in azimuth) and sectoral antennas, which can be applied in the case of multiple interference entries or time-varying interference entries.

On the other hand, for use in spatial statistical analysis of the interference, e.g. from a few GSO satellite systems into a large number of interfered-with FWS, a mathematical model is required for generic radiation patterns as given in the later sections in this Annex.

It should be noted that these mathematical models based on the sinusoidal functions, when applied in multiple entry interference calculations, may lead to biased results unless the interference sources are distributed over a large range of azimuth/elevation angles. Therefore, use of these patterns is recommended only in the case stated above.

2 Mathematical model for omnidirectional antennas

In case of spatial analysis of the interference from one or a few GSO satellite systems into a large number of FS stations, the following average side-lobe patterns should be used for elevation angles that range from 0° to 90° (see Annex 1):

$$G(\theta) = \begin{cases} G_0 - 12\left(\dfrac{\theta}{\theta_3}\right)^2 & \text{for} \quad 0 \le \theta < \theta_4 \\ G_0 - 12 + 10\ \log(k+1) + F(\theta) & \text{for} \quad \theta_4 \le \theta < \theta_3 \\ G_0 - 12 + 10\ \log\left[\left(\dfrac{|\theta|}{\theta_3}\right)^{-1.5} + k\right] + F(\theta) & \text{for} \quad \theta_3 \le \theta \le 90° \end{cases} \qquad (43a)$$

with:

$$F(\theta) = 10\ \log\left(0.9 \sin^2\left(\frac{3\pi\theta}{4\theta_3}\right) + 0.1\right) \qquad (43b)$$

where θ, θ_3, θ_4, G_0 and k are defined and expressed in *recommends* 2.1 in the main text.

NOTE 1 – In cases involving typical antennas operating in the 1-3 GHz range, the parameter k should be 0.7.

NOTE 2 – In cases involving antennas with improved side-lobe performance in the 1-3 GHz range, and for all antennas operating in the 3-70 GHz range, the parameter k should be 0.

3 Mathematical model for sectoral antennas

In case of spatial analysis of the interference from one or a few GSO satellite systems into a large number of FS stations, the following average side-lobe patterns should be used for elevation angles that range from 0° to 90° and for azimuth angles from –180° to 180°:

$$G(\varphi, \theta) = G_{ref}(x) \qquad (44)$$

where:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \le x < 1.396$$

$$G_{ref}(x) = G_0 - 12 - 15\ \log(x) + F_{ref}(x) \qquad \text{for} \quad 1.396 \le x$$

$$F_{ref}(x) = 10\ \log(0.9 \sin^2(0.75\pi x) + 0.1)$$

$$\alpha = \arctan\left(\frac{\tan\theta}{\sin\varphi}\right)$$

$$\psi_\alpha = \frac{1}{\sqrt{\left(\frac{\cos\alpha}{\varphi_3}\right)^2 + \left(\frac{\sin\alpha}{\theta_3}\right)^2}}$$

$$= \varphi_3 \cdot \theta_3 \sqrt{\frac{(\sin\theta)^2 + (\sin\varphi \cdot \cos\theta)^2}{(\varphi_3 \cdot \sin\theta)^2 + (\theta_3 \cdot \sin\varphi \cdot \cos\theta)^2}} \qquad \text{degrees}$$

$$\psi = \arccos(\cos\varphi \cdot \cos\theta) \qquad \text{degrees}$$

$$x = \psi / \psi_\alpha$$

where all variables and parameters are as defined in *recommends* 3.1 in the main text.

NOTE 1 – In cases involving sectoral antennas with a 3 dB beamwidth in the azimuthal plane less than about 120°, the relationship between the maximum gain and the 3 dB beamwidth in both the azimuthal plane and the elevation plane, on a provisional basis, is (see Annex 3):

$$\theta_3 = \frac{31\,000 \times 10^{-0.1\,G_0}}{\varphi_3}$$

where all parameters are as defined in *recommends* 3.1 in the main text.

Annex 6

Rationale used to develop equations for sectoral peak and average antennas between 1 GHz and about 6 GHz

1 Development of the equations for sectoral antennas between 1 GHz and 6 GHz

1.1 Rationale of the development

In order to appropriately model measured antenna pattern data at frequencies around 2 GHz a *k* parameter was introduced to account for side-lobe level performance into the equations used for sectoral antennas, similarly to the equations used for omnidirectional antennas.

It was found that a sectoral antenna pattern with a *k* parameter greater than zero agrees with measured antenna patterns whose first side-lobe levels are not well-estimated when using equations for ominidirectional antennas.

Equation (45) applies for peak sectoral antenna patterns:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \le x < 1$$

$$G_{ref}(x) = G_0 - 12 + 10\log(x^{-1.5} + k) \qquad \text{for} \quad 1 \le x \qquad (45)$$

The definition of all parameters is the same as in the main text of this Recommendation (*recommends* 3.1).

Note that there is a small discontinuity at $x = 1$ in equation (45).

When k = 0.7, for example, the lower formula becomes $G_{ref}(x) = G_0 - 9.7$ while the upper one remains as $G_{ref}(x) = G_0 - 12$ (about 2 dB difference). This discontinuity becomes smaller for smaller values of k.

In order to define more precisely the breakpoint between these two equations, it is found that with small approximations (k<<1 and x_k the breakpoint close to 1):

$$G_{ref}(x_k) = G_0 - 12 + 10\ \log(x_k^{-1.5} + k) = G_0 - 12x_k^2$$

$$\Rightarrow -12x_k^2 = -12 + \frac{10}{\ln(10)}\ln(x_0^{-1.5} + k) \approx -12 + \frac{10}{\ln(10)}k$$

$$\Rightarrow x_k \approx \sqrt{1 - \frac{5k}{6\ln 10}}$$

Thus, the breakpoint "1" can be replaced by a floating breakpoint x_k.

In that case, equation (45) becomes:

$$G_{ref}(x) = G_0 - 12x^2 \qquad \text{for} \quad 0 \leq x < x_k$$

$$G_{ref}(x) = G_0 - 12 + 10\ \log(x^{-1.5} + k) \qquad \text{for} \quad x_k \leq x \tag{46}$$

with:

$$x_k = \sqrt{1 - 0.36k}$$

1.2 Determination of the domain for which the equations are valid

It was determined, from evaluating measured antenna patterns, that different equations are needed for antennas operating from 1 GHz to about 6 GHz.

1.3 Studies on a value of a *k* parameter

Results of the analysis with respect to the k parameter and measured antenna patterns are summarized in Table 3, which also shows a trend of side-lobe performance improvement over a 10-year period.

1.4 Impact of the *k* parameter on sectoral antenna pattern

It is shown in general, that a peak antenna pattern with a typical value of k = 0.7 (see § 2 of this Annex) is appropriate in most cases (typical side-lobe case).

Note that k = 0 allows these antenna patterns to fit with sectoral antenna patterns with improved side-lobe performance.

TABLE 3

Value of parameter *k* in this Recommendation

Recommendation ITU-R F.1336			**1997**	**2000**	***2006 (used in this Recommendation)***
Omnidirectional antenna	Typical side lobe	1-3 GHz	$k = 1.5$	$k = 0.7$	$k = 0.7$
		3-70 GHz	$k = 1.5$	$k = 0$	$k = 0$
	Improved side lobe	1-70 GHz	$k = 0$	$k = 0$	$k = 0$
Sectoral antenna	Typical side lobe	1-3 GHz	–	$k = 0.7$	Peak: $k = 0.7$ Average: $k = 0.2$ (Note 1)
		3-6 GHz	–	$k = 0$	
		6-70 GHz	–	$k = 0$	$k = 0$
	Improved side lobe	1-70 GHz	–	$k = 0$	$k = 0$

NOTE 1 – For sectoral antennas, equations (46) are used.

Equation (47) should be used for peak sectoral antenna patterns:

$$
\begin{aligned}
G_{ref}(x) &= G_0 - 12x^2 && \text{for} \quad 0 \le x < x_k \\
G_{ref}(x) &= G_0 - 12 + 10\log(x^{-1.5} + k) && \text{for} \quad x_k \le x < 4 \\
G_{ref}(x) &= G_0 - \lambda_k - 15\log(x) && \text{for} \quad x \ge 4
\end{aligned}
\qquad (47)
$$

with $\lambda_k = 12 - 10\ \log(1 + 8k)$ and $x_k = \sqrt{1 - 0.36k}$

and

x_k: breakpoint which ensures continuity between the main lobe and the first side lobes

λ_k: needed attenuation factor below the antenna gain which ensures continuity between side lobes and back lobes for $x = 4$.

Equation (47) is used in *recommends* 3.1.1 and 3.2.1.

2 Consideration on parameter *k* for sectoral antennas in the 1-6 GHz range

In order to evaluate an appropriate value for *k,* the total difference was calculated between the reference pattern and measured antenna patterns provided by several countries for both fixed and mobile applications. These measured patterns provided the gain for many values of elevation angle.

For the peak pattern, the experimental data were compared directly with equation (47) with a *k* factor equal to 0.7.

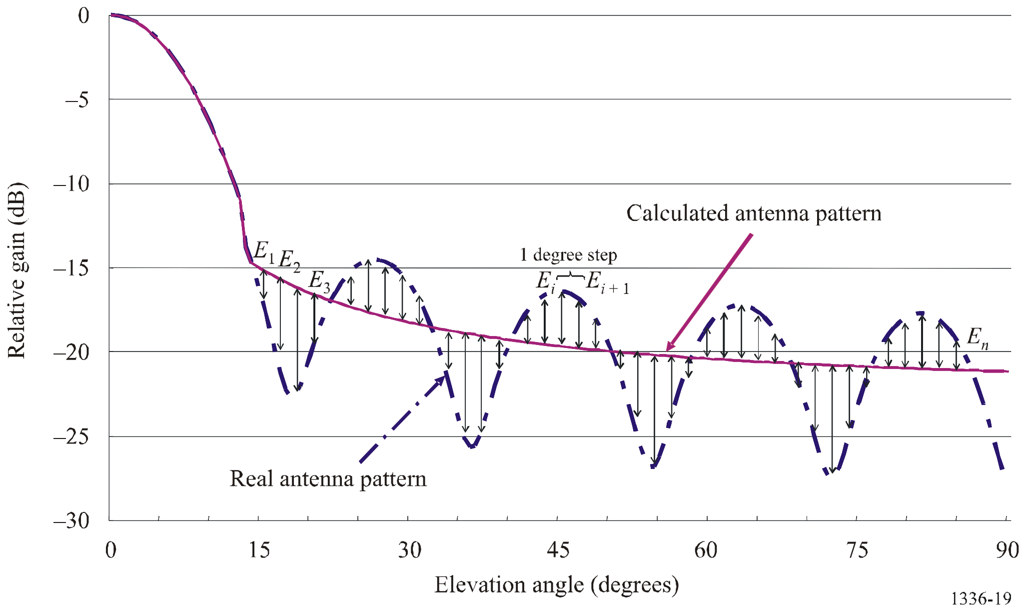
For the average pattern, the calculation was conducted only for the side-lobe range (not for the main-lobe portion); differences between the computed and real antenna patterns were sampled every one degree to determine the "Total error".

Total error is defined as below. E_i is calculated for a real value and not for a dB value.

$$\text{Total error} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} E_i^2}$$

FIGURE 19

Total error calculation to evaluate *k* parameter for the average pattern



The total error was calculated for each pattern using several *k* parameters between 0 and 0.3. The results are shown in Fig. 19. One could regard the value of the *k* parameter that provides the minimum total error as the optimum value. Based on this analysis, the value *k* = 0.2 should be used for average antenna patterns.

Another important factor to consider is "Sigma value" which is defined by the total power integration over the range of angles.

The basic idea is that:

– for peak envelope patterns, Sigma value should be in the range 2-4 dB;

– for average side-lobe patterns, Sigma value should be in the range 0-1 dB.

Calculation results of the Sigma values for the equations recommended for representative examples of typical antennas are given in Table 4.

For the peak envelope patterns, the Sigma value for *k* = 0.7 is within the permissible level. In addition, *k* = 0.2 will be a possible value for the average side-lobe patterns.

FIGURE 20

Example of evaluation of an optimum *k* value for the average pattern

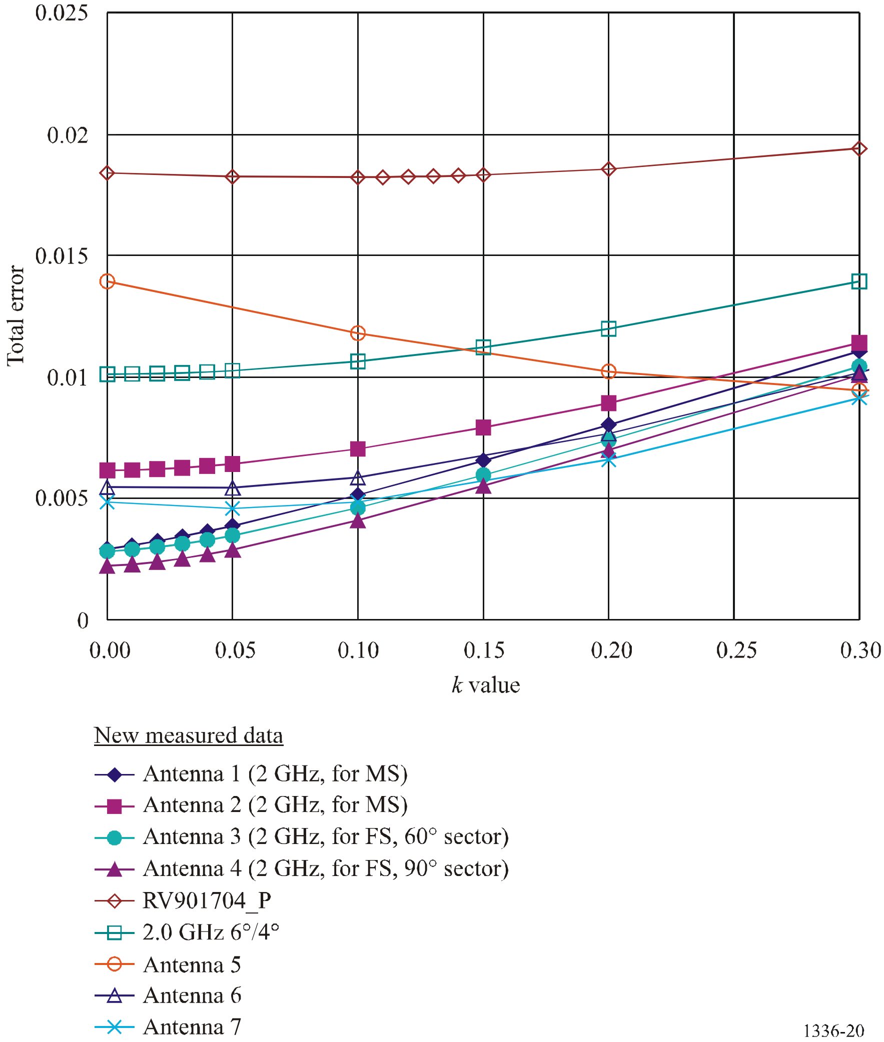


TABLE 4

Calculation results of the Sigma values

	Pattern	Equations	*k* parameter	Sigma value	
				16 dBi, 60° sector	16 dBi, 120° sector
Typical antennas in 1-6 GHz range	Peak envelope	*recommends* 3.1.1	$k = 0.7$	3.8 dB	2.55 dB
	Average side lobe	*recommends* 3.2.1	$k = 0.2$	0.8 dB	0.12 dB
			$k = 0.4$	1.43 dB	0.57 dB
			$k = 0.6$	1.93 dB	0.97 dB