RECOMMENDATION ITU-R F.1765

Methodology for determining the aggregate equivalent isotropically radiated power from point-to-point high-density applications in the fixed service operating in bands above 30 GHz

(2006)

Scope

This Recommendation provides methodologies which may be used to derive the aggregate equivalent isotropically radiated power (a.e.i.r.p) for transmitting point-to-point (P-P) high density applications in the fixed service (HDFS) stations in bands above 30 GHz which may be used by administrations wishing to assess the potential interference from P-P HDFS stations to other interfered-with services.

The ITU Radiocommunication Assembly,

considering

a) that an estimate of the aggregate equivalent isotropically radiated power (a.e.i.r.p.) from a deployment of point-to-point (P-P) high density applications in the fixed service (HDFS) transmitting stations referred to a central point may be required by administrations to assess the potential interference from P-P HDFS stations to other victim services on a national and bilateral basis;

b) that using automatic transmitter power control (ATPC) in P-P transmitters would reduce the aggregate radiated power;

c) that it is also necessary to determine the a.e.i.r.p. as a function of the elevation angle to be evaluated, taking account of mode (2) propagation mechanisms,

recognizing

1 that No. 5.547 of the Radio Regulations (RR) identifies the bands 31.8-33.4 GHz, 37-40 GHz, 40.5-43.5 GHz, 51.4-52.6 GHz, 55.78-59 GHz and 64-66 GHz as being available for high-density applications in the fixed service (HDFS),

noting

a) that Resolution 75 (WRC-2000) invites ITU-R to develop, as a matter of urgency, the technical basis for determining the coordination area for coordination of a receiving earth station in the space research service (deep space) with HDFS transmitting stations in the 31.8-32.3 GHz and 37-38 GHz bands;

b) that Resolution 79 (WRC-2000) invites ITU-R to conduct studies on the coordination distance between radio astronomy stations operating in the 42.5-43.5 GHz band and HDFS systems, *recommends*

1 that the following mathematical models may be provisionally used to derive the a.e.i.r.p. for transmitting P-P HDFS stations under the assumption that the elevation angles of all HDFS transmitting antennas are 0° (see Notes 1, 2, 3, 5, 6 and 9):

1.1 when the elevation angle of the direction of the a.e.i.r.p to be evaluated is 0°:

$$a.e.i.r.p. = P_t + 1.061\,(\log N_t)^2 + (-0.1164\,G_t + 6.103)\,\log N_t + 0.9428\,G_t - 2.62 \text{ dBW}$$

1.2 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 2.5°:

$$a.e.i.r.p. = P_t - 0.13743\,(\log N_t)^3 + 1.8243\,(\log N_t)^2 + 1.5569\,\log N_t + 0.0052917\,G_t^3 - 0.57530\,G_t^2 + 19.985\,G_t - 200.77 \text{ dBW}$$

1.3 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 5°:

$$a.e.i.r.p. = P_t + 0.54858\,(\log N_t)^2 + 5.6488\,\log N_t - 0.0036218\,G_t^3 + 0.42380\,G_t^2 - 16.645\,G_t + 227.44 \text{ dBW}$$

1.4 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 10°:

$$a.e.i.r.p. = P_t + 9.086\,\log N_t - 0.25\,G_t + 8.30 \text{ dBW}$$

1.5 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 15°:

$$a.e.i.r.p. = P_t + 9.344\,\log N_t - 0.25\,G_t + 5.19 \text{ dBW}$$

1.6 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 20°:

$$a.e.i.r.p. = P_t + 9.522\,\log N_t - 0.25\,G_t + 3.19 \text{ dBW}$$

1.7 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 25°:

$$a.e.i.r.p. = P_t + 9.663\,\log N_t - 0.25\,G_t + 1.78 \text{ dBW}$$

1.8 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 30°:

$$a.e.i.r.p. = P_t + 9.775\,\log N_t - 0.25\,G_t + 0.74 \text{ dBW}$$

where:

P_t: transmitter power at the antenna input (dBW)

N_t: number of transmitters

G_t: antenna gain (dBi);

N_t: number of transmitters;

2 that the following mathematical models may be provisionally used to derive the a.e.i.r.p. for transmitting P-P HDFS stations under the assumption that the HDFS transmitting antennas have variable elevation angles as described in Annex 1 (see Notes 1, 2, 4, 5, 6, 8 and 9):

2.1 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 0°:

$$a.e.i.r.p. = P_t + 0.82096\,(\log N_t)^3 + (-0.15210\,G_t - 0.92771)\,(\log N_t)^2 + (0.024504\,G_t^2 - 1.0198\,G_t + 27.270)\,\log N_t - 0.077296\,G_t^2 + 5.1982\,G_t - 73.62 \text{ dBW}$$

2.2 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 2.5°:

$$a.e.i.r.p. = P_t + 0.93906\,(\log N_t)^3 + (-0.31918\,G_t + 3.4110)\,(\log N_t)^2 + (0.023524\,G_t^2 + 0.096937\,G_t - 4.8156)\,\log N_t + 0.0011791\,G_t^3 - 0.21452\,G_t^2 + 8.5619\,G_t - 82.88 \text{ dBW}$$

2.3 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 5°:

$$a.e.i.r.p. = P_t + (-0.10457\ G_t + 3.0618)\ (\log N_t)^3 + (0.027889\ G_t^2 - 1.1358\ G_t + 9.7775)\ (\log N_t)^2 + (-0.15803\ G_t^2 + 9.3247\ G_t - 132.36) \log N_t + 0.20619\ G_t^2 - 13.901\ G_t + 247.30 \text{ dBW}$$

2.4 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 10°:

$$a.e.i.r.p. = P_t + 9.263 \log N_t - 0.2511\ G_t + 8.43 \text{ dBW}$$

2.5 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 15°:

$$a.e.i.r.p. = P_t + 9.299 \log N_t - 0.25\ G_t + 5.45 \text{ dBW}$$

2.6 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 20°:

$$a.e.i.r.p. = P_t + 9.497 \log N_t - 0.25\ G_t + 3.32 \text{ dBW}$$

2.7 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 25°:

$$a.e.i.r.p. = P_t + 9.651 \log N_t - 0.25\ G_t + 1.84 \text{ dBW}$$

2.8 when the elevation angle of the direction of the a.e.i.r.p. to be evaluated is 30°:

$$a.e.i.r.p. = P_t + 9.767 \log N_t - 0.25\ G_t + 0.79 \text{ dBW;}$$

3 that for a different elevation angle of the direction of the a.e.i.r.p. to be evaluated for which the formula is not given in *recommends* 1 or 2, the a.e.i.r.p. should be estimated by means of interpolation;

4 that the distance to the interfered-with station should be generally measured from the centre of the HDFS deployment area (see Note 7).

NOTE 1 – Annex 1 describes a method for determining a.e.i.r.p. values given in *recommends* 1 and 2. The a.e.i.r.p. values corresponding to 0 or low elevation angles of the directions to be evaluated will be useful for estimating interference through mode (1) propagation mechanisms, while those corresponding to high elevation angles of the directions to be evaluated will be useful for estimating interference through mode (2) propagation mechanisms.

NOTE 2 – The formulae in *recommends* 1 and 2 were derived as approximations for $G_t = 28$ to 46 dBi and $N_t = 32$ to 8 192. The probability of the a.e.i.r.p. exceeding the values in *recommends* 1 and 2 is 5% (that is, the confidence level of the calculations is 95%). The maximum errors of the approximations are typically in the order of 0.5 dB, but about 1 dB in some cases of complicated approximation formulae using the third order polynomials of G or $\log N_t$. Determination of the most appropriate confidence level requires further study.

NOTE 3 – The formulae in *recommends* 1 are based on an assumption that the azimuth angles of HDFS antennas are uniformly distributed over 0° to 360° and their elevation angles are 0°.

NOTE 4 – The formulae in *recommends* 2 are based on an assumption that the azimuth angles of HDFS antennas are uniformly distributed over 0° to 360° and their elevation angles are variable as described in § 2.3 of Annex 1. Further study is required in order to establish the most appropriate probability distribution function of HDFS antenna elevation angles to be used in each frequency band.

NOTE 5 – The formulae in *recommends* 1 and 2 may over estimate the actual a.e.i.r.p. since no consideration is given to potential clutter loss. Further study is required to assess the magnitude of this factor.

NOTE 6 – In the case of HDFS systems employing ATPC, P_t in the formulae of *recommends* 1 and 2 should be the transmitter power under the normal condition where there is no precipitation. Generally speaking, the interference to the victim station will be less significant during precipitation.

NOTE 7 – In general, the distance defined in *recommends* 4 will be appropriate for evaluating the a.e.i.r.p. provided the distance between the victim receiver and the HDFS deployment area is not too short compared to the radius area of HDFS deployment area (see § 1.3 of Annex 1).

NOTE 8 – Recommendation ITU-R F.1498 contains other distributions of elevation angles of HDFS antennas operating in the 37-40 GHz range. Further study is required to extend this Recommendation to cover such distributions.

NOTE 9 – In order to facilitate computer implementation of this Recommendation, Appendix 1 to Annex 1 presents the approximate formulae in *recommends* 1 and 2 in a tabular form.

Annex 1

Methodology for determining the aggregate interference power from P-P HDFS

1 Simulation method

1.1 Introduction

Resolution 75 (WRC-2000) requests the development of the technical basis for determining the coordination area for coordination between receiving earth stations in the space research service (deep space) and transmitting stations of high-density applications in the fixed service (HDFS), in the 31.8-32.3 GHz and 37-38 GHz frequency bands. In addition, Resolution 79 (WRC-2000) invites ITU-R to conduct studies on the coordination distance between radio astronomy stations operating in the 42.5-43.5 GHz band and HDFS systems.

This Recommendation provides methodologies which may be used to derive the a.e.i.r.p. for transmitting P-P HDFS stations which may be used by administrations wishing to assess the potential interference from P-P HDFS stations to other interfered-with services in their national and bilateral discussions. The methodologies in this Recommendation may be used as a basis for further study by administrations wishing to answer the *resolves* under Resolutions 75 (WRC-2000) and 79 (WRC-2000).

Using the 38 GHz band as an example, simulations of P-P networks of HDFS have been used to develop a mathematical model from which to assess the equivalent aggregated interference power radiated from such networks. However, the calculation results are not frequency dependent. The aggregated power is expressed in terms of the number of transmitters, antenna gains and transmitter power levels and is found to aggregate (logarithmically) at a lower rate than the 10 log N, where N is the number of transmitters.

This section describes a methodology for estimating aggregated radiated power from a distribution of P-P HDFS using computer simulation.

In order to determine the aggregated radiated power equivalent to a single transmitter at the edge of the network closest to the interfered-with station receiver, P-P transmitters of HDFS have been simulated by varying the number of transmitters, the antenna gains, elevation angles and the antenna azimuths. In this context, the total radiated power is defined in terms of an aggregate equivalent isotropically radiated power (a.e.i.r.p.). For the purpose of this simulation, this represents the sum of the radiated powers from a network of transmitters, distributed over an area, and received at a distant point, corrected for the free-space path loss between that point and the closest transmitter, i.e.:

$$a.e.i.r.p. = \sum_{all\ directions} P_{received} + L_{fs} \qquad \text{dBW}^{*} \qquad (1)$$

where:

L_{fs} : free-space path loss.

1.2 System parameters

An extensive survey of P-P HDFS was undertaken, including Recommendation ITU-R F.758, documentation submitted to ITU-R and from other sources, from which generic set of system parameters were derived and which were used in the simulations.

Three antenna gains, 28, 36 and 44 dBi, were considered as input parameters to the model. Recommendation ITU-R F.1245 was used as a typical antenna radiation pattern. Transmitter power of 20 dBW was used in the simulations, but the absolute value of the power is not important. The effects of polarization were not taken into account.

Test receiving stations with isotropic antennas with antenna gain of 0 dBi, to aggregate together signals from all the P-P transmitters were located at distances of 50, 100 and 150 km from the edge of the network.

1.3 Analytical simulations

Simulations have been made with varying numbers of transmitters whose antennas are rotating in azimuth at random scan rates between 0 and 1 degree/s, while the starting azimuths were also set randomly between 0° and 360°. By sampling the aggregated power over a period of time, distributions are obtained which describe the probability that the antennas point in a given direction and which can then provide estimates of the worst-case power levels for a certain degree of risk. Examination of the power levels received at the three test receivers, at 50, 100 and 150 km from the edge of the network, indicated little difference when corrected for free-space path loss. Test receivers were located only in one direction from the network since circular symmetry was ensured through the azimuthal rotation of all the transmitting antennas. The transmitters were distributed uniformly over a circular area with a diameter of 25 km and some of the simulations were repeated with transmitters distributed over circular areas with diameters of 15 and 35 km. Figure 1 shows the cumulative distribution of power levels from a single transmitter with antenna gain of 44 dBi and antenna elevation angle of 0°, and shows clearly the antenna radiation pattern, as expected.

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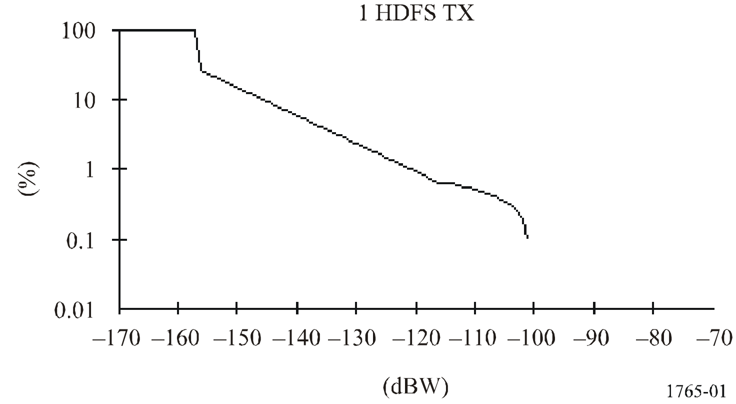
* Power received by an isotropic antenna (0 dBi antenna gain).

FIGURE 1

Cumulative probability distribution of power from a single randomly located antenna



As more antennas are included, the probability distribution changes. Figure 2 shows the distribution from 12 randomly-rotating antennas. Figure 2 illustrates that two distributions are forming: power from the main lobe of a single antenna is combining with power from the side lobes of the other antennas in the network to yield a power level of about −90 dBW (equivalent to about 68 dBW when corrected for free-space path loss), while the side lobes of all antennas are combining together to form a larger skewed distribution at lower power levels. As the number of transmitters increases, this lower peak from the antenna side lobes increases in magnitude until it eventually subsumes the main-lobe peak, and the distribution approaches a log normal distribution, as shown in the examples in Fig. 3.

FIGURE 2

Distribution of power levels from 12 randomly located antennas

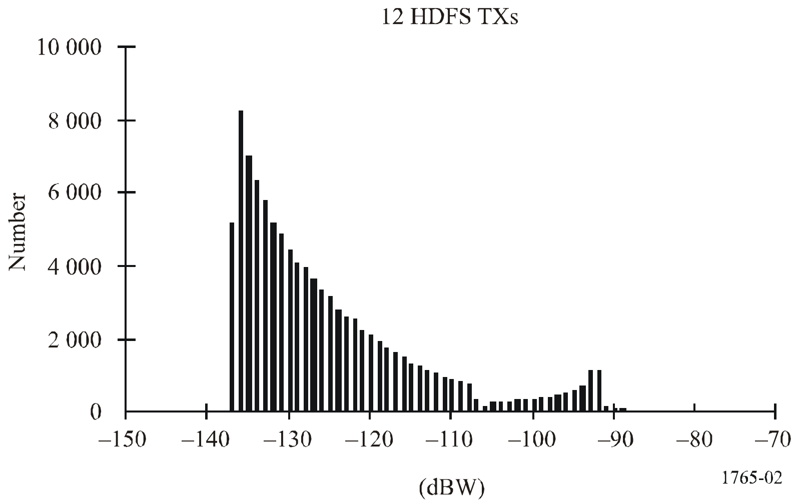
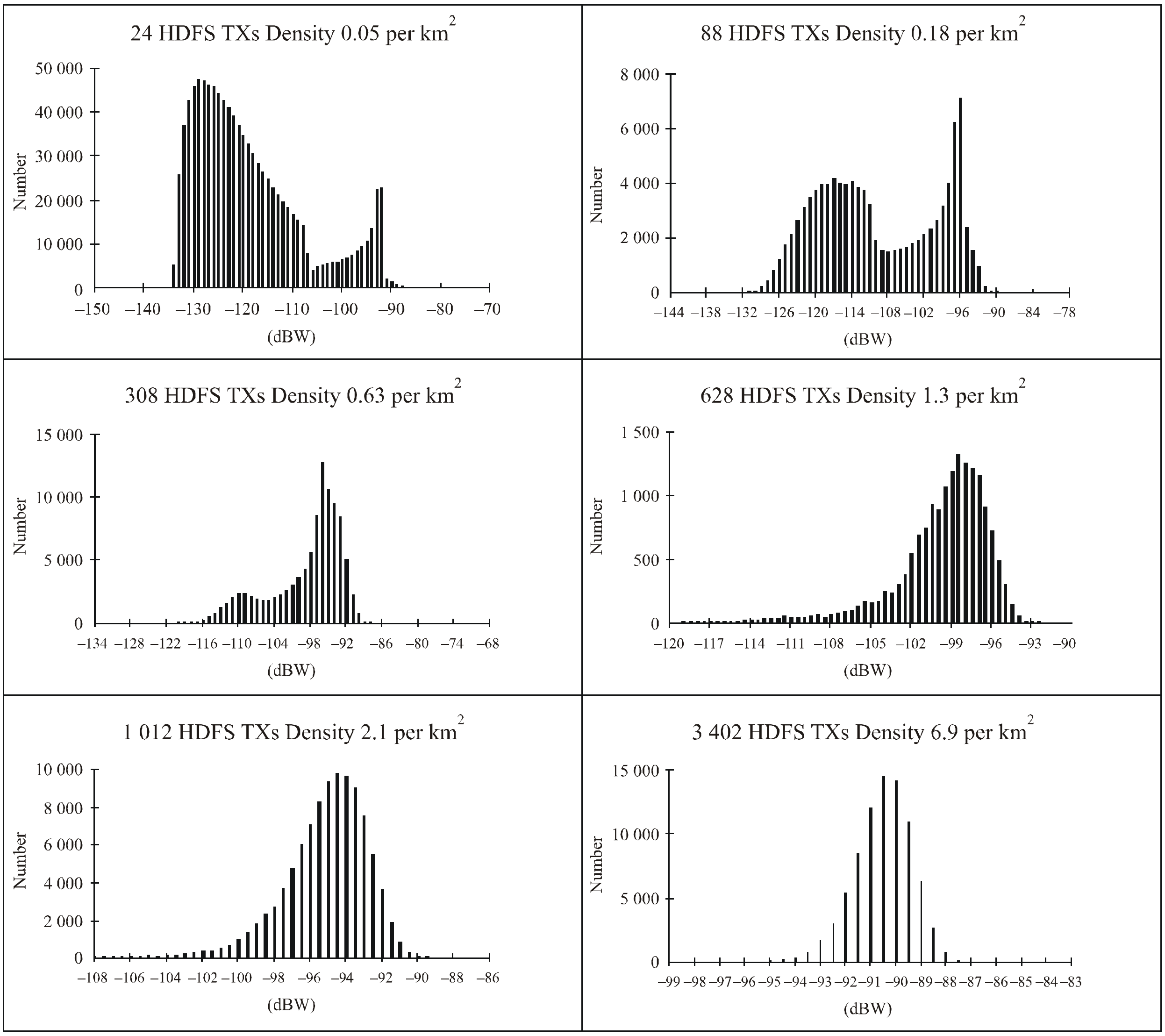


FIGURE 3

Examples of distributions of power levels from increasing numbers of P-P transmitters



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The transmitters were distributed, in varying numbers, in a uniform grid superimposed on circular areas with diameters of 25 km, with some simulations repeated for 15 km and 35 km diameter areas.

In order to obtain an estimate of the likely aggregated power levels for interference calculations, it is appropriate to consider the worst values of interference produced in the simulations, which, when corrected for free-space path loss, is equivalent to the worst-case aggregated radiated power from the network of transmitters. Since the magnitude of these distributions continues to increase as the simulation progresses, the worst-case interference power level also increases, and an extensive series of tests was carried out into the extent to which the worst-case value continues to increase with simulation time. After an initial sharp increase, this was found to increase more slowly. Simulations were carried out with time steps of 1 s and 1 min, with little difference being found between the two after a certain number of time steps. As more transmitters are added to the simulation, the simulation speed reduces considerably. The simulations used in the analysis all ran for 100 000 steps of 1 min, in order to ensure that the results represented comparable risk.

1.4 Effects of HDFS antenna elevation angles

In the preceding section, an assumption was made that all HDFS antenna elevation angles are 0°. However, the a.e.i.r.p. may vary with a range of HDFS antenna elevation angles. Therefore, the distribution of elevation angles for 8 539 United Kingdom fixed links operating in the 38 GHz band has been analysed. This yields the proportion of transmitters in four ranges of elevation angles, shown in Table 1, and a further simulation was set up with four groups of transmitters in the same proportion, totalling 1 950. In each group, the antenna elevation angle was set randomly within the range shown in Table 1.

The simulation was run for two cases, with the elevation angles set randomly within the groups listed in Table 1, and with all elevation angles set to zero. The results are compared in Table 2, which lists the calculated aggregated radiated power.

FIGURE 4

Distribution of 38 GHz link elevation angles in the United Kingdom
(Note the logarithmic scale on the vertical axis)

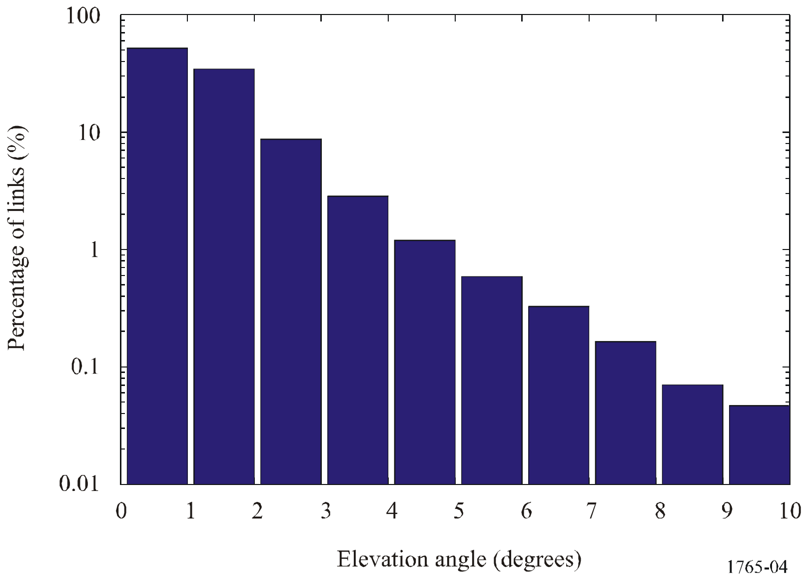


TABLE 1

Distribution of antenna elevation angles

Elevation angle range (degrees)	Percentage of 38 GHz links in range, from Fig. 4	No. of transmitters in each simulation group
0-1	51.7	1 012
1-2	34.4	665
2-5	12.7	249
5-10	1.2	24

TABLE 2

Antenna gain (dBi)	a.e.i.r.p. (dBW)		Difference (dB)
	Variable elevation angle	Zero elevation angle	
28 36 44	64.9 68.9 73.3	65.4 70.1 75.2	0.5 1.2 1.9

For the realistic distributions of elevation angles considered, there is no significant difference between these results. However, it should be noted that Table 2 shows the case where only the a.e.i.r.p. towards the horizon (0°) was evaluated. Section 2 discusses the cases where the a.e.i.r.p. towards the directions with elevation angles higher than 0°. In these cases, differences between the variable elevation angle situation and the zero elevation angle situation may be significant depending on HDFS antenna gain.

The above results were obtained from networks of P-P HDFS transmitters distributed over a 25 km diameter circular area. Simulations have been repeated for 15 km and 35 km diameter areas, to investigate the dependence on link density.

There was very little difference between the aggregated radiated power when the transmitters are distributed over different areas, at least between 15 and 35 km diameter. The power levels are essentially independent of the area over which the transmitters are spread, within ±0.5 dB. It is therefore not considered necessary to include area as a parameter in the model for the a.e.i.r.p. from P-P transmitters of HDFS.

2 Theoretical calculations by means of convolution integral

2.1 A general method for analysis and the confidence level of calculations

The preceding section described a simulation method for calculating the a.e.i.r.p. from a number of HDFS transmitters. However, it may be pointed out that, in general, a simulation method is time consuming in arriving at reliable results and should be avoided, if other methods are available and that the magnitude of calculation errors inherent in the results cannot be accurately quantified, in particular, for a small percentage for which the values may be exceeded.

In fact, the issue is a purely mathematical problem in which a probability density function (pdf) of the aggregate radiated power from a number of HDFS transmitters is to be investigated. A theory tells us that an exact pdf can be analytically obtained as follows:

Assuming that $p(N_t, x)$ is the pdf of the aggregate radiated power, x (expressed in a numerical value), from N_t transmitters, the pdf for the a.e.i.r.p. from $M_t + N_t$ transmitters can be calculated as follows by a convolution integral:

$$p(M_t + N_t, x) = \int_0^x p(M_t, u) \cdot p(N_t, x-u) \cdot \mathrm{d}u \qquad (2)$$

Equation (2) assumes that the azimuth angle of each transmitter is randomly located over 0°-360° and that some differences in the locations of HDFS transmitters within the deployment area can be ignored. By repeating this convolution integral, the pdf can be calculated for any number of transmitters. In actual calculations, M_t was chosen as equal to N_t and the pdfs were calculated for $N_t = 1, 2, 4, 8, 16, ..., 32\,768$ transmitters.

For $N_t = 1$, first of all, let us assume that the elevation angle and the azimuth angle of an HDFS transmitting antenna are ε_f and α_f, respectively. The elevation angle and the azimuth angle of the direction of the interference to be evaluated will be designated as ε_u and α_u, respectively. In this case, the separation angle, φ, between the HDFS antenna direction and the direction of the interference to be evaluated can be calculated by:

$$\varphi = \arccos\left(\cos\varepsilon_f \cdot \cos\varepsilon_u \cdot \cos(\alpha_f - \alpha_u) + \sin\varepsilon_f \cdot \sin\varepsilon_u\right) \tag{3}$$

Without loss of generality, it is possible to assume that $\alpha_u = 0$ and that ε_u is a given value (0 or a positive value). In addition, it can be assumed that α_f is uniformly distributed over 0° ~ 360°. The distribution of ε_f needs to be defined which will be discussed in more detail in the succeeding sections.

If the value of φ is determined by equation (3), the HDFS antenna gain towards the direction of interference to be evaluated will be calculated according to the antenna radiation pattern defined in Recommendation ITU-R F.1245. Hence, the pdf of the a.e.i.r.p. for $N_t = 1$ can be determined.

Then, for $N_t = 2$, the pdf (0.01 dB step) can be calculated according to equation (2). This can be repeated for $N_t = 4, 8, 16, ..., 32\,768$.

For the purpose of this study, a confidence level of 95% was generally used. This means that the probability that the a.e.i.r.p. may exceed the calculation results is as small as 5°. In addition, some results, i.e. Table 3b), are presented for a confidence level of 99.9°.

2.2 Case of HDFS antennas with zero elevation angle

In this case, the elevation angles of all HDFS antennas are assumed to be 0° ($\varepsilon_f = 0$ for all HDFS antennas). For $N_t = 1$, the azimuth range of 180° was divided into 10 000 portions. Thus, the probability for each 0.01 dB interval was calculated. Since the transmitter power level is not an important factor, it was assumed to be 0 dBW.

For N_t larger than 1, the pdf was calculated according to equation (2). Figure 5a shows the results for $G_t = 28$ dB and $\varepsilon_u = 0, 2.5, 5, 10, 15, 20$ degrees. Figure 5b) is the case of $G_t = 44$ dB. P_t is assumed as 0 dBW in all cases.

It is noted that in Fig. 5a, the difference between the curves of $\varepsilon_u = 0°$ and $\varepsilon_u = 2.5°$ is very small because the beamwidth of 28 dBi gain antenna is as large as 6.7° while in Fig. 5b, the difference between the curves of $\varepsilon_u = 0°$ and $\varepsilon_u = 2.5°$ is very large because the beamwidth of 44 dBi gain antenna is as small as 1.1°.

It is also noted that in Fig. 5b, at $\varepsilon_u = 0°$, the increase of the a.e.i.r.p. is rapid for a smaller number of HDFS transmitters and becomes slower at a larger number of HDFS transmitters. The a.e.i.r.p. values (dBW) as a function of G_t (dBi) and the number of HDFS transmitters (N_t) are shown in Table 3a for the case of $\varepsilon_u = 0°$ at confidence level of 95%.

Table 3b shows the results at confidence level of 99.9%.

FIGURE 5a

a.e.i.r.p. in the direction of various elevation angles as a function of the total number of HDFS transmitters (G_t = 28 dB and all HDFS antenna elevation angles are 0°)

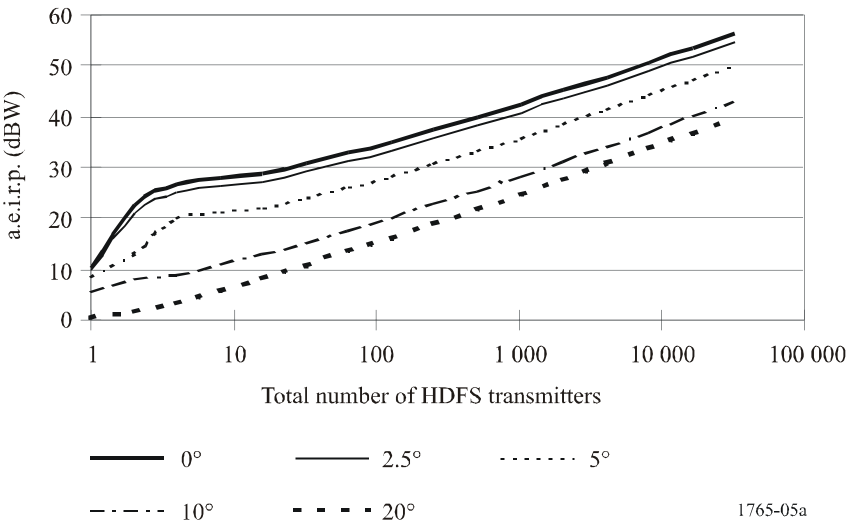


FIGURE 5b

a.e.i.r.p. in the direction of various elevation angles as a function of the total number of HDFS transmitters (G_t = 44 dB and all HDFS antenna elevation angles are 0°)

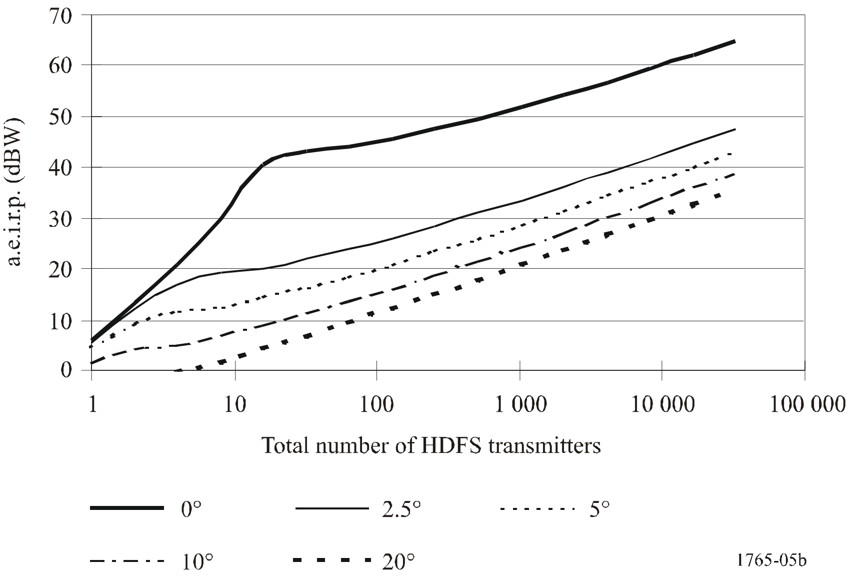


TABLE 3a

a.e.i.r.p. in dBW as a function of G_t (dBi) and the number of transmitters (N_t)
(95% confidence level)

G_t (dB)	a.e.i.r.p. (dBW) for various numbers of HDFS transmitters (N_t = 32 ~ 32 768)										
	32	**64**	**128**	**256**	**512**	**1024**	**2 048**	**4 096**	**8 192**	**16 384**	**32 768**
28	30.86	32.81	34.97	37.29	39.75	42.34	45.04	47.82	50.66	53.54	56.46
30	32.35	34.18	36.25	38.51	40.92	43.47	46.14	48.89	51.72	54.58	57.49
32	33.69	35.49	37.54	39.74	43.11	44.61	47.24	49.96	52.76	55.62	58.52
34	34.89	36.89	38.84	41.00	43.31	45.77	48.36	51.05	53.83	56.67	59.55
36	36.10	38.38	40.20	42.27	44.53	46.94	49.49	52.15	54.90	57.72	60.59
38	37.98	39.72	41.51	43.56	45.76	48.13	50.63	53.26	55.98	58.78	61.63
40	39.84	40.92	42.90	44.86	47.01	49.33	51.79	54.38	57.07	59.84	62.68
42	41.62	42.12	44.39	46.22	48.29	50.54	52.96	55.50	58.16	60.91	63.73
44	43.24	43.98	45.74	47.53	49.58	51.78	54.14	56.65	59.27	61.99	64.79
46	44.72	45.85	46.94	48.92	50.88	53.03	55.34	57.80	60.39	63.08	65.86

TABLE 3b

a.e.i.r.p. in dBW as a function of G_t (dB) and the number of transmitters (N_t)
(99.9% confidence level)

G_t (dB)	a.e.i.r.p. in dBW for various numbers of transmitters (N_t = 32 ~ 32 768)										
	32	**64**	**128**	**256**	**512**	**1 024**	**2 048**	**4 096**	**8 192**	**16 384**	**32 768**
28	33.59	35.11	36.85	38.79	40.92	43.24	45.71	48.31	51.02	53.81	56.65
30	35.13	36.60	38.26	40.13	42.20	44.46	46.88	49.44	52.11	54.87	57.70
32	36.67	38.10	39.70	41.50	43.50	45.70	48.06	50.58	53.22	55.95	58.76
34	38.34	39.64	41.16	42.89	44.82	46.95	49.26	51.73	54.33	57.03	59.82
36	39.94	41.18	42.64	44.30	46.16	48.23	50.48	52.90	55.46	58.13	60.89
38	41.44	42.71	44.14	45.73	47.53	49.52	51.72	54.08	56.60	59.23	61.96
40	43.00	44.37	45.67	47.19	48.91	50.84	52.97	55.28	57.75	60.35	63.05
42	44.85	45.98	47.21	48.67	50.32	52.18	54.25	56.50	58.91	61.47	64.14
44	46.66	47.48	48.73	50.16	51.75	53.54	55.54	57.73	60.10	62.61	65.24

An attempt was made to search an appropriate formula which approximates the a.e.i.r.p. values in Table 3a by applying the mini-max approximation method for two variables and the following formula was derived as a reasonable approximation for G_t = 28 to 46 dBi and N_t = 32 to 8 192:

$$a.e.i.r.p. = P_t + 1.061\,(\log N_t)^2 + (-0.1164\,G_t + 6.103)\log N_t + 0.9428\,G_t - 2.62 \quad \text{dBW} \qquad (4)$$

This is the formula presented in *recommends* 1.1 of the main text. The maximum approximation error of this formula is 0.52 dB. The same approximation method was applied to other values of ε_u and the formulae in *recommends* 1.2 ~ 1.8 of the main text were derived. When $\varepsilon_u = 2.5°$ or 5°, the curves are more complicated and, hence, the approximation formulae require polynomials of a higher order.

2.3 Case of HDFS antennas with variable elevation angles

The assumption of zero elevation angle adopted in the preceding section is somewhat hypothetical. In actual situations, HDFS antennas will have variable elevation angles. However, it is a difficult issue to establish a typical pdf of elevation angles. In the analysis here, Fig. 4 will be used as an example pdf of HDFS antenna elevation angles. However, it should be noted that all elevation angles in Fig. 4 are 0 or positive. In reality, it is reasonable to assume that the elevation angles may be either positive or negative, and that the pdf is symmetric with respect to 0 elevation angles. Therefore, the pdf of Fig. 4 is converted to a symmetric distribution as presented in Fig. 6 and Table 4.

FIGURE 6

Cumulative distribution curve of HDFS antenna elevation angles corresponding to Fig. 4

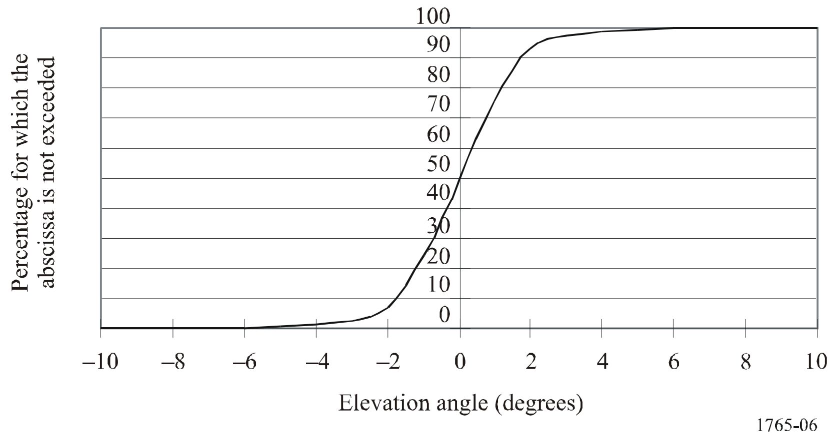


TABLE 4

Cumulative distribution of HDFS antenna elevation angles corresponding to Fig. 4

Degrees	–10	–9	–8	–7	–6	–5	–4	–3	–2	–1	0
Percentage	0	0.023	0.06	0.145	0.31	0.6	1.2	2.7	6.95	24.15	50

Degrees	1	2	3	4	5	6	7	8	9	10
Percentage	75.85	93.05	97.3	98.8	99.4	99.69	99.855	99.94	99.977	100

Under this assumption, the pdf of the a.e.i.r.p. was calculated for $N_t = 1$. For larger values of N_t, the pdfs were calculated according to equation (2). The results are presented in Figs 7a and 7b. P_t is assumed as 0 dBW in all cases.

FIGURE 7a

a.e.i.r.p. in the direction of various elevation angles as a function of the total number of HDFS transmitters (G_t= 28 dB and HDFS antenna variable elevation angles)

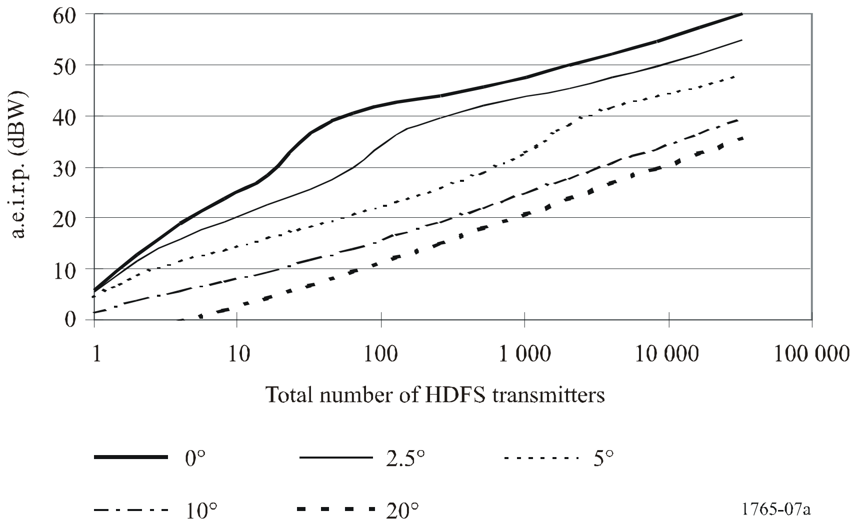
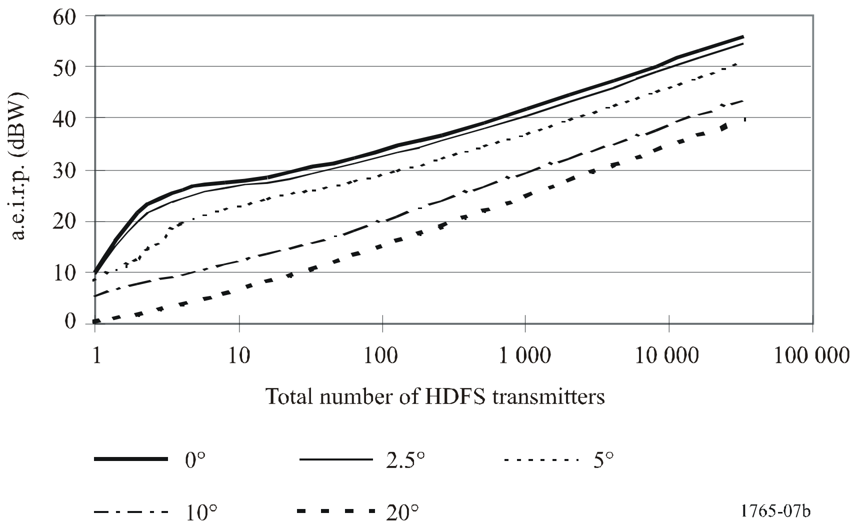


FIGURE 7b

a.e.i.r.p. in the direction of various elevation angles as a function of the total number of HDFS transmitters (G_t= 44 dB and HDFS antenna variable elevation angles)



If Figs 5a and 7a are compared, it will be found that the differences are very small. This will mean that, when the HDFS antenna gain is small, the assumption of variable elevation angles will cause little effects on the a.e.i.r.p., because the antenna beamwidth is fairly wide. On the other hand, the differences between Figs 5b and 7b are large, because the antenna beamwidth is very small. This indicates that the effects of variable elevation angles are different depending on HDFS antenna gain.

A number of approximation formulae were derived for the range of $G_t = 28$ to 46 dBi and $N_t = 32$ to 8 192 at various values of ε_u. The results are presented in *recommends* 2 of the main text. It may be noted that when $\varepsilon_u = 0°$, 2.5° or 5°, the curves are complicated and, therefore, higher order polynomials are required in order to give a good approximation.

3 Comparison between the analytical and probabilistic simulation

A simulation was conducted in order to compare the results obtained in § 2. This simulation was based on a square cell of 1 km size with a given density (equal in this case to the number) of HDFS emitters (UT) without any power control, a maximum antenna gain of 44 dBi with an antenna pattern following Recommendation ITU-R F.1245 and an elevation angle of 0°, a receiver located in a random azimuth with regard to the HDFS cell and a distance of 100 km, with an antenna gain of 0 dBi.

In this case, the power received at the receiver is given by equation (5).

$$P_r = AEIRP + 20\ \log\left(\frac{\lambda}{4\pi d}\right) \tag{5}$$

where:

P_r: power received at a 0 dBi antenna located at distance d from the HDFS cell (dBW)

$AEIRP$: aggregate e.i.r.p. produced by the UT of the cell (dBW)

λ: wavelength (m)

d: distance between the cell and the 0 dBi antenna where the power is evaluated (m).

For one single UT, P_r is given by equation (6).

$$P_r = P_e + G_e + 20\ \log\left(\frac{\lambda}{4\pi d}\right) \tag{6}$$

where:

P_r: power received at a 0 dBi antenna located at distance d from the HDFS cell (dBW)

P_e: emission power produced by the only UT of the cell (dBW)

G_e: antenna gain of the UT in the direction of the 0 dBi antenna receiver

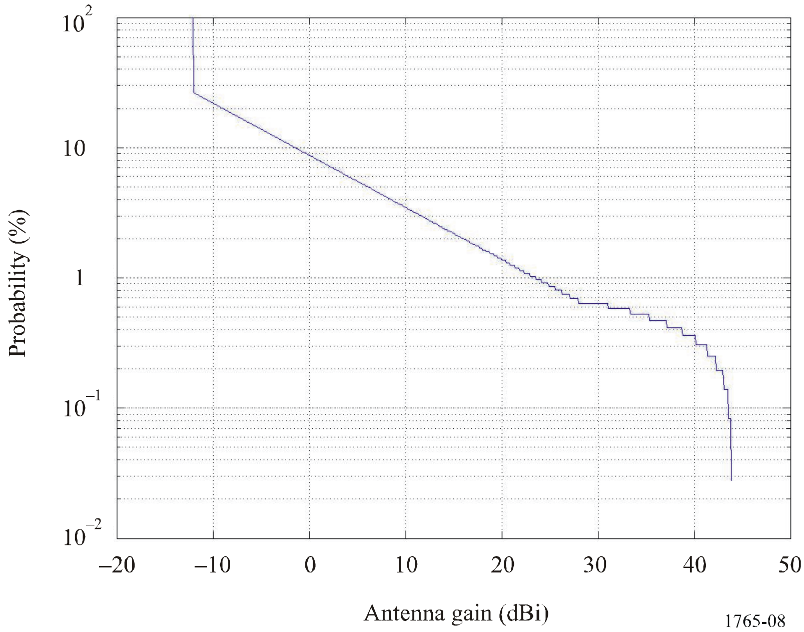
λ: wavelength (m)

d: distance between the cell and the 0 dBi antenna where the power is evaluated (m).

The antenna gain is the only variable and varies between −12 and 44 dBi according to the azimuth following the cdf curve given in Fig. 8.

FIGURE 8

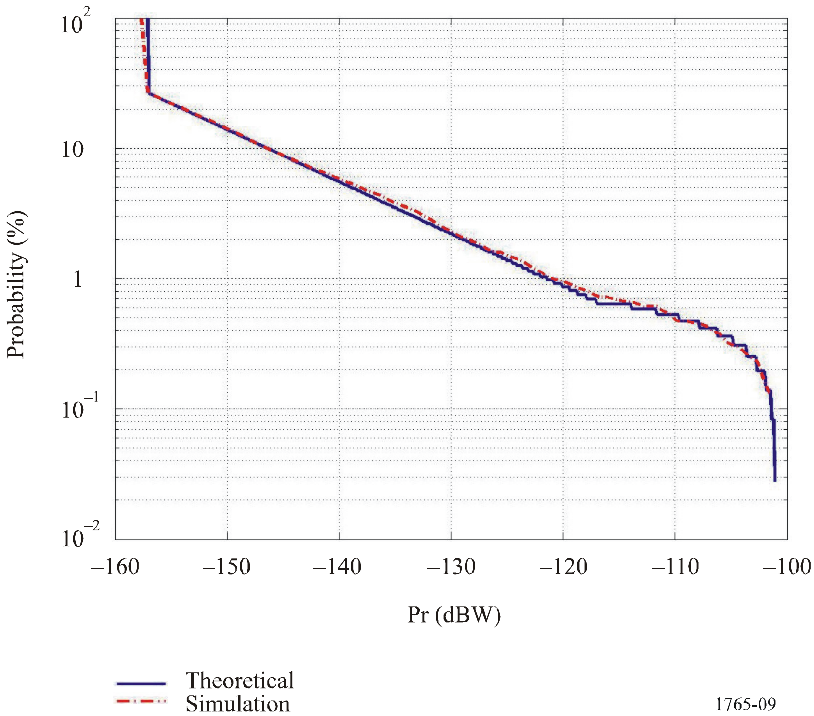
Recommendation ITU-R F.1245 antenna gain cdf for a maximum antenna gain of 44 dBi



The theoretical power received by a 0 dBi receiver is therefore given in Fig. 9, as well as the power obtained for one single HDFS transmitter.

FIGURE 9

Power received at a 0 dBi antenna located at 100 km from the HDFS UT with a 20 dBW power



Assuming this time for an emission power of 0 dBW for all UT, it is possible to compare the results obtained by the analytical simulation given in § 2 of this Annex and the ones obtained by the probabilistic approach, for a confidence level of 95% calculated over 10 000 trials (see Table 3a).

TABLE 5

HDFS maximum antenna gain of 44 dBi

	a.e.i.r.p. (dBW) for various numbers of HDFS transmitters (N_t = 32 to 2 048)						
	32	64	128	256	512	1 024	2 048
Analytical simulation	43.24	43.98	45.74	47.53	49.58	51.78	54.14
Probabilistic simulation	43.33	43.94	45.73	47.37	49.59	51.81	54.19

TABLE 6

HDFS maximum antenna gain of 28 dBi

	a.e.i.r.p. (dBW) for various numbers of HDFS transmitters (N_t = 32 to 2 048)						
	32	64	128	256	512	1 024	2 048
Analytical simulation	30.86	32.81	34.97	37.29	39.75	42.34	45.04
Probabilistic simulation	30.84	32.78	34.97	37.28	39.74	42.37	45.04

The comparison of results obtained by the probabilistic and the analytical method show a very good concordance between them.

Appendix 1
to Annex 1

Approximate formulae for a.e.i.r.p.

This Appendix presents the approximate formulae of *recommends* 1 and 2 of this Recommendation in a tabular form in order to facilitate computer implementation.

Formulae in *recommends* 1 (elevation angles of all HDFS antennas are 0°)

Elevation angle of the direction of the a.e.i.r.p. to be evaluated: 0°, 2.5° or 5°:

$$a.e.i.r.p. = P_t + a_{30}\,(\log N_t)^3 + a_{20}\,(\log N_t)^2 + (a_{11}\,G_t + a_{10})\log N_t$$
$$+\, a_{03}\,G_t^3 + a_{02}\,G_t^2 + A_{01}\,G_t + a_{00}$$

TABLE 7a

Elevation angle to be evaluated (degrees)	a_{30}	a_{20}	a_{11}	a_{10}	a_{03}	a_{02}	a_{01}	a_{00}
0	0	1.061	–0.1164	6.103	0	0	0.9428	–2.62
2.5	–0.13743	1.8243	0	1.5569	0.0052917	–0.57530	19.985	–200.77
5	0	0.54858	0	5.6488	–0.0036218	0.42380	–16.645	227.44

Elevation angle of the direction of the a.e.i.r.p. to be evaluated: 10°, 15°, 20°, 25° or 30°:

$$a.e.i.r.p. = P_t + a_{10} \log N_t + a_{01} G_t + a_{00}$$

TABLE 7b

Elevation angle to be evaluated (degrees)	a_{10}	a_{01}	a_{00}
10	9.086	–0.25	8.30
15	9.344	–0.25	5.19
20	9.522	–0.25	3.19
25	9.633	–0.25	1.78
30	9.775	–0.25	0.74

Formulae in *recommends* 2 (elevation angles of all HDFS antennas are variable)

Elevation angle of the direction of the a.e.i.r.p. to be evaluated: 0°, 2.5° or 5°:

$$a.e.i.r.p. = P_t + (a_{31} G_t + a_{30}) (\log N_t)^3 + (a_{22} G_t^2 + a_{21} G_t + a_{20}) (\log N_t)^2 + (a_{12} G_t^2 + a_{11} G_t + a_{10}) \log N_t + a_{03} G_t^3 + a_{02} G_t^2 + a_{01} G_t + a_{00}$$

TABLE 8a

Elevation angle to be evaluated (degrees)	a_{31}	a_{30}	a_{22}	a_{21}	a_{20}	a_{12}
0	0	0.82096	0	–0.15210	0.92771	0.024504
2.5	0	0.93906	0	–0.31918	3.4110	0.023524
5	–0.10457	3.0618	0.027889	–1.1358	9.7775	–0.15803

Elevation angle to be evaluated (degrees)	a_{11}	a_{10}	a_{03}	a_{02}	a_{01}	a_{00}
0	–1.0198	27.270	0	–0.077296	5.1982	–73.62
2.5	0.096937	–4.8156	0.0011791	–0.21452	8.5619	–82.88
5	9.3247	–132.36	0	0.20619	–13.901	247.30

Elevation angle of the direction of the a.e.i.r.p. to be evaluated: 10°, 15°, 20°, 25° or 30°:

$$a.e.i.r.p. = P_t + a_{10} \log N_t + a_{01}\, G_t + a_{00}$$

TABLE 8b

Elevation angle to be evaluated (degrees)	a_{10}	a_{01}	a_{00}
10	9.263	–0.2511	8.43
15	9.299	–0.25	5.45
20	9.497	–0.25	3.32
25	9.651	–0.25	1.84
30	9.767	–0.25	0.79