RECOMMENDATION ITU-R P.1239*

ITU-R REFERENCE IONOSPHERIC CHARACTERISTICS**

(Questions ITU-R 212/3 and ITU-R 223/3)

(1997)

The ITU Radiocommunication Assembly,

considering

a) that long-term reference ionospheric data and prediction methods are needed for radio-circuit design, service planning and frequency band selection,

recommends

1 that for the prediction of ionospheric characteristics, use should be made of the formulations contained in Annex 1.

ANNEX 1

Ionospheric characteristics

1 Introduction

Expressions are provided for the evaluation of the monthly median of foF2, M(3000)F2, foE, foF1, h'F and h'F,F2 and of the monthly median, upper decile and lower decile of foEs and fbEs. Also included are representations of the percentage of occurrence of spread-F. These formulations yield values for any location, month and time-of-day for different solar epochs. In the case of foE and foF1, empirical formulae in terms of solar-zenith angle are presented. For the other ionospheric characteristics a numerical mapping technique based on orthogonal Fourier functions is applied.

2 Mapping functions

The general form of the numerical map function, $\Omega(\lambda, \theta, T)$ is the Fourier time series:

$$\Omega(\lambda, \theta, T) = a_0(\lambda, \theta) + \sum_{j=1}^{H} \left[a_j(\lambda, \theta) \cos jT + b_j(\lambda, \theta) \sin jT \right]$$
 (1)

where:

 Ω : ionospheric characteristic to be mapped

 λ : geographic latitude ($-90^{\circ} \le \lambda \le 90^{\circ}$)

 θ : east geographic longitude (0° $\leq \theta \leq 360$ °) (θ in degrees East of the Greenwich meridian)

T: universal time (UTC) expressed as an angle $(-180^{\circ} \le T \le 180^{\circ})$

H: maximum number of harmonics used to represent the diurnal variation.

^{*} Radiocommunication Study Group 3 made editorial amendments to this Recommendation in 2000 in accordance with Resolution ITILR 44

^{**} Computer programs associated with the prediction procedures and data described in this Recommendation are available from that part of the ITU-R website dealing with Radiocommunication Study Group 3.

The Fourier coefficients, $a_j(\lambda, \theta)$ and $b_j(\lambda, \theta)$, vary with the geographic coordinates, and are represented by series of the form:

$$a_j(\lambda, \theta) = \sum_{k=0}^K U_{2j,k} G_k(\lambda, \theta), \quad j = 0, 1, 2, \dots, H$$
 (2a)

$$b_j(\lambda, \theta) = \sum_{k=0}^{K} U_{2j-1,k} G_k(\lambda, \theta), \quad j = 1, 2, ..., H$$
 (2b)

The particular choice of the functions, $G_k(\lambda, \theta)$ is determined by specifying the integers k ($k_0, k_1, k_2, \ldots, k_i, \ldots, k_m$; $k_m = K$), where i is the order in longitude. Therefore, a numerical map can be written more explicitly in the form:

$$\Omega(\lambda, \theta, T) = \sum_{k=0}^{K} U_{0k} \ G_k(\lambda, \theta) + \sum_{j=1}^{H} \left[\cos j T \sum_{k=0}^{K} U_{2j,k} \ G_k(\lambda, \theta) + \sin j T \sum_{k=0}^{K} U_{2j-1,k} \ G_k(\lambda, \theta) \right]$$
(3)

 $U_{2j,k}$ and $U_{2j-1,k}$ in equations (2a), (2b) and (3), can be written as $U_{s,k}$, where s is either 2j or 2j-1.

In the numerical mapping technique, the modified magnetic dip:

$$X = \arctan\left(\frac{I}{\sqrt{\cos \lambda}}\right) \tag{4}$$

has been used, where I is the magnetic dip and λ is the geographic latitude. Since X is a function of both geographic latitude and longitude, the formal expression of Ω (λ , θ , T), equation (3), is unchanged. Table 1 shows the geographic functions, $G_k(\lambda, \theta)$.

TABLE 1 Geographic coordinate functions $G_k(\lambda, \mathbf{\theta})$

(X is a function of λ and θ , m is the maximum order in longitude)

$$q_0 = k_0; \ q_i (i = 1, m) = \frac{k_i - k_{i-1} - 2}{2}$$

k	Main latitude variation	k	First order longitude	k	Second order longitude	 k	mth order longitude
0	1	$k_0 + 1$	cos λ cos θ	$k_1 + 1$	$\cos^2\lambda\cos 2\theta$	 $k_{m-1} + 1$	$\cos^m \lambda \cos m \theta$
1	sin X	$k_0 + 2$	$\cos\lambda\sin\theta$	$k_1 + 2$	$\cos^2\lambda\sin 2\theta$	 $k_{m-1} + 2$	$\cos^m \lambda \sin m \theta$
2	$\sin^2 X$	$k_0 + 3$	$\sin X \cos \lambda \cos \theta$	$k_1 + 3$	$\sin X \cos^2 \lambda \cos 2 \theta$	 $k_{m-1} + 3$	$\sin X \cos^m \lambda \cos m \theta$
		$k_0 + 4$	$\sin X \cos \lambda \sin \theta$	$k_1 + 4$	$\sin X \cos^2 \lambda \sin 2 \theta$	 $k_{m-1} + 4$	$\sin X \cos^m \lambda \sin m \theta$
k_0	$\sin^{q_0} X$	$k_1 - 1$	$\sin^{q_1} X \cos \lambda \cos \theta$	k ₂ - 1	$\sin^{q_2} X \cos^2 \lambda \cos 2 \theta$	 $k_m - 1$	$\sin^{q_m} X \cos^m \lambda \cos m \theta$
		k_1	$\sin^{q_1} X \cos \lambda \sin \theta$	k_2	$\sin^{q_2} X \cos^2 \lambda \sin 2 \theta$	 k_m	$\sin^{q_m} X \cos^m \lambda \sin m \theta$

A model of the Earth's magnetic field for epoch 1960 based on a sixth-order spherical-harmonic analysis is employed in order to determine modified magnetic dip and gyrofrequency required in the evaluation of the numerical maps. The 1960 epoch must be used, rather than some other epoch of interest because it is that which is used in generating the values of the numerical coefficients.

The magnetic induction F_x , F_y and F_z (gauss) along the geographic North, East and vertically downwards directions respectively, is given by:

$$F_{x} = \sum_{n=1}^{6} \sum_{m=0}^{n} x_{n}^{m} \left[g_{n}^{m} \cos m \theta + h_{n}^{m} \sin m \theta \right] R^{n+2}$$
 (5a)

$$F_{y} = \sum_{n=1}^{6} \sum_{m=0}^{n} y_{n}^{m} \left[g_{n}^{m} \sin m \theta - h_{n}^{m} \cos m \theta \right] R^{n+2}$$
 (5b)

$$F_{z} = \sum_{n=1}^{6} \sum_{m=0}^{n} z_{n}^{m} \left[g_{n}^{m} \cos m \theta + h_{n}^{m} \sin m \theta \right] R^{n+2}$$
 (5c)

where:

$$x_n^m = \frac{d}{d\,\phi} \left(P_{n,m} \left(\cos \phi \right) \right) \tag{6a}$$

$$y_n^m = m \cdot \frac{P_{n, m}(\cos \phi)}{\sin \phi}$$
 (6b)

$$z_n^m = -(n+1) P_{n, m}(\cos \varphi)$$
 (6c)

with:

φ: northern co-latitude (= $90^{\circ} - \lambda$), where λ is the geographic latitude (degrees) (North positive, $-90^{\circ} \le \lambda \le 90^{\circ}$)

 $P_{n,m}(\cos \varphi)$: associated Legendre function defined as:

$$P_{n,m}(\cos \varphi) = \sin^m \varphi \left[\cos^{n-m} \varphi - \frac{(n-m)(n-m-1)}{2(2n-1)} \cos^{n-m-2} \varphi + \right]$$

$$\frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{(2)(4)(2n-1)(2n-3)}\cos^{n-m-4}\varphi + \dots$$
 (7)

 $g^{m,n}$ and $h^{m,n}$: numerical coefficients for the field model (gauss)

R: height-dependent scaling factor given as:

$$R = \frac{6371.2}{6371.2 + h_r} \tag{8}$$

where:

 h_r : height at which the field is evaluated (taken as 300 km).

The total magnetic field, F, is given as:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \tag{9}$$

The magnetic dip, I, and gyrofrequency, f_H (MHz) are determined from:

$$I = \tan^{-1} \left(\frac{F_Z}{\sqrt{F_X^2 + F_y^2}} \right)$$
 (10)

and:

$$f_H = 2.8 F$$
 (11)

3 Prediction of foF2 and M(3000)F2

The F2-layer numerical maps are based on vertical incidence soundings of the ionosphere at a large number of ground stations all over the world. The sets of numerical coefficients defining the diurnal and geographical variations of the monthly median of foF2 and M(3000)F2 are based on a linear relationship with solar activity. The coefficients are the values of $U_{s,k}$ (see equations (2) and (3)) that define the function $\Omega(\lambda, \theta, T)$, of the numerical map of the given characteristic for the indicated month and level of solar activity. The coefficients are available for each month of the year, and for two levels of solar activity, $R_{12} = 0$ and $R_{12} = 100$. R_{12} is the twelve month running mean value of the monthly sunspot numbers and is used as an index of the level of solar activity.

For most purposes it is adequate to assume a linear relationship with R_{12} for both foF2 and M(3000)F2. However, the relationship between foF2 and R_{12} becomes non-linear at a level of solar activity which is a function of geographic location, time of day and season. The most noticeable departure from linearity is for values of R_{12} above approximately 150. For values of R_{12} greater than 150, the error is reduced by assuming that higher values are effectively 150. The relationship of M(3000)F2 with R_{12} is also taken to be linear over the range of values up to $R_{12} = 150$. For higher values of R_{12} , M(3000)F2 is taken to be the value obtained for $R_{12} = 150$.

4 Prediction of foE

The method for predicting the monthly median foE is based on all published data over the years 1944-1973 from 55 ionospheric stations.

foE (MHz) is given by:

$$(foE)^4 = ABCD \tag{12}$$

where:

A: solar activity factor, given as:

$$A = 1 + 0.0094 \,(\Phi - 66) \tag{13}$$

 Φ : monthly mean 10.7 cm solar radio flux expressed in units of 10⁻²² W m⁻² Hz⁻¹. For prediction purposes, it is appropriate to approximate Φ by an estimate of Φ_{12} , the twelve-monthly smoothed value (see Recommendation ITU-R P.371)

B: seasonal factor, given as:

$$B = \cos^m N \tag{14}$$

where:

$$N = \lambda - \delta = 80^{\circ}$$

 λ : geographic latitude and is taken as positive in the Northern Hemisphere

 δ : solar declination and is taken as positive for northern declinations.

The exponent m is a function of geographic latitude, λ :

$$m = -1.93 + 1.92 \cos \lambda$$
 for $|\lambda| < 32^{\circ}$ (15a)

or:

$$m = 0.11 - 0.49 \cos \lambda \qquad \text{for } |\lambda| \ge 32^{\circ} \tag{15b}$$

C: main latitude factor, given as:

$$C = X + Y \cos \lambda \tag{16a}$$

where:

$$X = 23, Y = 116$$
 for $|\lambda| < 32^{\circ}$ (16b)

or:

$$X = 92, Y = 35$$
 for $|\lambda| \ge 32^{\circ}$ (16c)

D: time-of-day factor.

1st Case: $\chi \le 73^{\circ}$

$$D = \cos^p \chi \tag{17a}$$

where χ is the solar zenith angle (degrees). For $|\lambda| \le 12^{\circ}$, p = 1.31; for $|\lambda| > 12^{\circ}$, p = 1.20.

2nd Case: $73^{\circ} < \chi < 90^{\circ}$

$$D = \cos^p \left(\chi - \delta \chi \right) \tag{17b}$$

where:

$$\delta \chi = 6.27 \times 10^{-13} (\chi - 50)^8$$
 degrees (17c)

and p is as in the 1st Case.

3rd Case: $\chi \ge 90^{\circ}$

The night-time value of D, for $\chi \ge 90^{\circ}$, is taken as the greater of those given by:

$$D = (0.072)^p \exp(-1.4 h) \tag{17d}$$

or:

$$D = (0.072)^p \exp(25.2 - 0.28 \chi) \tag{17e}$$

where h is the number of hours after sunset ($\chi = 90^{\circ}$). In polar winter conditions, when the Sun does not rise, equation (17e) should be used. p has the same value as in the 1st Case.

The minimum value of foE, is given by:

$$(\text{foE})^4_{minimum} = 0.004 (1 + 0.021 \,\Phi)^2 \tag{18}$$

where Φ may be approximated by an estimated value of Φ_{12} .

At night, if foE, when calculated by equations (12) to (17e), is less than that calculated by equation (18) the latter value should be taken.

Tests of the accuracy of the prediction method give for a data base of over 80 000 hourly comparisons for the 55 stations a median r.m.s. deviation of 0.11 MHz.

5 Prediction of foF1

Expressions for monthly median foF1 are based on data recorded from 1954 to 1966 at 39 ionospheric stations located in both hemispheres.

foF1 (MHz) is given by:

$$foF1 = f_s \cos^n \chi \tag{19}$$

where:

$$\begin{split} f_s &= f_{s_0} + 0.01 \, (f_{s_{100}} - f_{s_0}) \, R_{12} \\ f_{s_0} &= 4.35 + 0.0058 \, \lambda - 0.000120 \, \lambda^2 \\ f_{s_{100}} &= 5.35 + 0.0110 \, \lambda - 0.000230 \, \lambda^2 \\ n &= 0.093 + 0.00461 \, \lambda - 0.0000540 \, \lambda^2 + 0.00031 \, R_{12} \end{split}$$

and where λ , the value of the geomagnetic latitude (degrees) taken as positive in both hemispheres, is given by:

$$\lambda = \left[\arcsin \left[\sin g_0 \cdot \sin g + \cos g_0 \cdot \cos g \cdot \cos (\theta_0 - \theta) \right] \right]$$

where:

g: geographic latitude of position of interest

g₀: geographic latitude of North geomagnetic pole (taken as 78.3° N)

 θ : geographic longitude of position of interest

 θ_0 : geographic longitude of North geomagnetic pole (taken as $69.0^{\circ}\,W$).

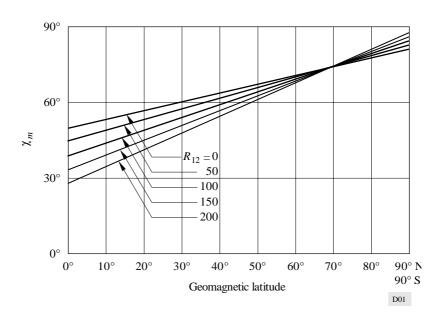
The maximum solar zenith angle at which the F1 layer is present (see also Figs. 1 and 2) is given by the following expressions:

$$\chi_m = \chi_0 + 0.01 (\chi_{100} - \chi_0) R_{12}$$
 degrees (20)

where:

$$\chi_0 = 50.0 + 0.348 \,\lambda$$

 $\chi_{100} = 38.7 + 0.509 \,\lambda$



6 Prediction of foEs and fbEs

A set of numerical coefficients defining the diurnal, geographical and monthly variations of the medians and upper and lower deciles of the foEs for a year of minimum and one of maximum solar activity, and a set of numerical coefficients defining the variations of the medians and upper and lower deciles of the fbEs (blanketing frequency of sporadic-E) for a year of minimum solar activity have been derived.

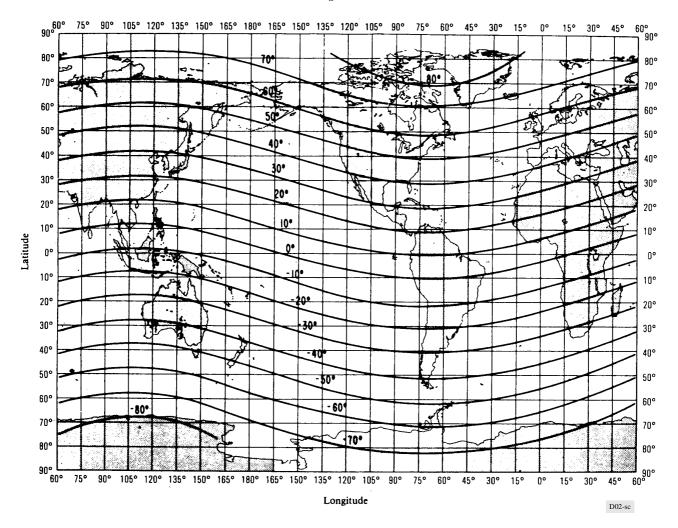
7 Prediction of h'F and h'F,F2

Numerical maps have been developed on a monthly basis for years of maximum and minimum solar activity of monthly median h'F, which is the minimum observed virtual height of reflection of vertical incidence signals from the F region (generally from the F2 layer at night and from the F1 layer in the daytime). Numerical maps have also been developed for years of maximum and minimum solar activity of h'F,F2. h'F,F2 is the combination of the minimum observed virtual height of reflection of vertical-incidence signals from both the F layer at night and the F2 layer in the daytime.

8 Prediction of the percentage of occurrence of spread-F

The percentage occurrence of spread-F has been determined from the ionospheric data from the world network of vertical-incidence ionosonde stations on a monthly basis for a year representative of high solar activity and for a year of low solar activity, and values have been represented numerically by means of a mapping technique.

FIGURE 2
Geomagnetic latitudes



9 Available computer programs and reference data

The procedures described in this Annex are implemented in the computer programs WOMAP and HRMNTH. The program WOMAP displays for locations in a specified geographic area, the values of the chosen ionospheric characteristic, for a given Universal Time, month and solar epoch. The complementary program HRMNTH displays the chosen ionospheric characteristic for a given location and year, as a function of the Universal Time, for each month and the associated solar epoch.
