RECOMMENDATION ITU-R P.1815

Differential rain attenuation

(Question ITU-R 208/3)

(2007)

Scope

This Recommendation predicts the joint differential rain attenuation statistics between a satellite and two locations on the surface of the Earth.

The ITU Radiocommunication Assembly,

considering

a) that it is necessary to have appropriate techniques to predict differential attenuation due to rain between satellite paths from a single satellite to multiple locations on the surface of the Earth for the purpose of sharing analyses;

b) that estimates of the spatial correlation of rain rate are available;

c) that methods have been developed to predict differential attenuation between space-Earth paths due to rain,

recommends

1 that the methods described in Annex 1 should be used to predict differential rain attenuation on satellite paths between a single satellite and multiple locations on the surface of the Earth.

Annex 1

Description of differential rain attenuation method

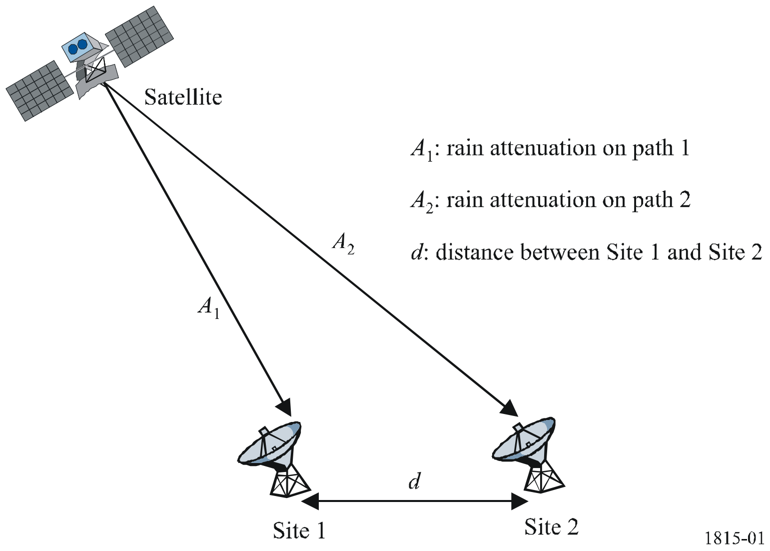
1 Introduction

The method described in this Annex predicts the joint differential rain attenuation statistics between a satellite and two locations on the surface of the Earth and is applicable to frequencies up to 55 GHz, elevation angles above approximately 10°, and site separations between 0 and at least 250 km.

This method considers the statistical and temporal characteristics of rain cell size, rain intensity and movement of rain cells related to differential rain attenuation.

FIGURE 1

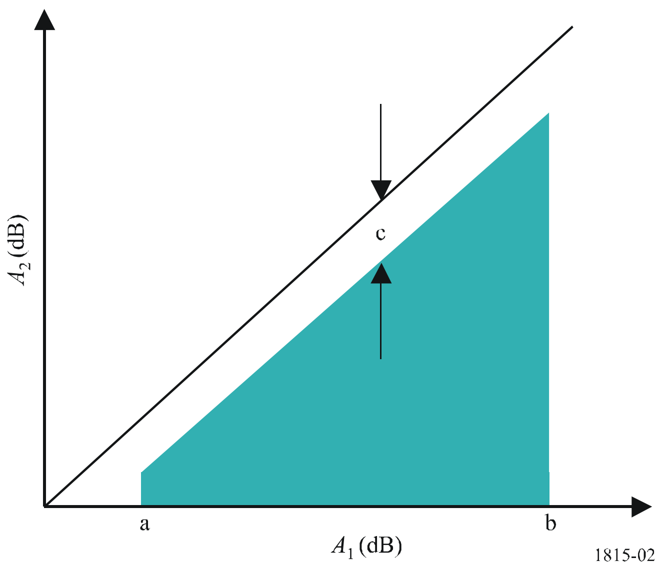
Differential attenuation geometry



The geometry is shown in Fig. 1, where A_1 and A_2, are the rain attenuations on path 1 and path 2, respectively. The desired statistic is the joint probability that the attenuation on the first path, A_1, is between a and b, and the attenuation on the second path, A_2, is less than or equal to $A_1 - c$; i.e. $\Pr\{a < A_1 \leq b,\ A_2 \leq A_1 - c\}$. This joint probability is shown graphically in Fig. 2 as the integrated probability within the shaded region.

FIGURE 2

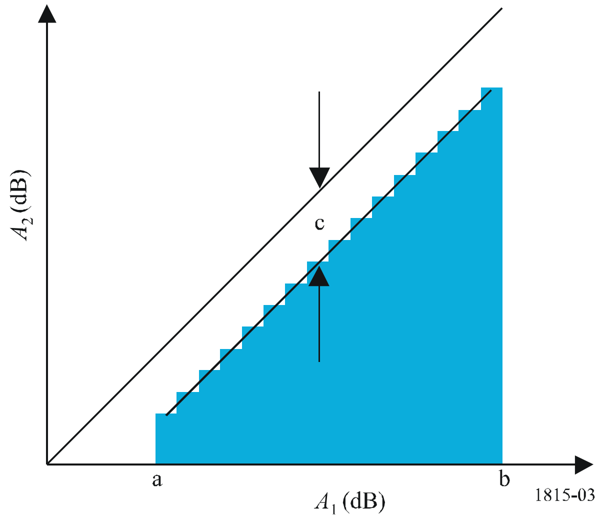
Desired joint probability distribution



The joint probability within the shaded region of Fig. 2 can be well-approximated as the sum of the integrated probabilities within the narrow vertical rectangular regions as illustrated in Fig. 3.

FIGURE 3

Approximation to desired joint probability distribution



The joint probability within the shaded region in Fig. 3 can then be computed as the difference between the joint probability within the shaded region in Fig. 4 and the joint probability within the shaded region in Fig. 5.

FIGURE 4

$\Pr(A_1 \geq a) - \Pr(A_1 \geq b)$

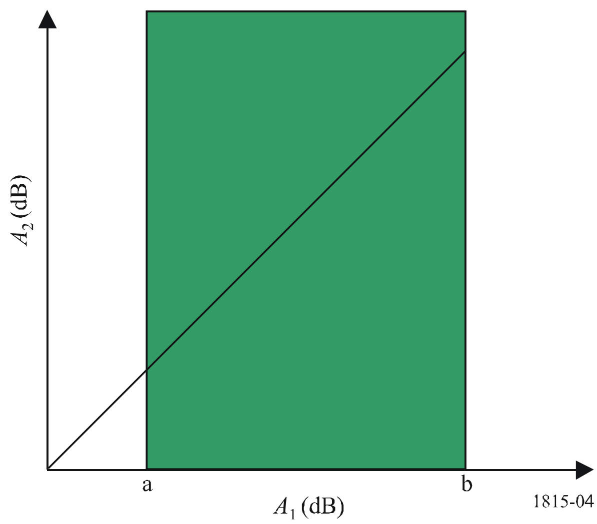
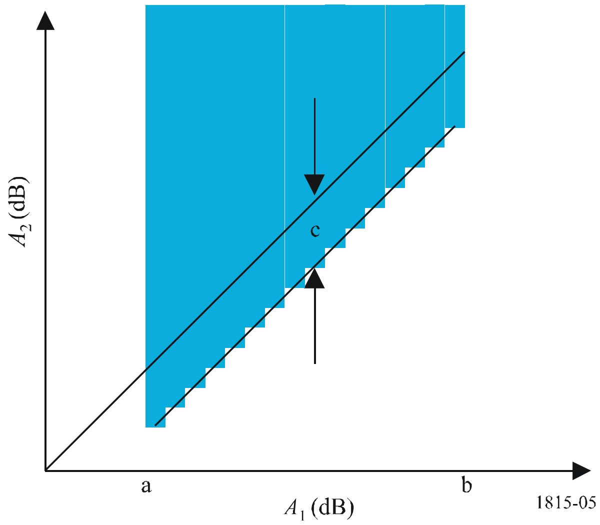


FIGURE 5

$$\sum_{i=1}^{n}\left\{\Pr\left(A_1 \ge a+(i-1)\delta-\frac{\delta}{2}, A_2 \ge a+(i-1)\delta-c\right)-\Pr\left(A_1 \ge a+(i-1)\delta+\frac{\delta}{2}, A_2 \ge a+(i-1)\delta-c\right)\right\}$$



From Figs. 4 and 5, the joint probability $\Pr\{a < A_1 \le b, A_2 \le A_1 - c\}$ can be well-approximated by:

$$\begin{aligned}&\Pr\{a < A_1 \le b, A_2 \le A_1 - c\}\\ &= \Pr(A_1 \ge a) - \Pr(A_1 \ge b)\\ &-\sum_{i=1}^{n}\left\{\Pr\left(A_1 \ge a+(i-1)\delta-\frac{\delta}{2}, A_2 \ge a+(i-1)\delta-c\right)-\Pr\left(A_1 \ge a+(i-1)\delta+\frac{\delta}{2}, A_2 \ge a+(i-1)\delta-c\right)\right\}\end{aligned}$$

where:

$$\delta = \frac{b-a}{n}$$

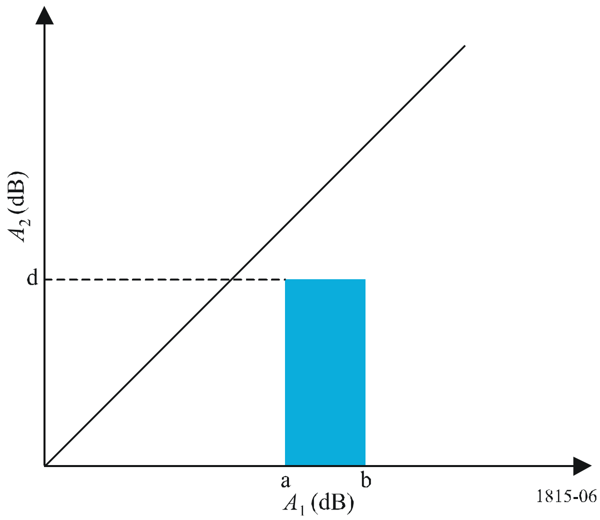
and the number of points, n, is selected so the approximation is sufficiently accurate. A step size, δ, of 0.01 dB generally provides sufficient accuracy.

This method can also be used to compute other desired joint probabilities. For example, the joint probability $\Pr\{a < A_1 \le b, A_2 \le d\}$ shown in the shaded region of Fig. 6 is:

$$\Pr\{a < A_1 \le b, A_2 \le d\} = \Pr\{A_1 \ge a\} - \Pr\{A_1 \ge b\} - [\Pr(A_1 \ge a, A_2 \ge d) - \Pr(A_1 \ge b, A_2 \ge d)]$$

FIGURE 6

$\Pr\{a < A_1 \le b, A_2 \le d\}$



2 Annual differential attenuation statistics

If annual differential attenuation statistics are required, the probability $\Pr\{A_1 \ge a, A_2 \ge b\}$ can be computed using the prediction method described in Annex 2, based on fitting the single-site rain attenuations vs. annual probabilities of occurrence, $\Pr\{A_1 \ge a\}$ and $\Pr\{A_2 \ge b\}$, to log-normal probability distributions. The rain attenuation vs. annual probability of occurrence can be predicted using the method described in § 2.2.1.1 of Recommendation ITU-R P.618.

Annual differential attenuation statistics can be obtained using the following procedure:

Step 1: Obtain the annual rain attenuation vs. probability of occurrence using the ITU-R rain attenuation prediction method described in § 2.2.1.1 of Recommendation ITU-R P.618.

Step 2: Apply the differential rain attenuation prediction method described in § 1, where the appropriate probabilities $\Pr(A_1 \ge a_1, A_2 \ge a_2)$ are calculated using the method described in Annex 2.

3 Worst-month differential attenuation statistics

If worst-month differential attenuation statistics are required, Recommendation ITU-R P.841 can be used to convert single-site annual rain attenuation statistics to single-site worst-month rain attenuation statistics.

Worst-month differential attenuation statistics can be obtained using the following procedure:

Step 1: Obtain the annual rain attenuation vs. probability of occurrence using the ITU-R rain attenuation prediction method described in § 2.2.1.1 of Recommendation ITU-R P.618.

Step 2: Convert the annual rain attenuation statistics to worst-month rain attenuation statistics using the ITU-R worst-month conversion method described in Recommendation ITU-R P.841.

Step 3: Apply the differential rain attenuation prediction method described in § 1, where the appropriate probabilities $\Pr(A_1 \ge a_1, A_2 \ge a_2)$ are calculated using the method described in Annex 2.

Annex 2

Description of differential rain attenuation prediction method

1 Analysis

The differential rain attenuation prediction method assumes a log-normal distribution of rain intensity and rain attenuation.

This method predicts $\Pr(A_1 \geq a_1, A_2 \geq a_2)$, the joint probability (%) that the attenuation on the path to the first site is greater than a_1 and the attenuation on the path to the second site is greater than a_2. $\Pr(A_1 \geq a_1, A_2 \geq a_2)$ is the product of two joint probabilities:

1 P_r, the joint probability that it is raining at both sites, and

2 P_a, the conditional joint probability that the attenuations exceed a_1 and a_2, respectively, given that it is raining at both sites; i.e:

$$\Pr(A_1 \geq a_1, A_2 \geq a_2) = 100 \times P_r \times P_a \qquad \% \tag{1}$$

These probabilities are:

$$P_r = \frac{1}{2\pi\sqrt{1-\rho_r^2}} \int_{R_1}^{\infty}\int_{R_2}^{\infty} \exp\left[-\left(\frac{r_1^2 - 2\rho_r r_1 r_2 + r_2^2}{2(1-\rho_r^2)}\right)\right] dr_1 dr_2 \tag{2}$$

where:

$$\rho_r = 0.7\exp(-d/60) + 0.3\exp\left[-(d/700)^2\right] \tag{3}$$

and

$$P_a = \frac{1}{2\pi\sqrt{1-\rho_a^2}} \int_{\frac{\ln a_1 - m_{\ln A_1}}{\sigma_{\ln A_1}}}^{\infty} \int_{\frac{\ln a_2 - m_{\ln A_2}}{\sigma_{\ln A_2}}}^{\infty} \exp\left[-\left(\frac{a_1^2 - 2\rho_a a_1 a_2 + a_2^2}{2(1-\rho_a^2)}\right)\right] da_1 da_2 \tag{4}$$

where:

$$\rho_a = 0.94\exp(-d/30) + 0.06\exp\left[-(d/500)^2\right] \tag{5}$$

and P_a and P_r are complementary bivariate normal distributions.

The parameter d is the separation between the two sites (km). The thresholds R_1 and R_2 are the solutions of:

$$P_k^{rain} = 100 \times Q(R_k) = 100 \times \frac{1}{\sqrt{2\pi}} \int_{R_k}^{\infty} \exp\left(-\frac{r^2}{2}\right) dr \tag{6}$$

i.e.:

$$R_k = Q^{-1}\left(\frac{P_k^{rain}}{100}\right) \tag{7}$$

where R_k is the threshold for the k-th site, respectively, P_k^{rain} is the probability of rain (%), Q is the complementary cumulative normal distribution, and Q^{-1} is the inverse complementary cumulative normal distribution. P_k^{rain} for a particular location can be obtained from Step 3 of Annex 1 of Recommendation ITU-R P.837 using either local data or the ITU-R rainfall rate maps.

The values of the parameters $m_{\ln A_1}, m_{\ln A_2}, \sigma_{\ln A_1}$, and $\sigma_{\ln A_2}$ are determined by fitting each single-site rain attenuation, A_i, vs. probability of occurrence, P_i, to the log-normal distribution:

$$P_i = P_k^{rain} Q\left(\frac{\ln A_i - m_{\ln A_i}}{\sigma_{\ln A_i}}\right) \tag{8}$$

These parameters can be obtained for each individual location, or a single location can be used. The rain attenuation vs. annual probability of occurrence can be predicted using the method described in § 2.2.1.1.

For each site, the log-normal fit of rain attenuation vs. probability of occurrence is performed as follows:

Step 1: Construct the set of pairs [P_i, A_i] where P_i (% of time) is the probability the attenuation A_i (dB) is exceeded.

Step 2: Transform the set of pairs to $\left[Q^{-1}\left(P_i / P_k^{rain}\right), \ln A_i\right]$.

Step 3: Determine the variables $m_{\ln A_i}$ and $\sigma_{\ln A_i}$ by performing a least-squares fit to $\ln A_i = \sigma_{\ln A_i} \, Q^{-1}\left(P_i / P_k^{rain}\right) + m_{\ln A_i}$ for all i.

(See Recommendation ITU-R P.1057 for a detailed description.)

An implementation of this prediction method in MATLAB and a reference to an approximation of the complementary bivariate normal distribution are available from the ITU-R website dealing with Radiocommunication Study Group 3.