International Telecommunication Union

ITU-R

Radiocommunication Sector of ITU

Recommendation ITU-R P.453-11

(07/2015)

The radio refractive index: its formula and refractivity data

P Series

Radiowave propagation

International Telecommunication Union

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Note: *This ITU-R Recommendation was approved in English under the procedure detailed in Resolution ITU-R 1.*

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RECOMMENDATION ITU-R P.453-11

The radio refractive index: its formula and refractivity data

(Question ITU-R 201/3)

(1970-1986-1990-1992-1994-1995-1997-1999-2001-2003-2012-2015)

Scope

Recommendation ITU-R P.453 provides methods to estimate the radio refractive index and its behaviour for locations worldwide; describes both surface and vertical profile characteristics; and provides global maps for the distribution of refractivity parameters and their statistical variation.

The ITU Radiocommunication Assembly,

considering

a) the necessity of using a single formula for calculation of the index of refraction of the atmosphere;

b) the need for reference data on refractivity and refractivity gradients all over the world;

c) the necessity to have a mathematical method to express the statistical distribution of refractivity gradients,

recommends

1 that the atmospheric radio refractive index, n, be computed by means of the formula given in Annex 1;

2 that refractivity data given on world charts and global numerical maps in Annex 1 should be used, except if more reliable local data are available;

3 that the statistical distribution of refractivity gradients be computed using the method given in Annex 1;

4 that in the absence of local data on temperature and relative humidity, the global numerical map of the wet term of the surface radio refractivity exceeded for 50% of the year described in Annex 1, § 2.2 be used (see Fig. 3).

Annex 1

1 The formula for the radio refractive index

The atmospheric radio refractive index, n, can be computed by the following formula:

$$n = 1 + N \times 10^{-6} \tag{1}$$

where the radio refractivity, N, is:

$$N = 77.6\frac{P_d}{T} + 72\frac{e}{T} + 3.75\times10^5\frac{e}{T^2} \qquad \text{(N-units)} \tag{2}$$

the dry term of the radio refractivity, N_{dry}, is:

$$N_{dry} = 77.6\frac{P_d}{T} \tag{3}$$

and the wet term of the radio refractivity, N_{wet}, is:

$$N_{wet} = 72\frac{e}{T} + 3.75\times10^5\frac{e}{T^2} \tag{4}$$

where:

P_d: dry atmospheric pressure (hPa)

P: total atmospheric pressure (hPa)

e: water vapour pressure (hPa)

T: absolute temperature (K)

and

$$P = P_d + e \tag{5}$$

Since $P_d = P - e$, equation (2) can be rewritten as:

$$N = 77.6\frac{P}{T} - 5.6\frac{e}{T} + 3.75 \text{ x } 10^5\frac{e}{T^2} \tag{6}$$

Equation (6) may be approximated with reduced accuracy as:

$$N = \frac{77.6}{T}\left(P + 4810\frac{e}{T}\right) \tag{7}$$

Equation (7) yields values of N within 0.02 percent of the value obtained from equation (2) for the temperature range from −50°C to +40°C. For representative profiles of temperature, pressure and water vapour pressure, see Recommendation ITU-R P.835.

For ready reference, the relationship between water vapour pressure e and relative humidity is given by:

$$e = \frac{H \cdot e_s}{100} \tag{8}$$

with:

$$e_s = EF \cdot a \cdot \exp\left[\frac{\left(b - \frac{t}{d}\right) \cdot t}{t + c}\right] \tag{9}$$

and:

$$EF_{water} = 1 + 10^{-4}\left[7.2 + P \cdot \left(0.00320 + 5.9 \cdot 10^{-7} \cdot t^2\right)\right]$$
$$EF_{ice} = 1 + 10^{-4}\left[2.2 + P \cdot \left(0.00382 + 6.4 \cdot 10^{-7} \cdot t^2\right)\right]$$

where:

- t: temperature (°C)
- P: pressure (hPa)
- H: relative humidity (%)
- e_s: saturation vapour pressure (hPa) at the temperature t (°C) and the coefficients a, b, c and d are:

for water	*for ice*
a = 6.1121	a = 6.1115
b = 18.678	b = 23.036
c = 257.14	c = 279.82
d = 234.5	d = 333.7
(valid between −40° to +50°)	(valid between −80° to 0°)

Vapour pressure e is obtained from the water vapour density ρ using the equation:

$$e = \frac{\rho T}{216.7} \qquad \text{hPa} \tag{10}$$

where ρ is given in g/m^3. Representative values of ρ are given in Recommendation ITU-R P.836.

2 Surface refractivity and height dependence

2.1 Refractivity as a function of height

It has been found that the long-term mean dependence of the refractive index n upon the height h is well expressed by an exponential law:

$$n(h) = 1 + N_0 \times 10^{-6} \times \exp(-h/h_0) \tag{11}$$

where:

- N_0: average value of atmospheric refractivity extrapolated to sea level
- h_0: scale height (km).

N_0 and h_0 can be determined statistically for different climates. For reference purposes a global mean of the height profile of refractivity may be defined by:

$$N_0 = 315$$
$$h_0 = 7.35 \text{ km}$$

These numerical values apply only for terrestrial paths.

This reference profile may be used to compute the value of refractivity N_s at the Earth's surface from N_0 as follows:

$$N_s = N_0 \exp(-h_s/h_0) \tag{12}$$

where:

h_s: height of the Earth's surface above sea level (km).

It is to be noted, however, that the contours of Figs. 1 and 2 were derived using a value of h_0 equal to 9.5 km. Figures 1 and 2 were derived from a 5-year data set (1955-1959) from about 1 000 surface stations. (Figures 1 and 2 are not available in numerical form.)

For Earth-satellite paths, the refractive index at any height is obtained using equations (1), (2) and (10) above, together with the appropriate values for the parameters given in Recommendation ITU-R P.835, Annex 1. The refractive indices thus obtained may then be used for numerical modelling of ray paths through the atmosphere.

(Note that the exponential profile in equation (12) may also be used for quick and approximate estimates of refractivity gradient near the Earth's surface and of the apparent boresight angle, as given in § 4.3 of Recommendation ITU-R P.834.)

2.2 Wet term of the surface refractivity

Figure 3 shows for easy reference the median value (50%) of the wet term of the surface refractivity exceeded for the average year. Data file ESANWET.TXT contains the numerical data.

The wet term of the surface refractivity was derived from two years (1992-1993) of initialization data of the numerical weather forecast of the European Centre for Medium-range Weather Forecast (ECMWF).

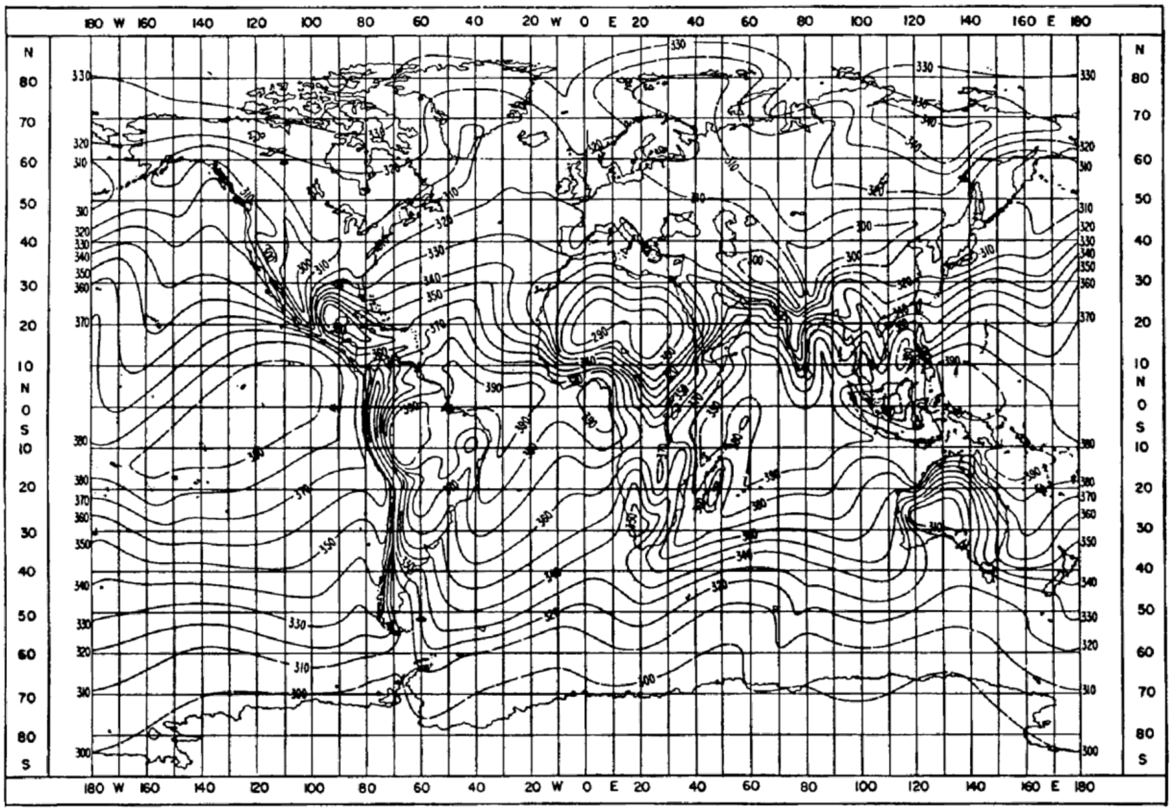
NOTE 1 – The data file ESANWET.TXT has a resolution of 1.5° in both latitude and longitude. The companion data files ESALAT.TXT and ESALON.TXT contain respectively the latitudes and longitudes of the corresponding entries (gridpoints) in data file ESANWET.TXT.

The data range from 0° to 360° in longitude and from +90° to –90° in latitude. For a location different from the gridpoints, the wet term of the refractivity at the desired location can be derived by performing a bi-linear interpolation on the values at the four closest gridpoints.

The data files can be obtained from the Radiocommunication Bureau (BR).

FIGURE 1

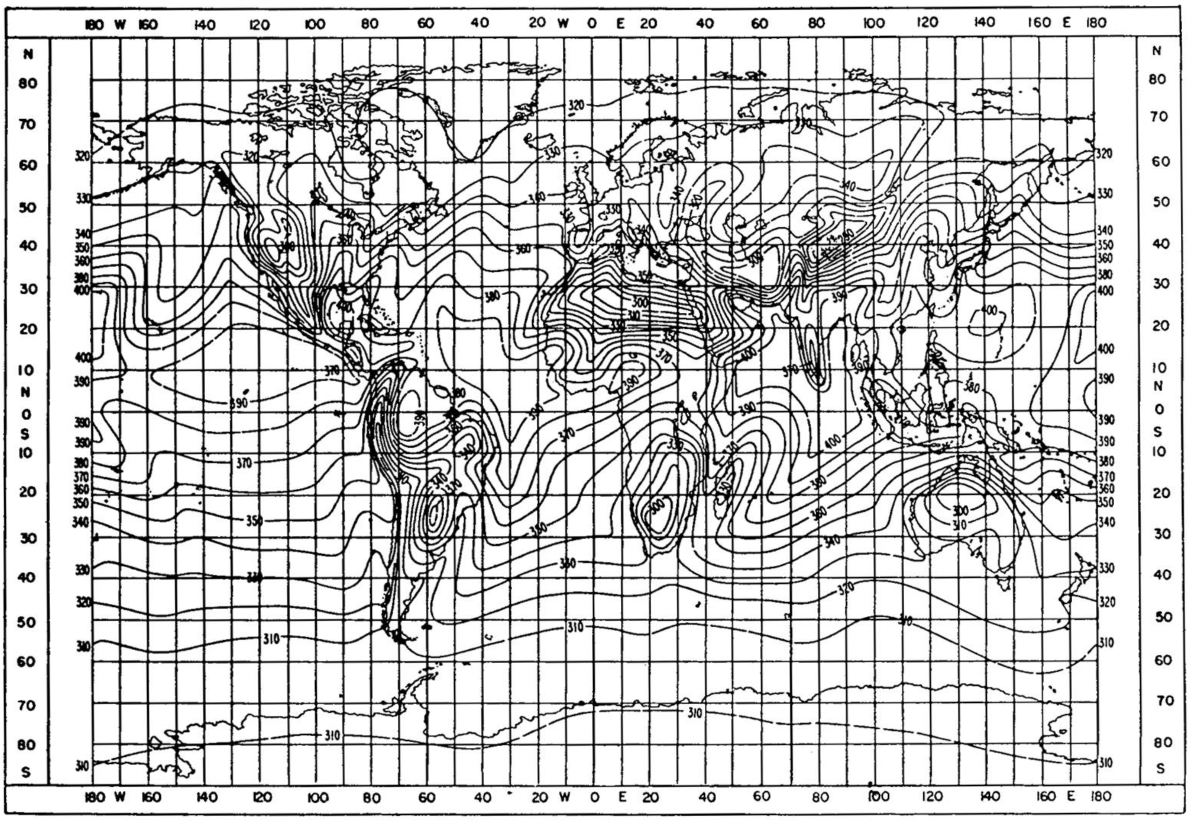
Monthly mean values of N_0: February



P.0453-01

FIGURE 2

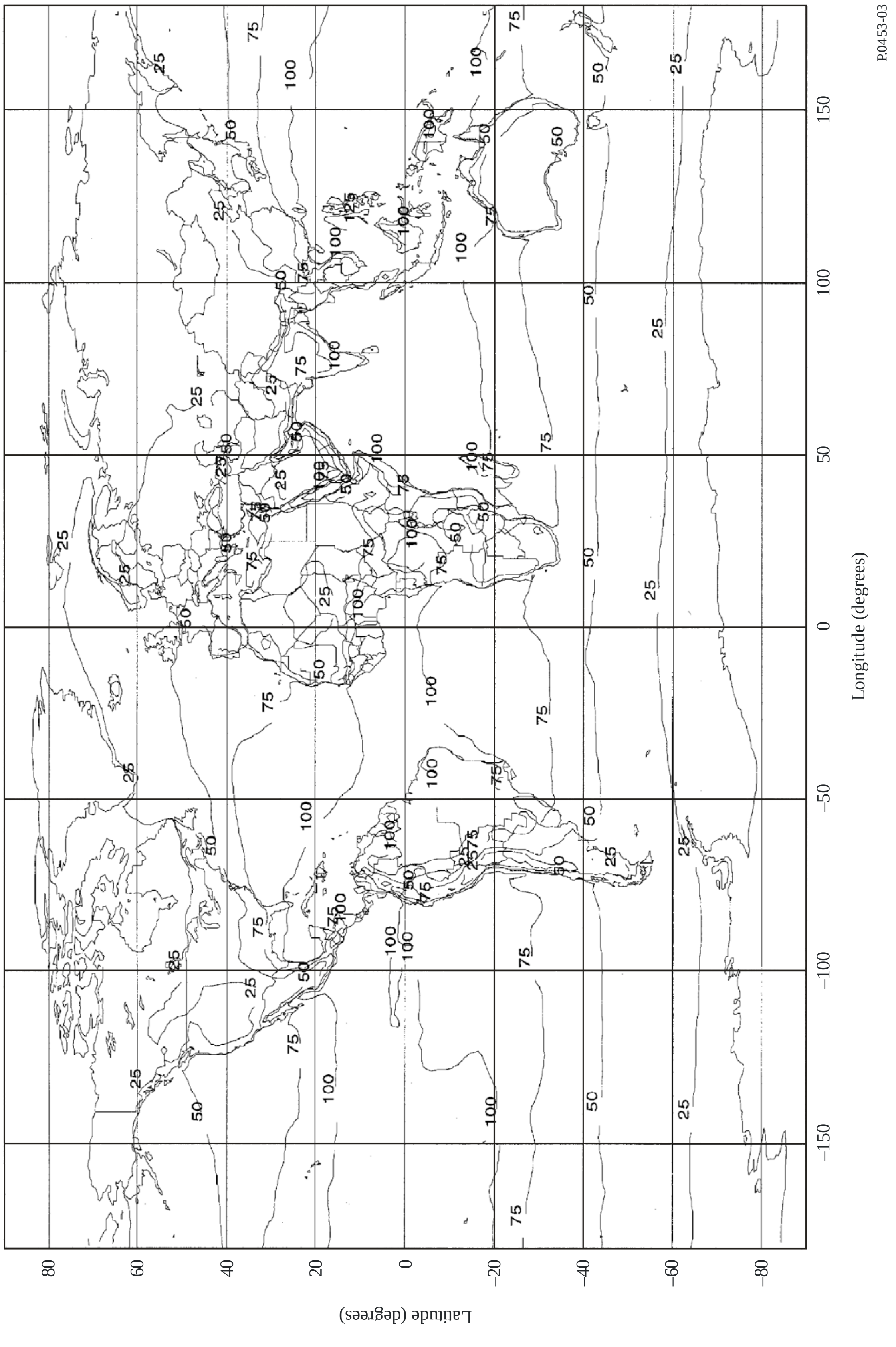
Monthly mean values of N_0: August



P.0453-02

FIGURE 3

Wet term of the surface refractivity (ppm) exceeded for 50% of the year



3 Vertical refractivity gradients

The statistics of the vertical gradient of radio refractivity in the lowest layer of the atmosphere are important parameters for the estimation of path clearance and propagation associated effects such as ducting on transhorizon paths, surface reflection and multipath fading and distortion on terrestrial line-of-sight links.

3.1 In the first kilometre of the atmosphere

Figures 4 to 7 present isopleths of monthly mean decrease (i.e. lapse) in radio refractivity over a 1 km layer from the surface. The change in radio refractivity, ΔN, was calculated from:

$$\Delta N = N_s - N_1 \tag{13}$$

where N_1 is the radio refractivity at a height of 1 km above the surface of the Earth. The ΔN values were not reduced to a reference surface. Figures 4 to 7 were derived from a 5-year data set (1955-1959) from 99 radiosonde sites. (Figures 4 to 7 are not available in numerical form.).

In addition, the annual values of ΔN, exceeded for 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 98, 99, 99.5, 99.8, 99.9 of an average year are an integral part of this Recommendation and are available in the form of digital maps and are provided in the Supplement. The monthly values of ΔN, exceeded for 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 98, 99, 99.5, 99.8, 99.9 of an average month are an integral part of this Recommendation and are available in the form of digital maps and are provided in the Supplement.

3.2 In the lowest atmospheric layer

Refractivity gradient statistics for the lowest 100 m from the surface of the Earth are used to estimate the probability of occurrence of ducting and multipath conditions. Where more reliable local data are not available, the charts in Figs. 8 to 11 give such statistics for the world which were derived from a 5-year data set (1955-1959) from 99 radiosonde sites. (Figures 8 to 11 are not available in numerical form.)

In addition the following parameters are an integral part of this Recommendation and are available in the form of digital maps and are provided in the Supplement:

– The annual values of the refractivity gradient in the lowest 65 m from the surface of the Earth, ΔN_{65m}, exceeded for 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 98, 99, 99.5, 99.8, 99.9 of an average year.

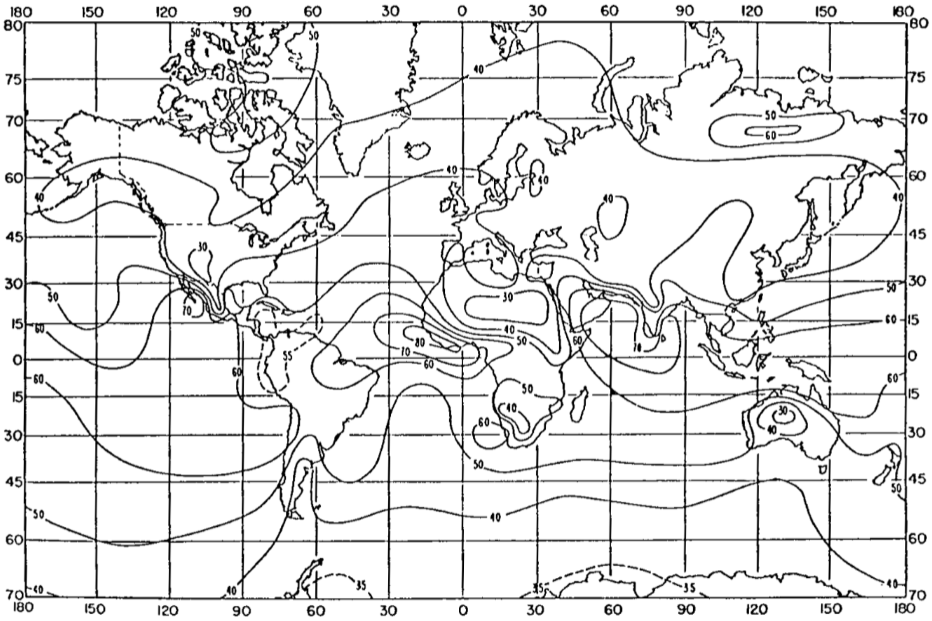
– The monthly values of the refractivity gradient in the lowest 65 m from the surface of the Earth, ΔN_{65m},, exceeded for 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 98, 99, 99.5, 99.8, 99.9 of an average month.

– The percentage of annual and monthly times for which refractivity gradient, ΔN over 100 m is lower than −100 N-unit/km, (%).

The data range from 0° to 360° in longitude and from +90° to −90° in latitude. For a location different from the gridpoints, the refractivity gradient at the desired location can be derived by performing a bi-linear interpolation on the values at the four closest gridpoints as described in Recommendation ITU-R P.1144.

FIGURE 4

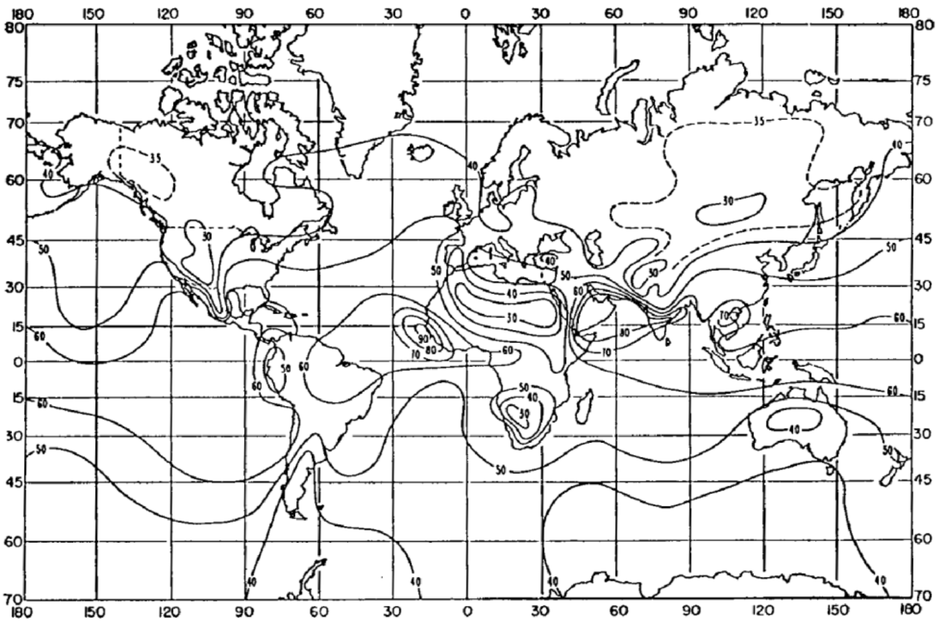
Monthly mean values of Δ*N*: February



P.0453-04

FIGURE 5

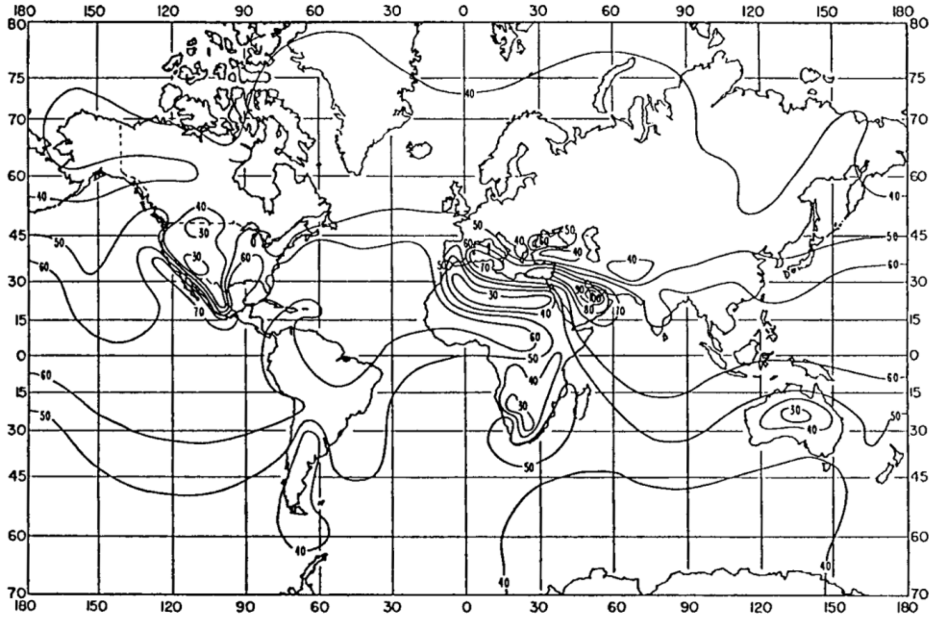
Monthly mean values of Δ*N*: May



P.0453-05

FIGURE 6

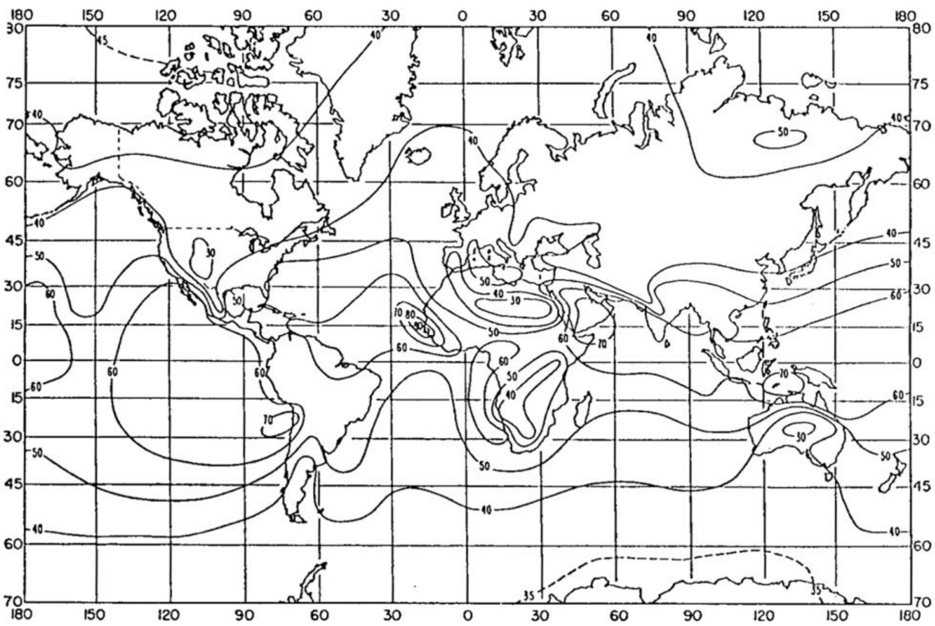
Monthly mean values of Δ*N*: August



P.0453-06

FIGURE 7

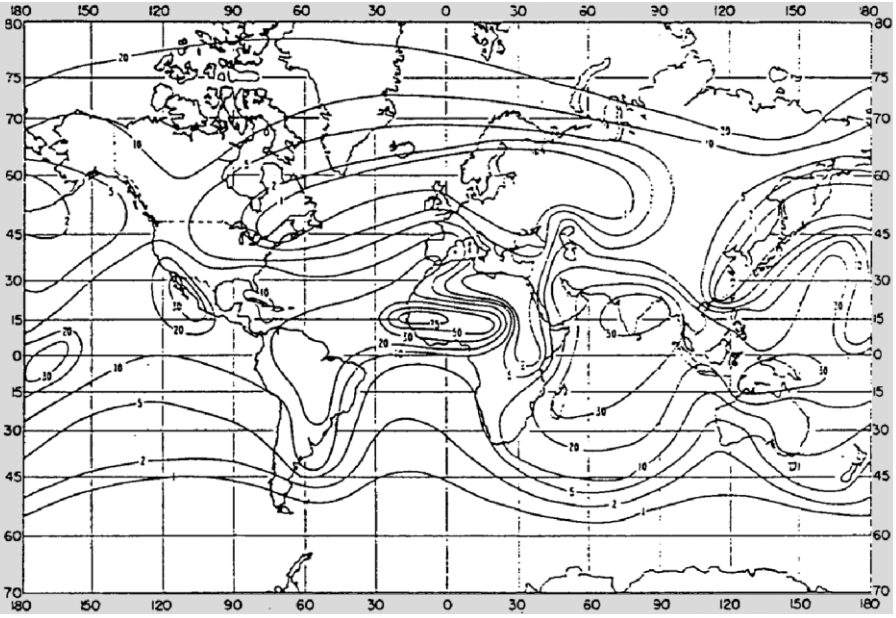
Monthly mean values of Δ*N*: November



P.0453-07

FIGURE 8

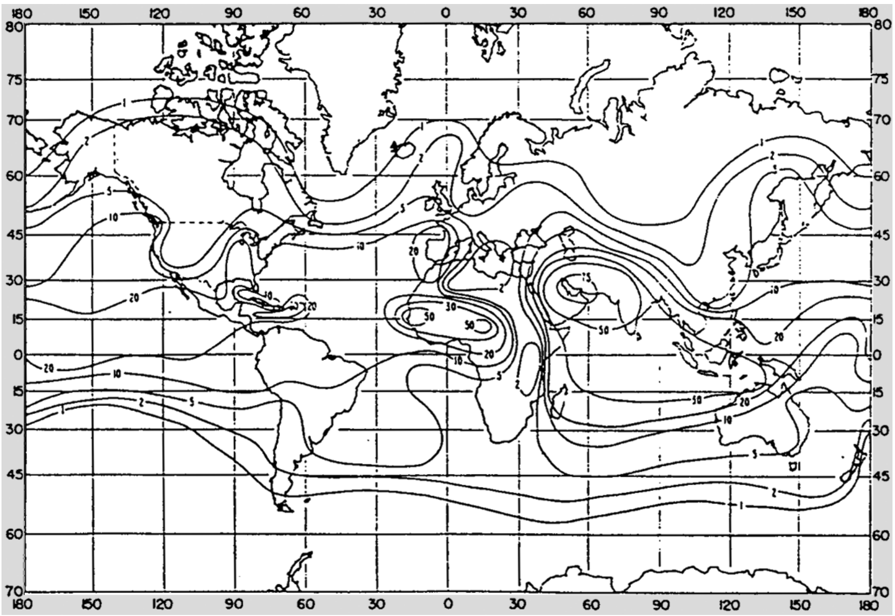
Percentage of time gradient ≤ −100 (N-units/km): February



P.0453-08

FIGURE 9

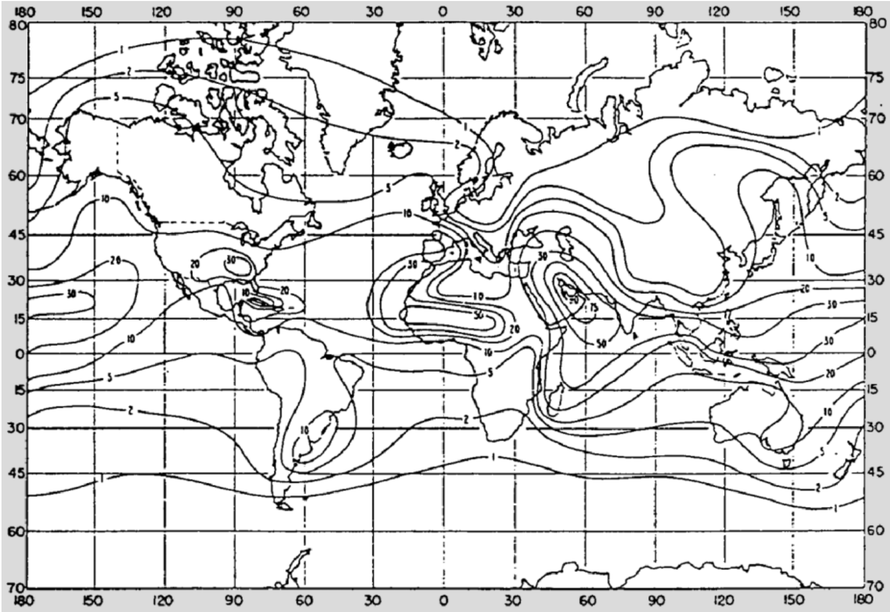
Percentage of time gradient ≤ −100 (N-units/km): May



P.0453-09

FIGURE 10

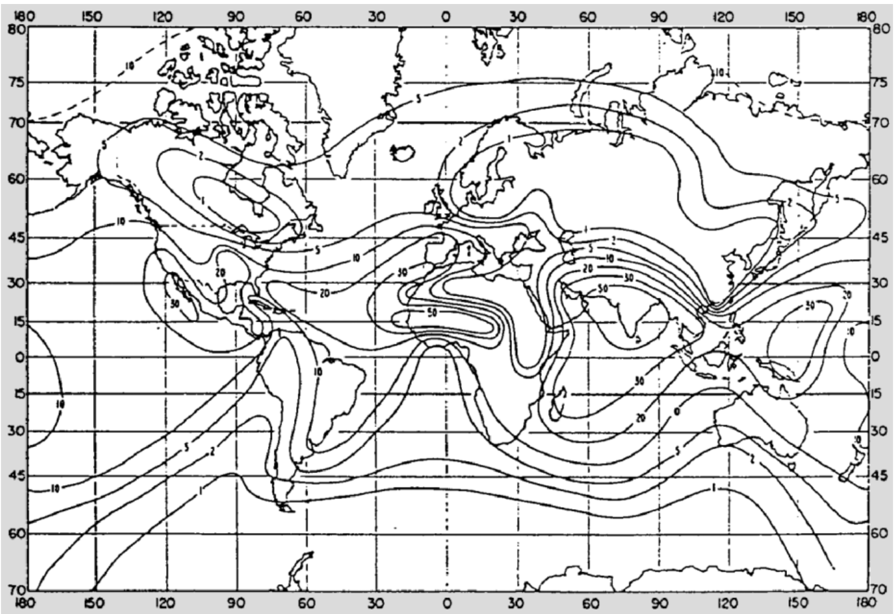
Percentage of time gradient ≤ –100 (N-units/km): August



P.0453-10

FIGURE 11

Percentage of time gradient ≤ –100 (N-units/km): November



P.0453-11

4 Statistical distribution of refractivity gradients

It is possible to estimate the complete statistical distribution of refractivity gradients near the surface of the Earth over the lowest 100 m of the atmosphere from the median value *Med* of the refractivity gradient and the ground level refractivity value, N_s, for the location being considered.

The median value, *Med*, of the refractivity gradient distribution may be computed from the probability, P_0, that the refractivity gradient is lower than or equal to D_n using the following expression:

$$Med = \frac{D_n + k_1}{(1/P_0 - 1)^{1/E_0}} - k_1 \qquad (14)$$

where:

$E_0 =$ $\log_{10}(\,|D_n|\,)$

$k_1 =$ 30.

Equation (14) is valid for the interval –300 N-units/km ≤ D_n ≤ –40 N-units/km. If this probability P_0 corresponding to any given D_n value of refractivity gradient is not known for the location under study, it is possible to derive P_0 from the world maps in Figs. 8 to 11 which give the percentage of time during which the refractivity gradient over the lowest 100 m of the atmosphere is less than or equal to −100 N-units/km.

Where more reliable local data are not available, N_s may be derived from the global sea level refractivity N_0 maps of Figs. 1 and 2 and equation (12).

For $D_n \le Med$, the cumulative probability P_1 of D_n may be obtained from:

$$P_1 = \frac{1}{1 + \left[\left(\frac{|D_n - Med|}{B} + k_2\right) k_3\right]^{E_1}} \qquad (15)$$

where:

$$B = \left|\frac{0.3\,Med - N_s + 210}{2}\right|$$

$$E_1 = \log_{10}(F + 1)$$

$$F = \frac{2 \times |D_n - Med|}{\left(\frac{B}{67}\right)^{6.5} + 1}$$

$$k_2 = \frac{1.6B}{120}$$

$$k_3 = \frac{120}{B}$$

Equation (15) is valid for values of $Med > 120$ N-units/km and for the interval –300 N-units/km < D_n < 50 N-units/km.

For $D_n > Med$, the cumulative probability P_2 of D_n is computed from:

$$P_2 = 1 - \frac{1}{1 + \left[\left(\frac{|D_n - Med|}{B} + k_2 \right) k_4 \right]^{E_1}} \tag{16}$$

where:

$$B = \left| \frac{0.3\,Med - N_s + 210}{2} \right|$$

$$E_1 = \log_{10}(F + 1)$$

$$F = \frac{2 \times |D_n - Med|}{\left(\frac{B}{67} \right)^{6.5} + 1}$$

$$k_4 = \left[\frac{100}{B} \right]^{2.4}$$

Equation (16) is valid for values of $Med > -120$ N-units/km and for the interval –300 N-units/km < D_n < 50 N-units/km.

5 Surface and elevated ducts

Atmospheric ducts may cause deep slow fading, strong signal enhancement, and multipath fading on terrestrial line-of-sight links and may also be the cause of significant interference on transhorizon paths. It is therefore of interest to describe the occurrence of ducts and their structure. This section gives statistics derived from 20 years (1977-1996) of radiosonde observations from 661 sites.

Ducts are described in terms of modified refractivity defined as:

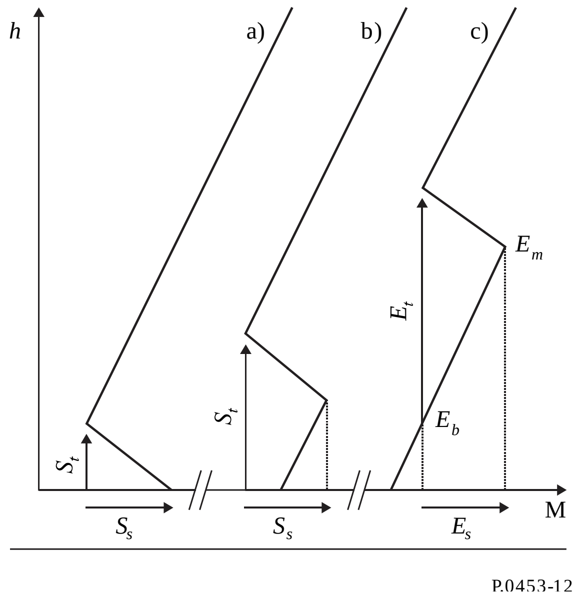
$$M(h) = N(h) + 157h \qquad \text{(M-units)} \tag{17}$$

where h (km) is the height.

Figure 12 illustrates the modified refractivity as a function of height above ground and the definitions of duct types. Ducts can be of three types: surface based, elevated-surface, and elevated ducts. Due to rather few cases of elevated-surface ducts in comparison with surface ducts, the statistics have been derived by combining these two types into one group called surface ducts. Surface ducts are characterized by their strength, S_s (M-units) or E_s (M-units), and their thickness, S_t (m) or E_t (m). Two additional parameters are used to characterize elevated ducts: namely, the base height of the duct E_b (m), and E_m (m), the height within the duct of maximum M.

FIGURE 12

Definition of parameters describing a) surface, b) elevated surface and c) elevated ducts



Figures 13 to 20 present, for easy reference, the data contained in the datafiles mentioned in the caption of the Figures. The surface and elevated-surface ducts have been combined in the statistics, due to the rather few cases of elevated-surface ducts.

The data range from 0° to 360° in longitude and from +90° to –90° in latitude with a 1.5° resolution. For a location different from the gridpoints, the parameter of interest at the desired location can be derived by performing a bi-linear interpolation on the values at the four closest gridpoints.

The data files can be obtained from the BR.

FIGURE 13

Filename: S_OCCURRENCE.TXT

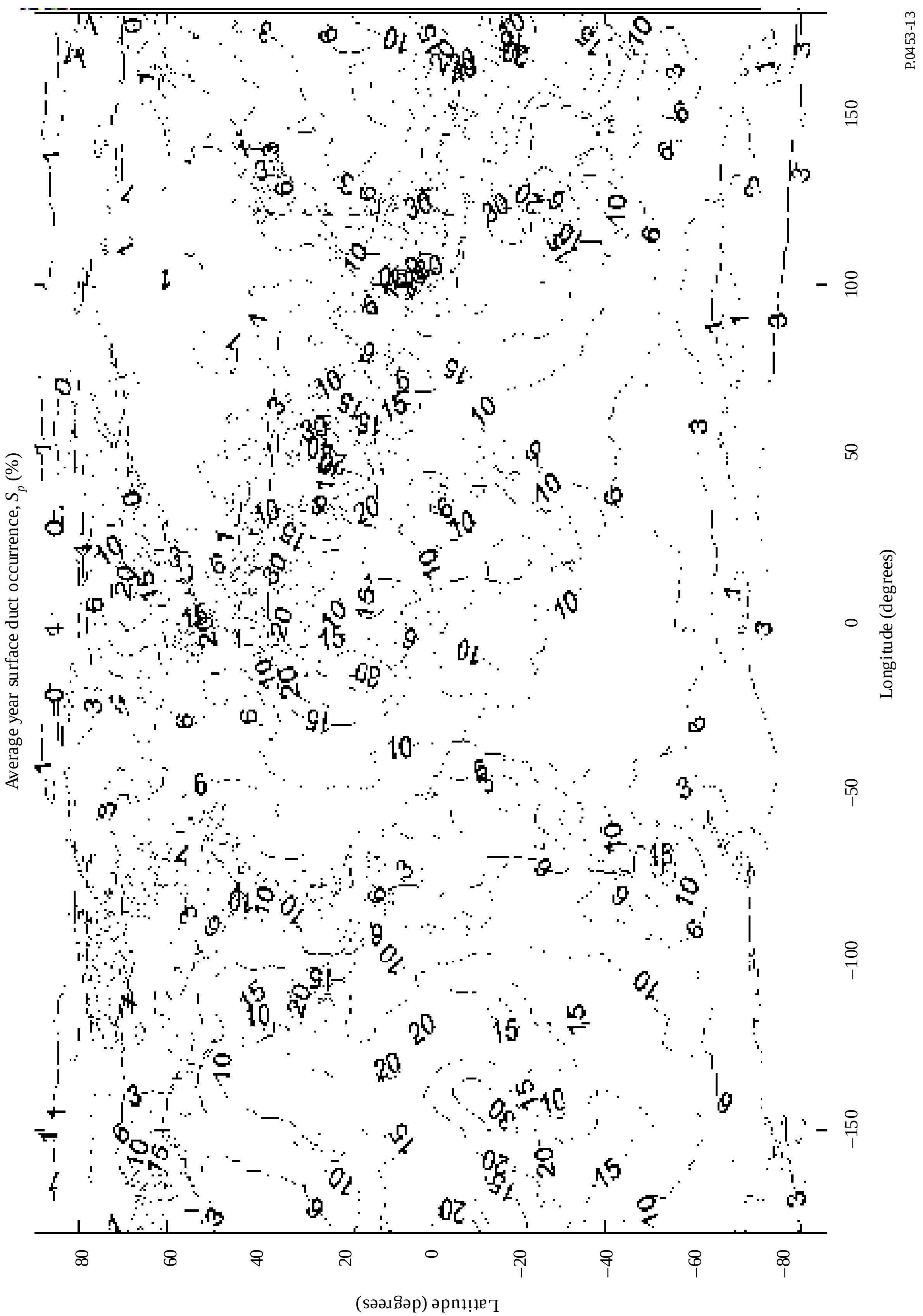


FIGURE 14

Filename: S_STRENGTH.TXT

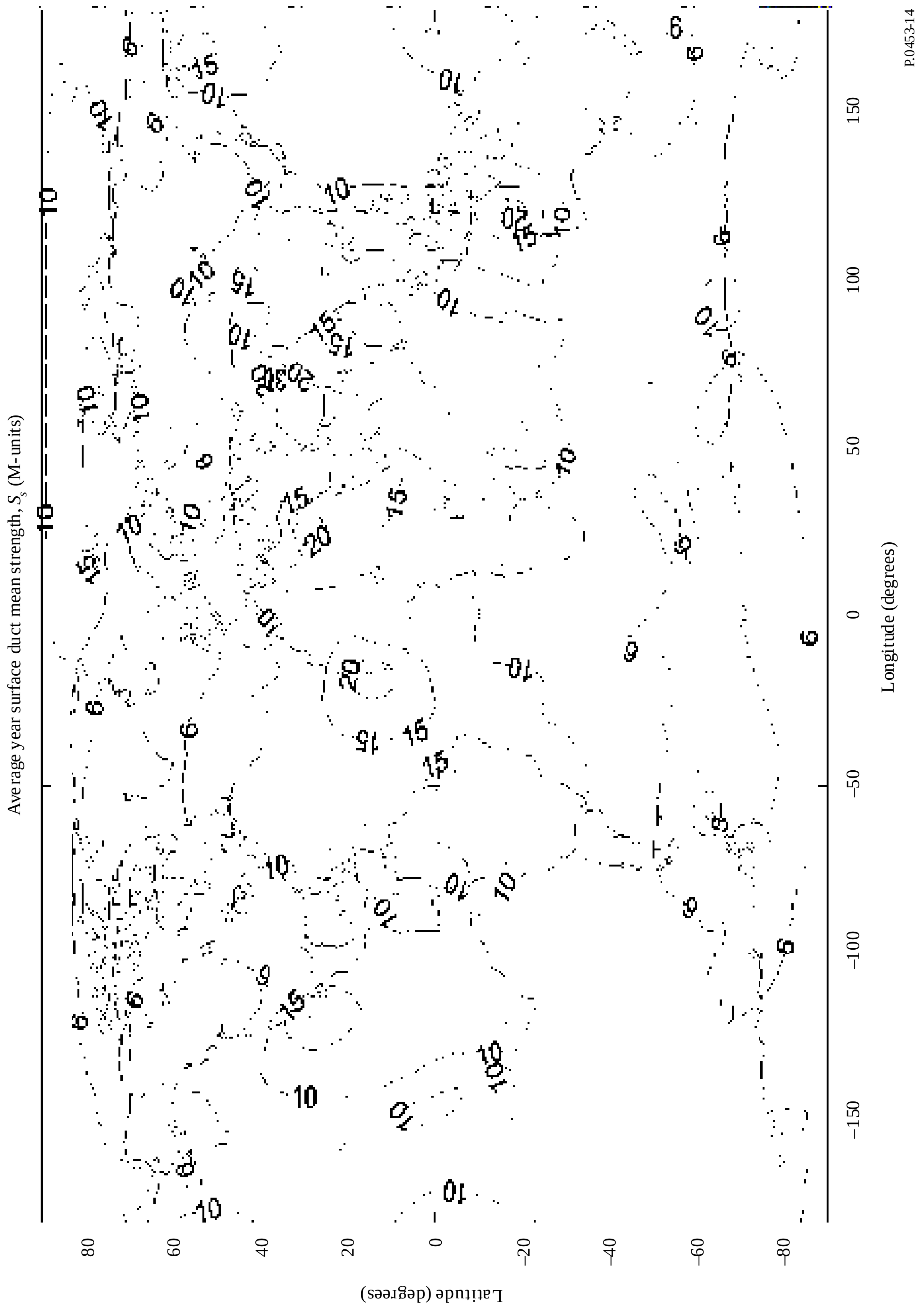


FIGURE 15

Filename: S_THICKNESS.TXT

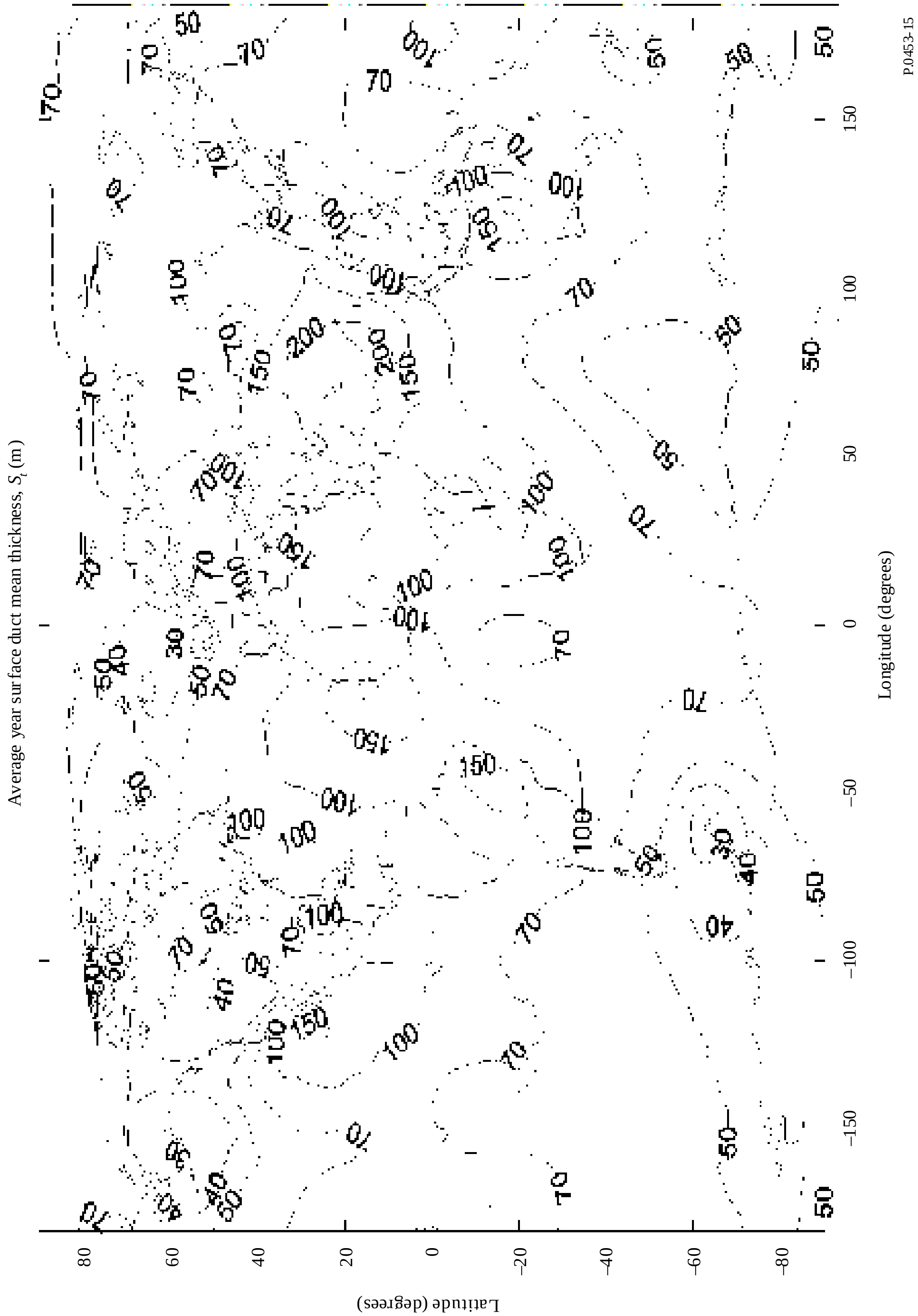


FIGURE 16

Filename: E_OCCURRENCE.TXT

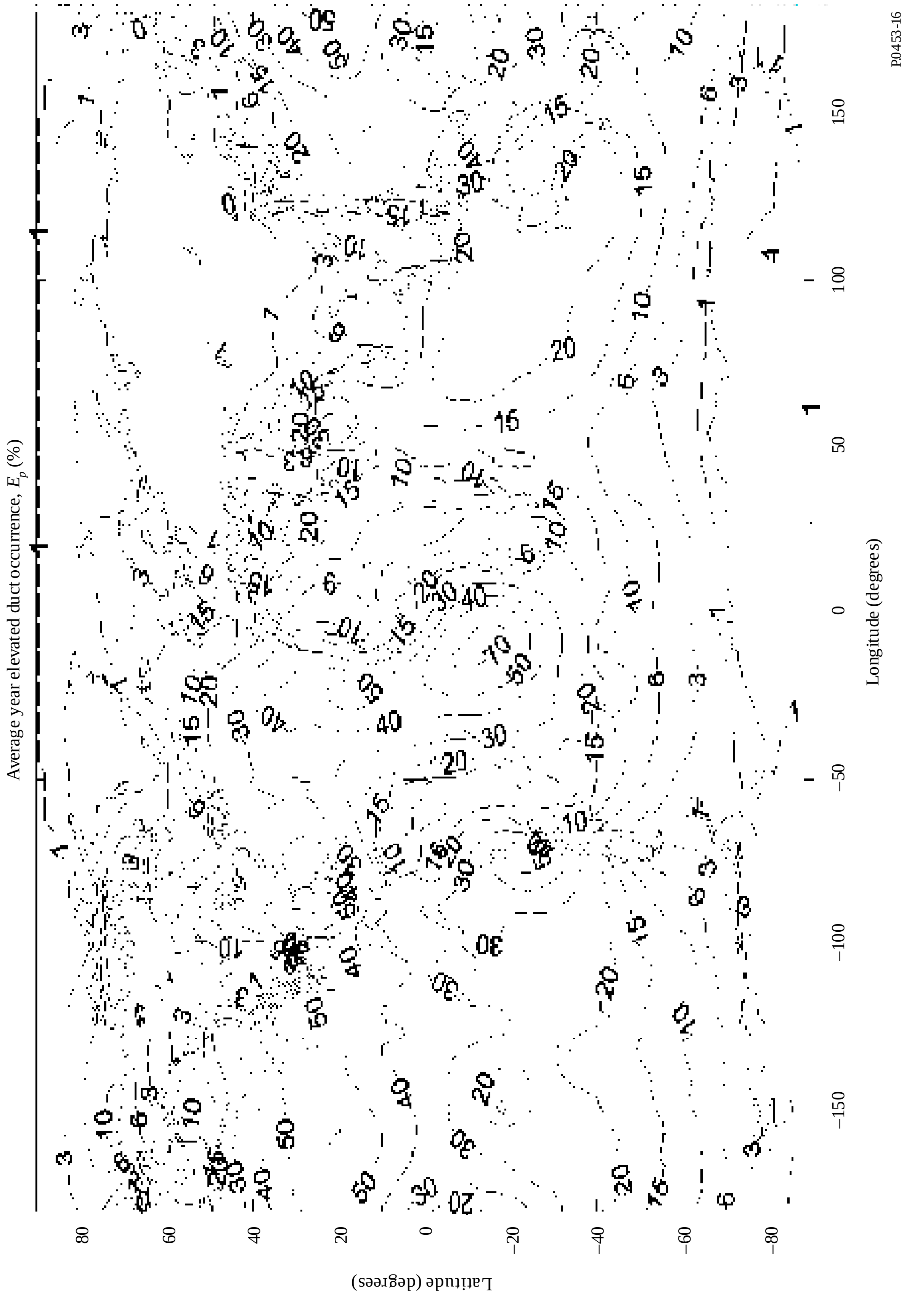


FIGURE 17

Filename: E_STRENGTH.TXT

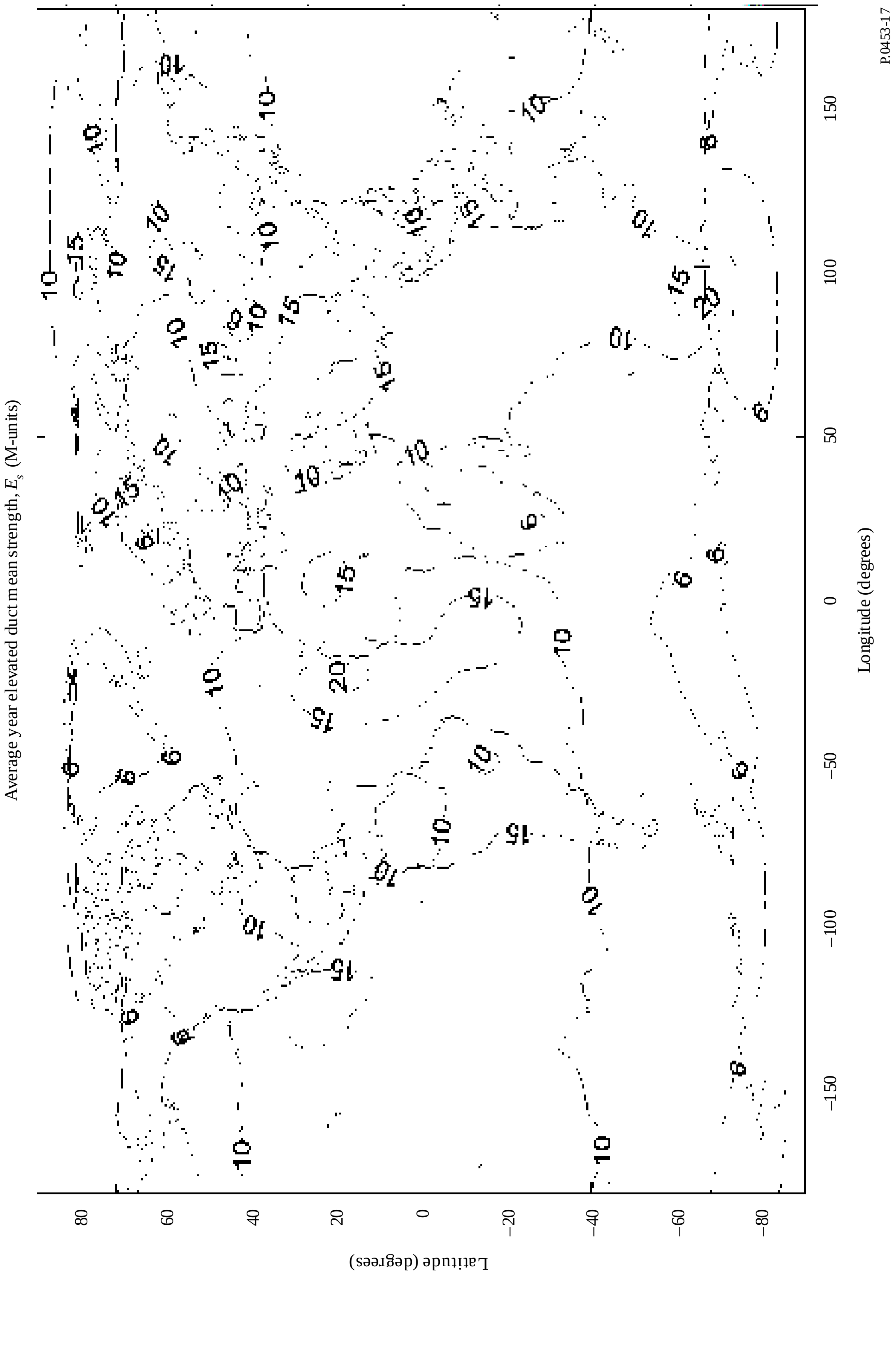


FIGURE 18

Filename: E_THICKNESS.TXT

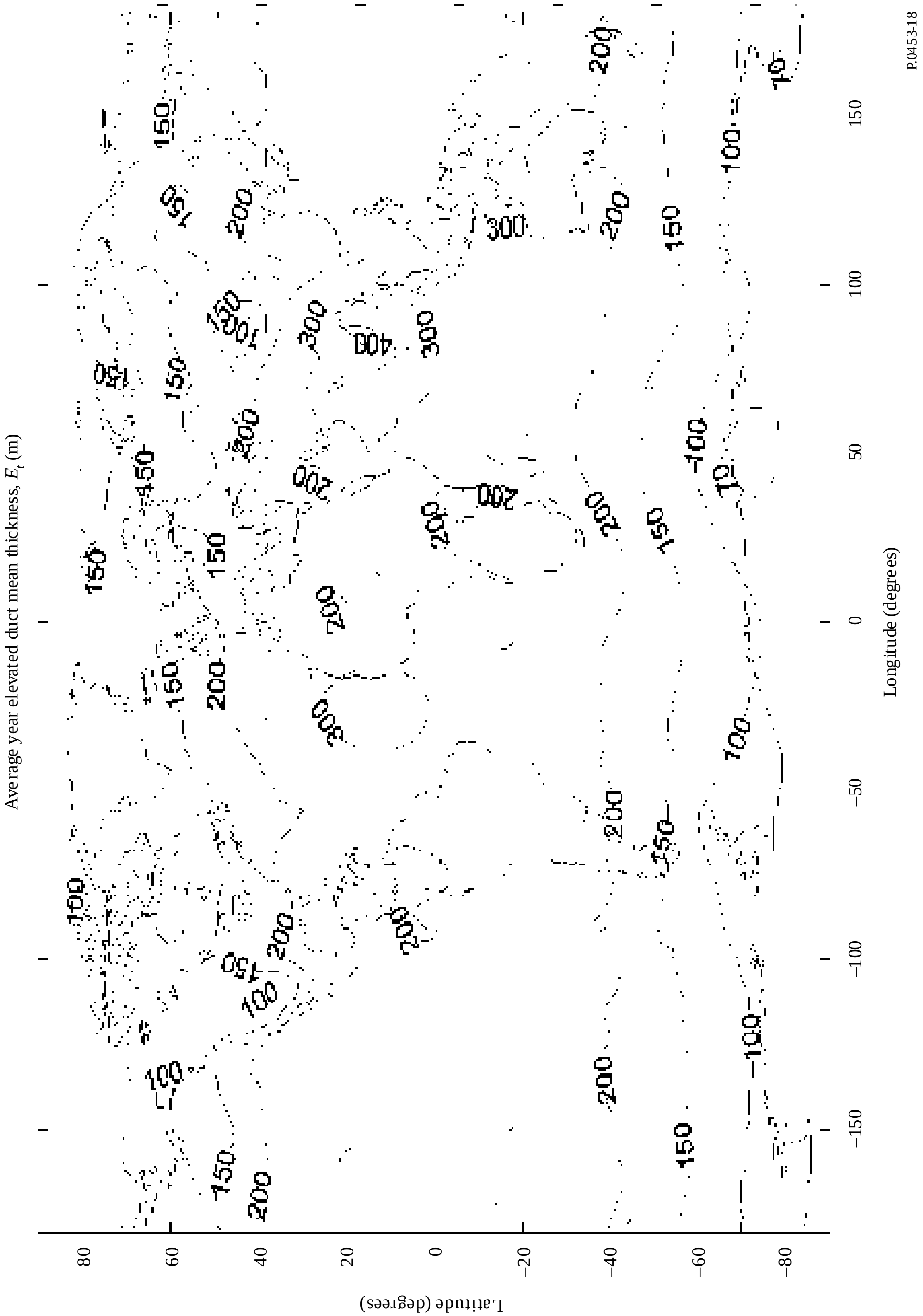


FIGURE 19

Filename: E_BASE.TXT

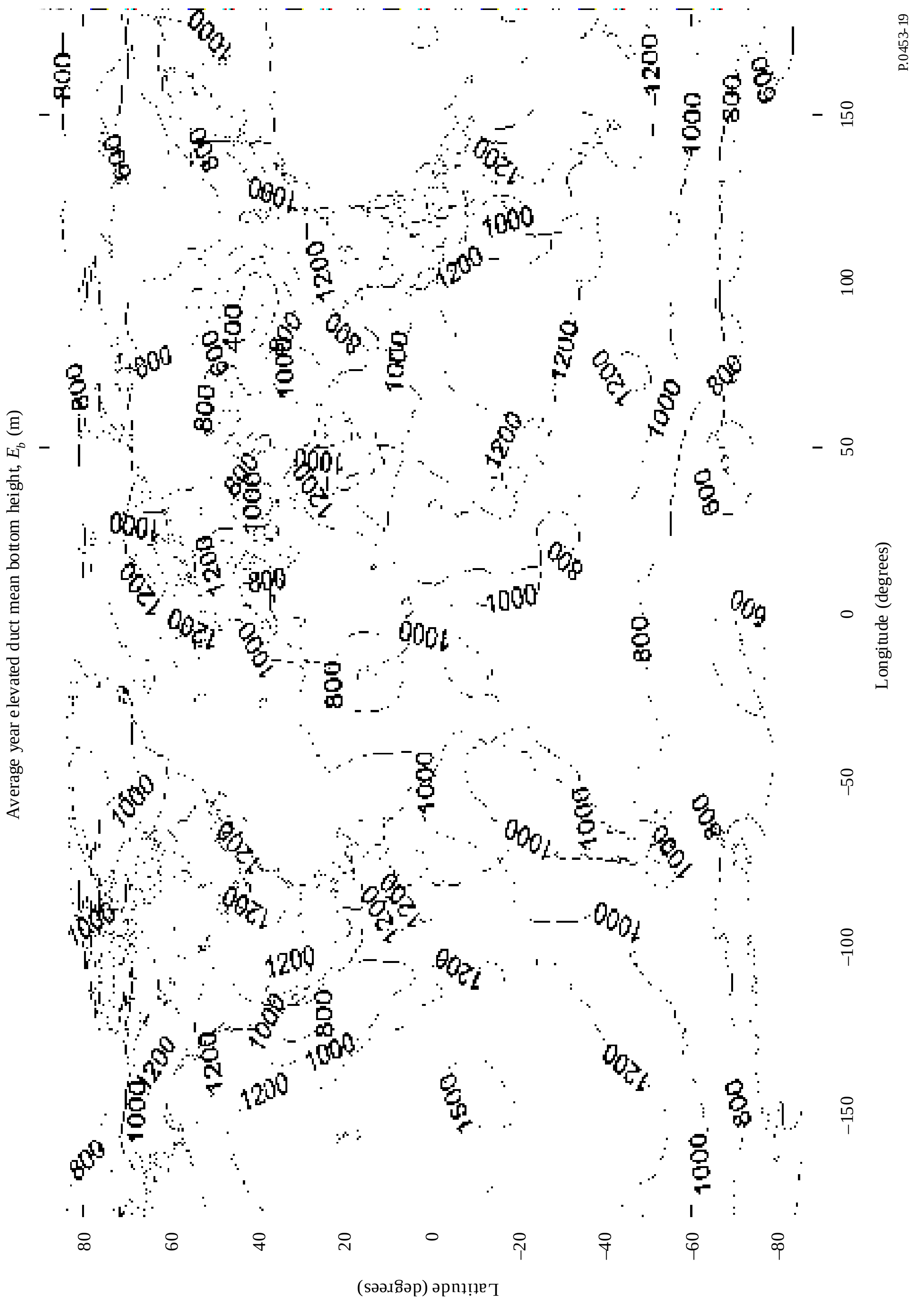


FIGURE 20

Filename: E_MAX_M.TXT