RECOMMENDATION ITU-R P.834-2

EFFECTS OF TROPOSPHERIC REFRACTION ON RADIOWAVE PROPAGATION

(Question ITU-R 201/3)

(1992-1994-1997)

The ITU Radiocommunication Assembly,

considering

a)   that for the proper planning of terrestrial and Earth-space links it is necessary to have appropriate calculation procedures for assessing the refractivity effects on radio signals;

b)   that procedures have been developed that allow the calculation of some refractive propagation effects on radio signals on terrestrial and Earth-space links,

recommends

1   that the information in Annex 1 be used for the calculation of large-scale refractive effects.

ANNEX 1

1 Ray bending

A radio ray passing through the lower (non-ionized) layer of the atmosphere undergoes bending caused by the gradient of the refractive index. Since the refractive index varies mainly with altitude, only the vertical gradient of the refractive index is generally considered. The curvature at a point is therefore contained in the vertical plane and is expressed by:

$$\frac{1}{\rho} = -\frac{\cos \varphi}{n} \frac{dn}{dh}$$

(1)

where:

ρ :   radius of curvature of the ray path

n :   refractive index of the atmosphere

dn/dh :   vertical gradient of refractive index

h :   height of the point above the Earth's surface

φ :   angle of the ray path with the horizontal at the point considered.

This ray curvature is defined as positive for ray bending towards the Earth's surface. This phenomenon is virtually independent of frequency, if the index gradient does not vary significantly over a distance equal to the wavelength.

2 Effective Earth radius on an approximately horizontal path

If the path is approximately horizontal, φ is close to zero. However, since n is very close to 1, equation (1) is simplified as follows:

$$\frac{1}{\rho} = -\frac{dn}{dh}$$

(2)

It is therefore clear that if the vertical gradient is constant, the trajectories are arcs of a circle.

A well-known transformation allows propagation to be considered as rectilinear above a hypothetical Earth of effective radius $R_e = k\,a$, where:

$$\frac{1}{k\,a} = \frac{1}{a} + \frac{\mathrm{d}n}{\mathrm{d}h} = \frac{1}{R_e} \tag{3}$$

where a is the actual Earth radius, and k is the effective earth radius factor. The exponential model of the refractive index (see Recommendation ITU-R P.453), used in the first kilometre of the atmosphere, can be approximated by a linear one corresponding to an effective Earth radius with $k = 4/3$.

3   Modified refractive index

For some applications, for example for ray tracing, a modified refractive index or refractive modulus is used, defined in Recommendation ITU-R P.310. The refractive modulus M is given by:

$$M = N + \frac{h}{a} \tag{4}$$

h being the height of the point considered expressed in metres and a the Earth's radius expressed in thousands of kilometres. This transformation makes it possible to refer to propagation over a flat Earth surmounted by an atmosphere whose refractivity would be equal to the refractive modulus M.

4   Apparent boresight angle on slant paths

4.1   Introduction

In sharing studies it is necessary to estimate the apparent elevation angle of a space station taking account of atmospheric refraction. An appropriate calculation method is given below.

4.2   Visibility of space station

As described in § 1 above, a radio beam emitted from a station on the Earth's surface (h (km) altitude and θ (degrees) elevation angle) would be bent towards the Earth due to the effect of atmospheric refraction. The refraction correction, τ (degrees), can be evaluated by the following integral:

$$\tau = -\int_{h}^{\infty} \frac{n'(x)}{n(x)\cdot\tan\varphi}\,\mathrm{d}x \tag{5}$$

where φ is determined as follows on the basis of Snell's law in polar coordinates:

$$\cos\varphi = \frac{c}{(r + x)\cdot n(x)} \tag{6}$$

$$c = (r + h)\cdot n(h)\cdot\cos\theta \tag{7}$$

   r :   Earth's radius (6 370 km)

   x :   altitude (km)

Since the ray bending is very largely determined by the lower part of the atmosphere, for a typical atmosphere the refractive index at altitude x may be obtained from:

$$n(x) = 1 + a\cdot\exp(-bx) \tag{8}$$

where:

   $a =$ 0.000315

   $b =$ 0.1361

This model is based on the exponential atmosphere for terrestrial propagation given in Recommendation ITU-R P.453. In addition, $n'(x)$ is the derivative of $n(x)$, i.e., $n'(x) = -a\,b\,\exp(-bx)$.

The values of $\tau(h, \theta)$ (degrees) have been evaluated under the condition of the reference atmosphere and it was found that the following numerical formula gives a good approximation:

$$\tau(h, \theta) = 1/[1.314 + 0.6437\,\theta + 0.02869\,\theta^2 + h\,(0.2305 + 0.09428\,\theta + 0.01096\,\theta^2) + 0.008583\,h^2] \quad (9)$$

The above formula has been derived as an approximation for $0 \leq h \leq 3$ km and $\theta_m \leq \theta \leq 10°$, where θ_m is the angle at which the radio beam is just intercepted by the surface of the Earth and is given by:

$$\theta_m = -\arccos\left(\frac{r}{r+h} \cdot \frac{n(0)}{n(h)}\right) \quad (10)$$

or, approximately, $\theta_m = -0.875\,\sqrt{h}$ (degrees).

Equation (9) also gives a reasonable approximation for $10° < \theta \leq 90°$.

Let the elevation angle of a space station be θ_0 (degrees) under free space propagation conditions, and let the minimum elevation angle from a station on the Earth's surface for which the radio beam is not intercepted by the surface of the Earth be θ_m. The refraction correction corresponding to θ_m is $\tau(h, \theta_m)$. Therefore, the space station is visible only when the following inequality holds:

$$\theta_m - \tau(h, \theta_m) \leq \theta_0 \quad (11)$$

4.3 Estimation of the apparent elevation angle

When the inequality in (11) holds, the apparent elevation angle, θ (degrees), can be calculated, taking account of atmospheric refraction, by solving the following equation:

$$\theta - \tau(h, \theta) = \theta_0 \quad (12)$$

and the solution of equation (12) is given as follows:

$$\theta = \theta_0 + \tau_s(h, \theta_0) \quad (13)$$

where the values of $\tau_s(h, \theta_0)$ are identical with those of $\tau(h, \theta)$, but are expressed as a function of θ_0.

The function $\tau_s(h, \theta_0)$ (degrees) can be closely approximated by the following numerical formula:

$$\tau_s(h, \theta_0) = 1/[1.728 + 0.5411\,\theta_0 + 0.03723\,\theta_0^2 + h\,(0.1815 + 0.06272\,\theta_0 + 0.01380\,\theta_0^2) + h^2\,(0.01727 + 0.008288\,\theta_0)] \quad (14)$$

The value of θ calculated by equation (13) is the apparent elevation angle.

4.4 Summary of calculations

Step 1: The elevation angle of a space station in free space propagation conditions is designated as θ_0.

Step 2: By using equations (9) and (10), examine whether (11) holds or not. If the answer is no, the satellite is not visible and, therefore, no further calculations are required.

Step 3: If the answer in Step 2 is yes, calculate θ by using equations (13) and (14).

4.5   Measured results of apparent boresight angle

Table 1 presents the average angular deviation values for propagation through the total atmosphere. It summarizes experimental data obtained by radar techniques, with a radiometer and a radiotelescope. There are fluctuations about the apparent elevation angle due to local variations in the refractive index structure.

TABLE 1

**Angular deviation values for propagation
through the total atmosphere**

Elevation angle, θ (degrees)	Average total angular deviation, $\Delta\theta$ (degrees)			
	Polar continental air	Temperate continental air	Temperate maritime air	Tropical maritime air
1	0.45	–	–	0.65
2	0.32	0.36	0.38	0.47
4	0.21	0.25	0.26	0.27
10	0.10	0.11	0.12	0.14
20		0.05	0.06	
30		0.03	0.04	
Day-to-day variation in $\Delta\theta$ (for columns 1 and 4 only)				
1	0.1 r.m.s.			
10	0.007 r.m.s.			

5   Beam spreading on slant paths

Signal loss may also result from additional spreading of the antenna beam caused by the variation of atmospheric refraction with the elevation angle. This effect should be negligible for all elevation angles above 3°. Figure 1 gives an estimate of the losses through the total atmosphere due to atmospheric refraction effects. Losses should be independent of frequency over the range 1-100 GHz where water vapour is contributing to the refractive profile.

6   Effective radio path length and its variations

Since the tropospheric refractive index is higher than unity varying as a function of altitude, a wave propagating between the ground and a satellite has a radio path length exceeding the geometrical path length. The difference in length can be obtained by the following integral:

$$\Delta L = \int_{A}^{B} (n - 1)\, \mathrm{d}s \tag{15}$$
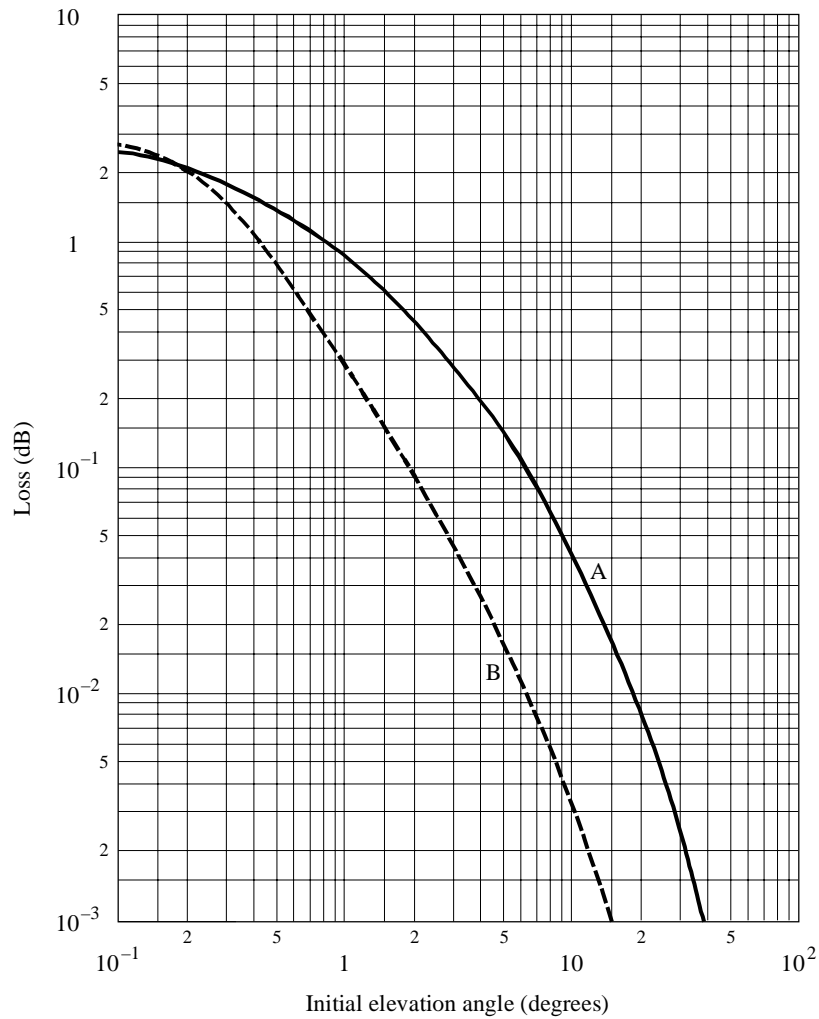
where:

   s: length along the path

   n: refractive index

A and B: path ends.

Equation (15) can be used only if the variation of the refractive index n along the path is known.

FIGURE 1

**An estimate of loss due to the additional spreading of a beam
and standard deviation about the average**

Curves A: average loss
       B: standard deviation

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When the temperature T, the atmospheric pressure P and the relative humidity H are known at the ground level, the excess path length ΔL will be computed using the semi-empirical method explained below, which has been prepared using the atmospheric radio-sounding profiles provided by a one-year campaign at 500 meteorological stations in 1979. In this method, the general expression of the excess path length ΔL is:

$$\Delta L = \frac{\Delta L_V}{\sin \varphi_0 (1 + k \cot^2 \varphi_0)^{1/2}} + \delta (\varphi_0, \Delta L_V) \tag{16}$$

where:

φ_0 :   elevation angle at the observation point

ΔL_V :   vertical excess path length

k and $\delta (\varphi_0, \Delta L_V)$:   corrective terms, in the calculation of which the exponential atmosphere model is used.

The k factor takes into account the variation of the elevation angle along the path. The $\delta (\varphi_0, \Delta L_V)$ term expresses the effects of refraction (the path is not a straight line). This term is always very small, except at very low elevation angle and is neglected in the computation; it involves an error of only 3.5 cm for a φ_0 angle of 10° and of 0.1 mm for a

φ_0 angle of 45°. It can be noted, moreover, that at very low elevation for which the δ term would not be negligible, the assumption of a plane stratified atmosphere, which is the basis of all methods of computation of the excess path length, is no longer valid.

The vertical excess path length (m) is given by:

$$\Delta L_V = 0.00227\, P + f(T)\, H \qquad (17)$$

In the first term of the right-hand side of equation (17), P is the atmospheric pressure (hPa) at the observation point.

In the empirical second term, H is the relative humidity (%); the function of temperature $f(T)$ depends on the geographical location and is given by:

$$f(T) = a\, 10^{bT} \qquad (18)$$

where:

T is in °C

a is in m/% of relative humidity

b is in °C^{-1}.

Parameters a and b are given in Table 2 according to the geographical location.

TABLE 2

Location	a (m/%)	b (°C^{-1})
Coastal areas (islands, or locations less than 10 km away from sea shore)	5.5×10^{-4}	2.91×10^{-2}
Non-coastal equatorial areas	6.5×10^{-4}	2.73×10^{-2}
All other areas	7.3×10^{-4}	2.35×10^{-2}

To compute the corrective factor k of equation (16), an exponential variation with height h of the atmospheric refractivity N is assumed:

$$N(h) = N_s \exp(-h/h_0) \qquad (19)$$

where N_s is the average value of refractivity at the Earth's surface (see Recommendation ITU-R P.453) and h_0 is given by:

$$h_0 = 10^6 \frac{\Delta L_V}{N_s} \qquad (20)$$

k is then computed from the following expression:

$$k = 1 - \left[\frac{n_s\, r_s}{n(h_0)\, r(h_0)}\right]^2 \qquad (21)$$

where n_s and $n(h_0)$ are the values of the refractive index at the Earth's surface and at height h_0 (given by equation (20)) respectively, and r_s and $r(h_0)$ are the corresponding distances to the centre of the Earth.

For Earth-to-satellite paths with elevation angle θ greater than 10°, the tropospheric excess path length (m) can be expressed as the sum of dry and wet components:

$$\Delta L = \Delta L_{dry} + \Delta L_{wet} = 0.00227\, P + \frac{1.79\, V}{T \sin \theta} \qquad \text{m} \qquad (22)$$

where P is the total pressure (hPa), T the temperature (K) at the ground level, θ is the elevation angle and V (kg/m^2 or, equivalently, mm of precipitable water) is the total columnar content of water vapour.

ΔL ranges from 2.2 to 2.7 m, at sea level in the zenith direction. By far the largest contribution, about 2.4 m, is due to the dry component. The wet component, which ranges from 0.05 to 0.6 m, is proportional to the total water vapour content along the atmospheric path and is highly variable.

Statistics of V are given in Recommendation ITU-R P.836. In the same Recommendation, guidance is given on how to retrieve the total atmospheric water vapour along the path of interest from radiometric measurements. In these cases the value V/sin θ is directly obtained. The accuracy achieved in estimating ΔL using radiometrically retrieved values of the total vapour content is around 1 cm.

7   Propagation in ducting layers

Ducts exist whenever the vertical refractivity gradient at a given height and location is less than –157 N/km.

The existence of ducts is important because they can give rise to anomalous radiowave propagation, particularly on terrestrial or very low angle Earth-space links. Ducts provide a mechanism for radiowave signals of sufficiently high frequencies to propagate far beyond their normal line-of-sight range, giving rise to potential interference with other services (see Recommendation ITU-R P.452). They also play an important role in the occurrence of multipath interference (see Recommendation ITU-R P.530) although they are neither necessary nor sufficient for multipath propagation to occur on any particular link.

7.1   Influence of elevation angle

When a transmitting antenna is situated within a horizontally stratified radio duct, rays that are launched at very shallow elevation angles can become "trapped" within the boundaries of the duct. For the simplified case of a "normal" refractivity profile above a surface duct having a fixed refractivity gradient, the critical elevation angle α (rad) for rays to be trapped is given by the expression:

$$\alpha = \sqrt{2 \times 10^{-6} \left|\frac{\mathrm{d}M}{\mathrm{d}h}\right| \Delta h} \tag{23}$$

where $\mathrm{d}M/\mathrm{d}h$ is the vertical gradient of modified refractivity $\left(\frac{\mathrm{d}M}{\mathrm{d}h} < 0\right)$ and Δh is the thickness of the duct which is the height of duct top above transmitter antenna.

Figure 2 gives the maximum angle of elevation for rays to be "trapped" within the duct. The maximum trapping angle increases rapidly for decreasing refractivity gradients below –157 N/km (i.e., increasing lapse rates) and for increasing duct thickness.

7.2   Minimum trapping frequency

The existence of a duct, even if suitably situated, does not necessarily imply that energy will be efficiently coupled into the duct in such a way that long-range propagation will occur. In addition to satisfying the maximum elevation angle condition above, the frequency of the wave must be above a critical value determined by the physical depth of the duct and by the refractivity profile. Below this minimum trapping frequency, ever-increasing amounts of energy will "leak" through the duct boundaries.

The minimum frequency for a wave to be trapped within a tropospheric duct can be estimated using a phase integral approach. Figure 3 shows the minimum trapping frequency for surface ducts (solid curves) where a constant (negative) refractivity gradient is assumed to extend from the surface to a given height, with a "standard" profile above this height. For the frequencies used in terrestrial systems (typically 8-16 GHz), a ducting layer of about 5 m to 15 m minimum thickness is required and in these instances the minimum trapping frequency, f_{min}, is a strong function of both the duct thickness and the refractive index gradient.

FIGURE 2

**Maximum trapping angle for a surface duct of constant
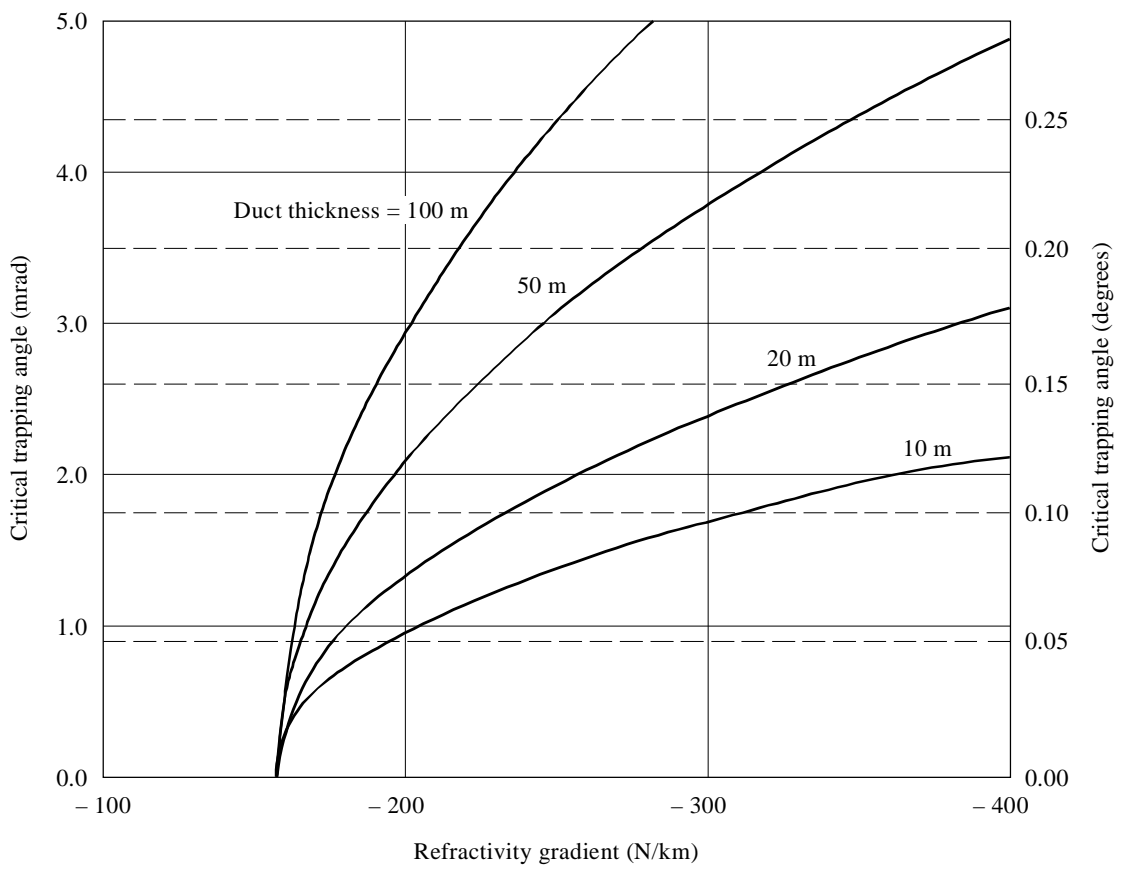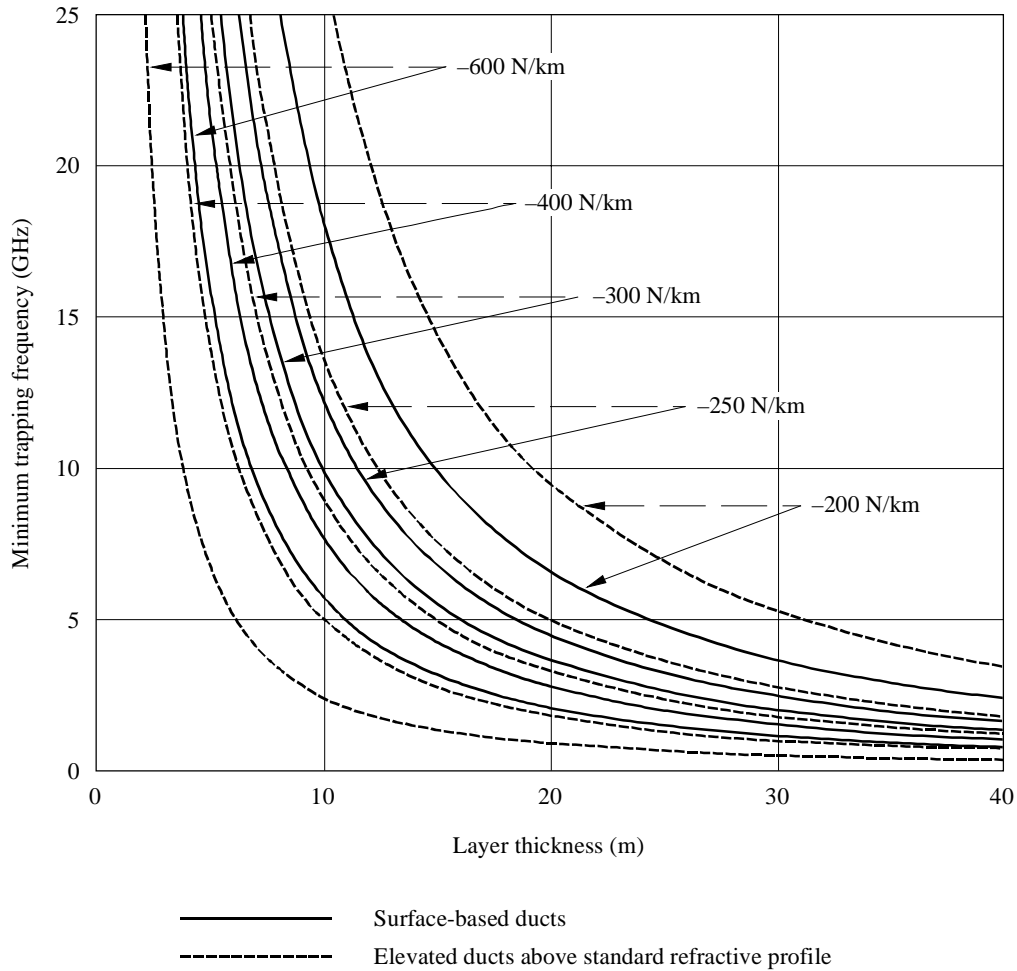refractivity gradient over a spherical Earth**

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FIGURE 3

**Minimum frequency for "trapping" in atmospheric radio ducts
of constant refractivity gradients**

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In the case of elevated ducts an additional parameter is involved even for the simple case of a linear refractivity profile. That parameter relates to the shape of the refractive index profile lying below the ducting gradient. The dashed curves in Fig. 3 show the minimum trapping frequency for a constant gradient ducting layer lying above a surface layer having a standard refractivity gradient of – 40 N/km.

For layers having lapse rates that are only slightly greater than the minimum required for ducting to occur, the minimum trapping frequency is actually increased over the equivalent surface-duct case. For very intense ducting gradients, however, trapping by an elevated duct requires a much thinner layer than a surface duct of equal gradient for any given frequency.

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