RECOMMENDATION ITU-R S.1062-4

Allowable error performance for a satellite hypothetical reference digital path operating below 15 GHz

(Question ITU-R 75-3/4)

(1994-1995-1999-2005-2007)

Scope

The fixed-satellite service (FSS) plays an important role in providing reliable international digital communications. Because of the integration with the terrestrial facilities, a satellite link should be designed to fulfil requirements that are compatible with terrestrial systems. ITU-T Recommendation G.826 specifies performance objectives for a satellite hop in the international portion of a hypothetical reference digital path (HRDP). In response to those objectives, this Recommendation gives guidance on bit error probability (BEP) or bit-error rate (BER) design masks which can fully comply with the requirements of ITU-T Recommendation G.826.

The ITU Radiocommunication Assembly,

considering

a) that satellites operating in the fixed-satellite service (FSS) play an important role in providing reliable international digital communications;

b) that satellite link performance must be sufficient to allow compliance with overall end-to-end performance objectives and end-user quality of service objectives;

c) that satellite link performance is generally distance independent;

d) that Recommendation ITU-R S.614 specifies satellite link performance objectives which comply with the objectives specified in ITU-T Recommendation G.821;

e) that the error performance for hypothetical reference digital paths (HRDPs) and hypothetical reference connections (HRX) have been specified in ITU-T Recommendation G.826;

f) that in defining error performance criteria, it is necessary to take into account all foreseeable error-inducing mechanisms, especially time-varying propagation conditions and interference,

noting

a) that Recommendation ITU-R S.1429 – Error performance objectives due to internetwork interference between GSO and non-GSO FSS systems for hypothetical reference digital paths operating at or above the primary rate carried by systems using frequencies below 15 GHz, specifies the error performance allowance due to interference between different satellite systems and that Recommendation ITU-R S.1323 – Maximum permissible levels of interference in a satellite network (GSO/FSS; non-GSO/FSS; non-GSO/MSS feeder links) in the fixed-satellite service caused by other codirectional FSS networks below 30 GHz, specifies how to calculate the operating margins to allow for both fading and interference, *recommends*

1 that future and, wherever possible, existing satellite links within the FSS should be designed to at least meet the specifications for a satellite hop in the international portion in ITU-T Recommendation G.826. An example set of design masks derived from ITU-T Recommendation G.826 parameters is presented in Note 1;

2 that the methodology explained in Annex 1 can be used to generate the necessary bit-error probability (BEP) (see Note 4) design masks specified in Note 1. The same methodology can be used at a 155 Mbit/s rate to derive the mask in Note 2;

NOTE 1 – In order to fully comply with the requirements of ITU-T Recommendation G.826, the BEP divided by the average number of errors per burst (BEP/α, see § 3 of Annex 1) at the output (i.e. at either end of a two-way connection) of a satellite HRDP forming part of an international portion of a connection or path should not exceed during the total time (including the worst month) the design masks defined by the values given in Table 1 and also in the BEP masks given in Fig. 4.

3 that the following Notes should be regarded as part of the Recommendation:

NOTE 2 – Although Note 1 assures full compliance with ITU-T Recommendation G.826, a more stringent mask may be desirable or necessary for certain services.

TABLE 1

Bit rate (Mbit/s)	Percentage of total time (worst month)	BEP/α
0.064	0.2 10.0	1.0×10^{-4} 1.0×10^{-8}
1.5	0.2 2.0 10.0	7×10^{-7} 3×10^{-8} 5×10^{-9}
2.0	0.2 2.0 10.0	7×10^{-6} 2×10^{-8} 2×10^{-9}
6.0	0.2 2.0 10.0	8×10^{-7} 1×10^{-8} 1×10^{-9}
51.0	0.2 2.0 10.0	4×10^{-7} 2×10^{-9} 2×10^{-10}
155	0.2 2.0 10.0	1×10^{-7} 1×10^{-9} 1×10^{-10}

In this case the BEP at the output (i.e. at either end of a two-way connection) of a satellite HRDP operating up to and including 155 Mbit/s should not exceed during the total time (worst month) the design mask defined by the values given in Table 2:

TABLE 2

Percentage of total time (worst month)	BEP/α	For α = 10 (BEP)
0.2	1×10^{-7}	1×10^{-6}
2	1×10^{-9}	1×10^{-8}
10	1×10^{-10}	1×10^{-9}

NOTE 3 – The HRDP referred to in this Recommendation is specified in Recommendation ITU-R S.521.

NOTE 4 – The BEP ratios given in Notes 1 and 2 could be estimated by BER measurement over a sufficiently long period of time. A method for measuring BERs as a function of percentage of time is given in Annex 1 of Recommendation ITU-R S.614.

NOTE 5 – For ease of application of this Recommendation the values for the objectives given in Notes 1 and 2 are given in terms of total time and represent the limits of a BEP performance model utilizing the method outlined in Annex 1. In arriving at the objectives given in Notes 1 and 2 the errors occurring during the unavailable time have been excluded from the calculation of the objectives. An explanation of the relationship between available time and total time is given in Note 7. The objectives for BEPs given in Note 1 are not unique in meeting the requirements of ITU-T Recommendation G.826. Other BEP masks may be used by the designer where appropriate as long as these masks satisfy ITU-T Recommendation G.826.

NOTE 6 – This Recommendation will find its primary application in satellite systems operating below 15 GHz. The extension of the performance requirements given in this Recommendation to systems operating at higher frequencies is the subject of further study.

NOTE 7 – A period of unavailable time begins at the onset of ten consecutive severely errored seconds (SES) events. These 10 s are considered to be part of unavailable time. A new period of available time begins at the onset of ten consecutive non-SES events. These 10 s are considered to be part of available time. Unavailability threshold values for BEP can be determined such that the unavailable state is reached with a probability = 0.5 as illustrated in Fig. 3.

NOTE 8 – The objectives given in Notes 1 and 2 are given in terms of percentage of the worst month. These monthly percentages correspond to the following yearly percentages:

– 10% of worst month 4.0% of year;
– 2% of worst month 0.6% of year;
– 0.2% of worst month 0.04% of year.

NOTE 9 – In order to comply with Notes 1 and 2 at frequencies greater than 10 GHz, it may be advantageous to make use of fade countermeasures including adaptive forward error-correction (FEC) coding, power control or site diversity. Information on site diversity operation is given in Annex 1, Recommendation ITU-R S.522.

NOTE 10 – The preferred method of verifying digital satellite performance is on the basis of in-service measurements. These measurements would utilize the block error detection schemes, which are related to the inherent block size and structure of the transmission system. FEC, scrambling and differential encoding have an impact on the interpretation of the measurements (see Annex 1, § 3).

NOTE 11 – The error performance described in Notes 1 and 2 was developed based on the use of an HRDP in the international portion of the link (e.g. switched international gateway-to-switched international gateway). Other applications of the HRDP within the connection are possible (e.g. end office-to-end office) and the error performance objectives can be adjusted accordingly.

NOTE 12 – The methods described in this Recommendation can be applied to the design of satellite links in private networks. The performance objectives will usually be agreed between the network operator and the network user via a service level agreement (SLA) as specified in ITU-T Recommendation E.800.

NOTE 13 – The performance objectives shall be met for the required transmission rate not necessarily for any higher rate created to support multiplexing or error correction. For instance, if the transmission rate over a satellite link is 6 Mbit/s and the contracted transmission rate specified in the SLA is 2 Mbit/s, then the objectives for 2 Mbit/s transmission shall apply.

Annex 1

1 General, history, definitions, parameters and objectives relating to ITU-T Recommendation G.826

The requirements of ITU-T Recommendation G.826 are given in terms of errored blocks as opposed to individual bit errors.

The purpose of this specification is to allow the verification of adherence to the performance requirements of ITU-T Recommendation G.826 on an in-service basis. The specification of performance in terms of block errors instead of bit errors has important consequences for systems where the errors tend to occur in groups, such as systems employing scrambling and FEC. The block used in ITU-T Recommendation G.826 is that group of contiguous bits that normally makes up the inherent monitoring block or frame of the transmission system being employed.

ITU-T Recommendation G.826 – End-to-end error performance parameters and objectives for international, constant bit-rate digital paths and connections, covers two types of transport system in detail and may be extended to other types where necessary. The two types are:

– the plesiochronous digital hierarchy (PDH) from 64 kbit/s to the primary rate; and
– the synchronous digital hierarchy (SDH) from the primary rate up to 3 500 Mbit/s.

The addition of the sub-primary speeds was made in the year 2002 to facilitate development at these speeds. However, to maintain stability for the very large installed base of PDH systems it was agreed not to change the long-standing ITU-T Recommendation G.821 which applies to these systems.

In SDH terminology an end-to-end circuit is referred to as a PATH.

In PDH terminology an end-to-end circuit is referred to as a CONNECTION.

Transport system performance is specified in terms of parameters called errored seconds (ESs) and severely errored seconds (SESs) in both PDH and SDH with SDH having an additional parameter called block errors to give a greater resolution for the higher transmission speeds. These blocks have a duration that is much shorter than a second.

An SDH block, whose size depends upon the transmission speed, is a set of consecutive bits that may not be contiguous if the block happens to bridge a container boundary, for example.

1.1 Definitions from ITU-T Recommendation G.826

1.1.1 Error performance events for paths

– *Errored block (EB)*

  A block in which one or more bits are in error.

– *Errored second (ES)*

  A 1 s period with one or more EBs.

– *Severely errored second (SES)*

  A 1 s period which contains ≥30% EBs or at least one defect (see ITU-T Recommendation G.826 for definition of defects).

  Note that SESs are a sub-set of ESs.

– *Background block error (BBE)*

  An EB not occurring as part of an SES.

1.1.2 Error performance events for connections

– *Errored second (ES)*

  A 1 s period in which one or more bits are in error or during which loss of signal or alarm indication signal is detected.

– *Severely errored second (SES)*

  A 1 s period which has a bit-error ratio of ≥1 in 10^{-3}.

1.2 Parameters

Error performance should only be evaluated while the path or connections is in the available state. For a definition of the entry/exit criteria for the unavailable state see Note 7 and Annex A of ITU-T Recommendation G.826.

– *Errored second ratio (ESR)*

  The ratio of ES to total seconds in available time during a fixed measurement interval.

– *Severely errored seconds ratio (SESR)*

  The ratio of SES to total seconds in available time during a fixed measurement interval.

– *Background block error ratio (BBER)*

  The ratio of EBs to total blocks during a fixed measurement interval, excluding all blocks during SES and unavailable time.

1.3 Monitoring blocks

Table 3 shows the block size and number of blocks/s for various transmission rates.

TABLE 3

Relationship between bit rate, block size and number of blocks/s

Bit rate (Mbit/s)	Block size (bits)	Number of blocks/s
1.544	4 632	333
2.048	2 048	1 000
6.312	3 156	2 000
44.736	4 760	9 398
51.84	6 480	8 000
155.52	19 440	8 000

1.4 Performance objectives

The end-to-end objectives defined in ITU-T Recommendation G.826 are reproduced for convenience in Table 4. The performance objectives are given as a function of transmission system bit rate. The ranges of block sizes accommodated at these bit rates are also given. As stated above, the block size will be that associated with the frame structure of the transmission system. These objectives are specified for available time.

TABLE 4

End-to-end performance objectives for a 27 500 km international digital HRDP or HRX from ITU-T Recommendation G.826

Rate (Mbit/s)	64 kbit/s to primary rate(1)	1.5 to 5	>5 to 15	>15 to 55	>55 to 160	>160 to 3 500
Bits/block	Not applicable	800-5 000	2 000-8 000	4 000-20 000	6 000-20 000	15 000-30 000(2)
ESR	0.04	0.04	0.05	0.075	0.16	(3)
SESR	0.002	0.002	0.002	0.002	0.002	0.002
BBER	Not applicable	2×10^{-4}(4)	2×10^{-4}	2×10^{-4}	2×10^{-4}	10^{-4}

(1) It is not required to apply these objectives to equipment that was designed prior to 2003. Performance objectives for such equipment are given in ITU-T Recommendation G.821.

(2) As currently defined, VC-4-4c (ITU-T Recommendation G.707) is a 601 Mbit/s path with a block size of 75 168 bits/block. Since this block size is outside the recommended range for 160-3 500 Mbit/s paths, performance on such VC-4-4c paths is outside this Table. The BBER objective for VC-4-4c using the 75 168 bit block size is 4×10^{-4}.

(3) ESR objectives tend to lose their significance at high bit rates and are therefore not specified for paths operating above 160 Mbit/s. However, for maintenance purposes, ES monitoring should be implemented.

(4) For systems designed prior to 1996, the BBER objective 3×10^{-4}.

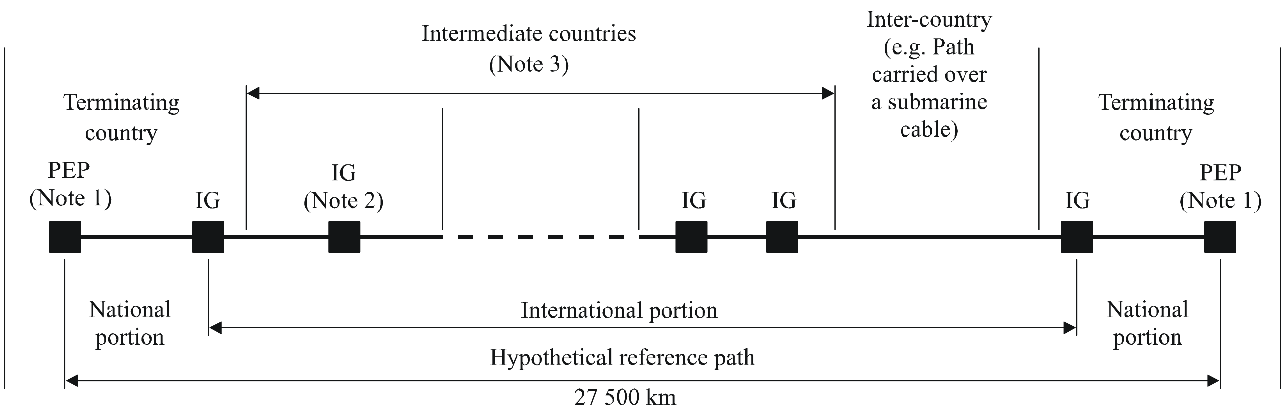
Digital paths and connections operating at bit rates covered by this Recommendation may be carried by transmission systems operating at higher bit rates. Such systems must be designed and implemented to objectives that will support the end-to-end objectives of their tributaries, current and anticipated. Under the assumption of random error distribution, meeting the allocated objectives in Table 1/G.826 for the higher bit rate systems should ensure that all tributaries will also be achieving their objectives.

1.5 Apportionment of the end-to-end objectives to portions of the path

The end-to-end performance objectives are apportioned between international and national portions of an HRDP using the allocation principles detailed in § 6.2 of ITU-T Recommendation G.828 (see Fig. 1).

FIGURE 1

HRDP



IG: International gateway
PEP: Path end point
Note 1 – If a path terminates at the IG, only the international portion allocation applies.
Note 2 – One or two IGs (entry or exit) may be defined per intermediate country.
Note 3 – Four "intermediate countries" are assumed for the terrestrial case and one satellite hop has been assumed In this Recommendation.

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1.6 Allocations for satellites

In communication transport systems operating at any bit rate covered by ITU-T Recommendation G.826, either above or below the primary rate, independent of the actual distance spanned, a satellite hop in the international portion receives a 35% allocation of all the end-to-end objectives.

If a satellite link provides a national portion then it receives an allocation of 42% of all the end-to-end objectives.

This is in contrast with the allocations in ITU-T Recommendation G.821 where the allocations are different for ESs and SESs. Satellites only receive a 20% allocation for ESs in the international portion but the ES end-to-end allowance is higher at 0.04 so the performance required by the satellite link is very similar. For SESs the satellite allocation is only 15% of 0.002 = 0.0003.

The performance objectives for satellites providing portions of a 27 500 km HRDP or HRX are given in Tables 5 and 6.

TABLE 5

Satellite performance objectives for an international portion

Rate (Mbit/s)	0.064 to 1.5	1.5 to 5	>5 to 15	>15 to 55	>55 to 160	>160 to 3 500
ESR	0.014	0.014	0.0175	0.0262	0.056	Not applicable
SESR	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
BBER	Not applicable	0.7×10^{-4}	0.7×10^{-4}	0.7×10^{-4}	0.7×10^{-4}	0.35×10^{-4}

TABLE 6

Satellite performance objectives for a national portion

Rate (Mbit/s)	0.064 to 1.5	1.5 to 5	>5 to 15	>15 to 55	>55 to 160	>160 to 3 500
ESR	0.0168	0.0168	0.021	0.0315	0.0672	Not applicable
SESR	0.00084	0.00084	0.00084	0.00084	0.00084	0.00084
BBER	Not applicable	0.84×10^{-4}	0.84×10^{-4}	0.84×10^{-4}	0.84×10^{-4}	0.42×10^{-4}

If a satellite provides the complete path or connection from end-to-end then the objectives in Table 4 would apply.

2 Bit error probability (BEP) mask derivation

The set of parameters and objectives defined in ITU-T Recommendation G.826 is not suitable for satellite system design. It must be transformed into a BEP versus percentage-of-time distribution, also called a BEP mask, in such a way that any satellite system designed to meet the mask would also meet the objectives of this Recommendation. The transform, however, does not result in a unique mask.

2.1 Probability of the basic events

It is well known that transmission errors over satellite links occur in bursts where the average number of errors per burst is, among other factors, a function of the scrambler and the FEC code. Consequently, a successful model of the digital performance over satellite links has to take into account this bursty nature. One statistical model that can adequately represent the random occurrence of bursts is the Neyman-A contagious distribution, where the probability of k errors occurring in N bits, $P(k)$, is:

$$P(k) = \frac{\alpha^k}{k!} e^{-\frac{BEP \cdot N}{\alpha}} \sum_{j=0}^{\infty} \frac{j^k}{j!} \left(\frac{BEP \cdot N}{\alpha} \right)^j e^{-j\alpha} \tag{1}$$

where:

α: average number of errored bits in a burst of errors

BEP: bit error probability.

If $N = N_B$ is taken as the number of bits in a block of data, then the probability of zero errors in a block is:

$$P(0) = \mathrm{e}^{-\frac{BEP \cdot N_B}{\alpha}} \sum_{j=0}^{\infty} \left(\left(\frac{BEP \cdot N_B}{\alpha} \right)^j \Big/ j! \right) \mathrm{e}^{-j\alpha} \cong \mathrm{e}^{-\frac{BEP \cdot N_B}{\alpha}} \text{ for all practical values of } \alpha. \qquad (2)$$

The probability of an EB, P_{EB}, is then given by:

$$P_{EB} = 1 - P(0) = 1 - \mathrm{e}^{-\frac{BEP \cdot N_B}{\alpha}} = 1 - \mathrm{e}^{-N_B \cdot BEP_{CRC}} \qquad (3)$$

where $BEP_{CRC} = BEP/\alpha$. The probability of an ES, P_{ES}, can then be expressed as:

$$P_{ES} = 1 - \mathrm{e}^{-n \cdot P_{EB}} \qquad (4)$$

where n is the number of blocks/s.

Since the probability of k errored blocks in a total of n blocks, $P_{n,k}$, is given by:

$$P_{n,k} = \frac{n!}{(n-k)!\, k!} \, (1 - P_{EB})^{n-k} \, P_{EB}^{k} \qquad (5)$$

then, the probability of an SES, P_{SES}, is:

$$P_{SES} = \sum_{k=0.3n}^{n} P_{n,k} = 1 - \sum_{k=0}^{0.3n-1} P_{n,k} = 1 - \sum_{k=0}^{0.3n-1} \frac{n!}{(n-k)!k!} (1 - P_{EB})^{n-k} \, P_{EB}^{k} \qquad (6)$$

2.2 Calculation of the ITU-T Recommendation G.826 parameters for a given mask of BEP cumulative distribution

Departing from the original definition for the ITU-T Recommendation G.826 parameters, we can write the following expressions for ESR, SESR and BBER:

$$ESR = \frac{N_{ES}}{N} \qquad (7)$$

$$SESR = \frac{N_{SES}}{N} \qquad (8)$$

$$BBER = \frac{N_{EB}}{N_B} \qquad (9)$$

where:

N_{ES}: number of errored seconds in the available time

N_{SES}: number of severely errored seconds in the available time

N_{EB}: number of errored blocks in the available time, excluding the severely errored seconds

N_B: number of blocks in the available time, excluding the severely errored seconds

N: total number of seconds in the available time.

The usual relative-frequency approximation for probabilities can be applied to the previous expressions to yield:

$$ESR \cong P_{ES} \tag{10}$$

$$SESR \cong P_{SES} \tag{11}$$

$$BBER \cong P_{EB} \tag{12}$$

The above probabilities should be interpreted as average probabilities in the respective observation interval. In practice, this average must be performed in time. Therefore, if we assume that a random BEP is observed in each second, we can define time-dependent probabilities for the basic events and then calculate their means through the following expressions:

$$ESR = \frac{\int_{T_a} P_{ES}(t)\mathrm{d}t}{T_a} \tag{13}$$

$$SESR = \frac{\int_{T_a} P_{SES}(t)\mathrm{d}t}{T_a} \tag{14}$$

For BBER, in order to consider the exclusion of the SESs, we have:

$$BBER = \frac{\int_{T_a} P_{EB}(t)\frac{1 - P_{SES}(t)}{1 - SESR}\mathrm{d}t}{T_a} \tag{15}$$

where T_a is the available time.

The time averages can be calculated through equivalent expressions in terms of the cumulative distribution function for BEP/α, defined as $F(x)$. The method is illustrated below to calculate ESR:

$$\frac{1}{T_a}\int_{T_a} P_{ES}(t)\mathrm{d}t = \int_0^{BEP_{th}/\alpha} P_{ES}(x)\mathrm{d}F(x) \tag{16}$$

where BEP_{th}/α is the threshold value above which the system is considered to be unavailable. Analogue derivations apply to the other parameters.

For a numerical calculation, a discrete approximation can be used as follows:

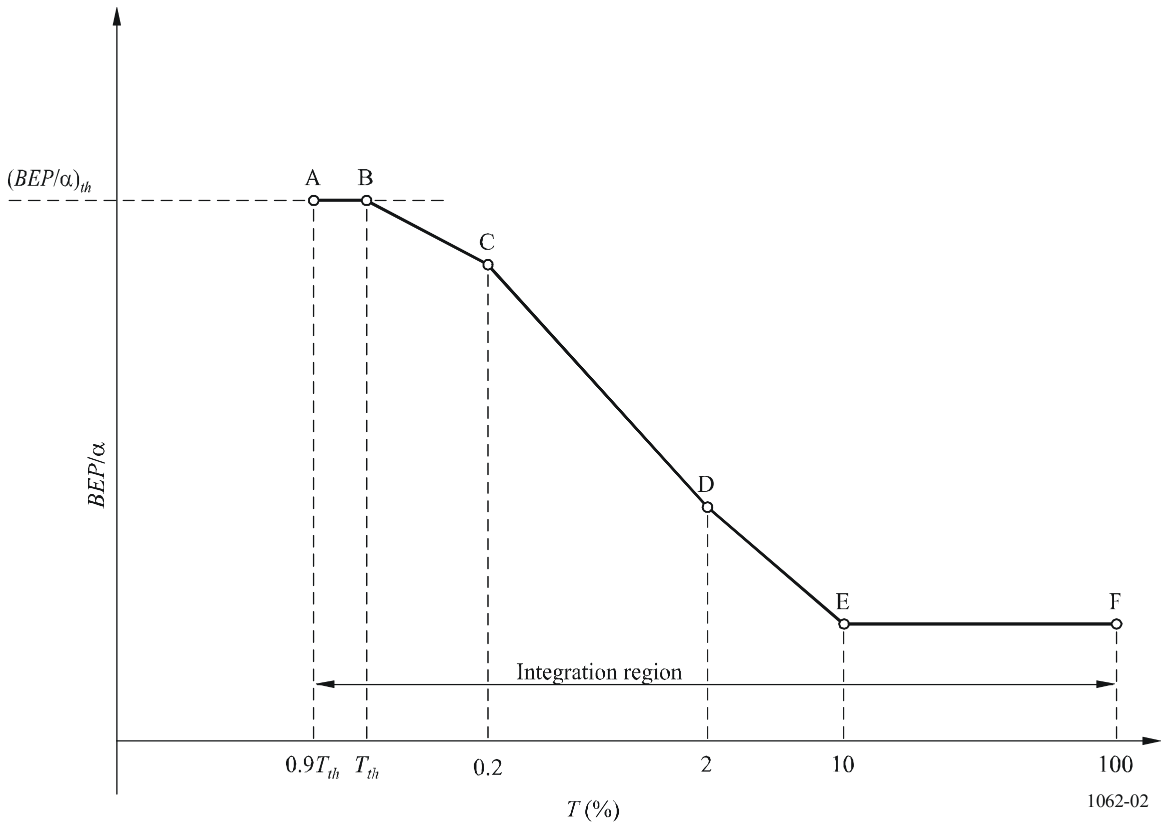
$$\frac{1}{T_a}\int_{T_a} P_{ES}(t)\mathrm{d}t \cong \sum_i P_{ES}(x_i)(F(x_{i+1}) - F(x_i)) \tag{17}$$

where the summation is performed for values x_i of BEP/α below BEP_{th}/α.

An infinite number of BEP/α cumulative distributions $F(x)$ can be found to meet the ITU-T Recommendation G.826 performance objectives. Therefore, a mask for $F(x)$ is assumed to have the form of Fig. 2. Note that $F(x)$ can be expressed as the percentage of time for which BEP/α does not exceed x and therefore $F(x)$ should be read as the complement of the values in the horizontal axis of Fig. 2.

FIGURE 2

General form of mask

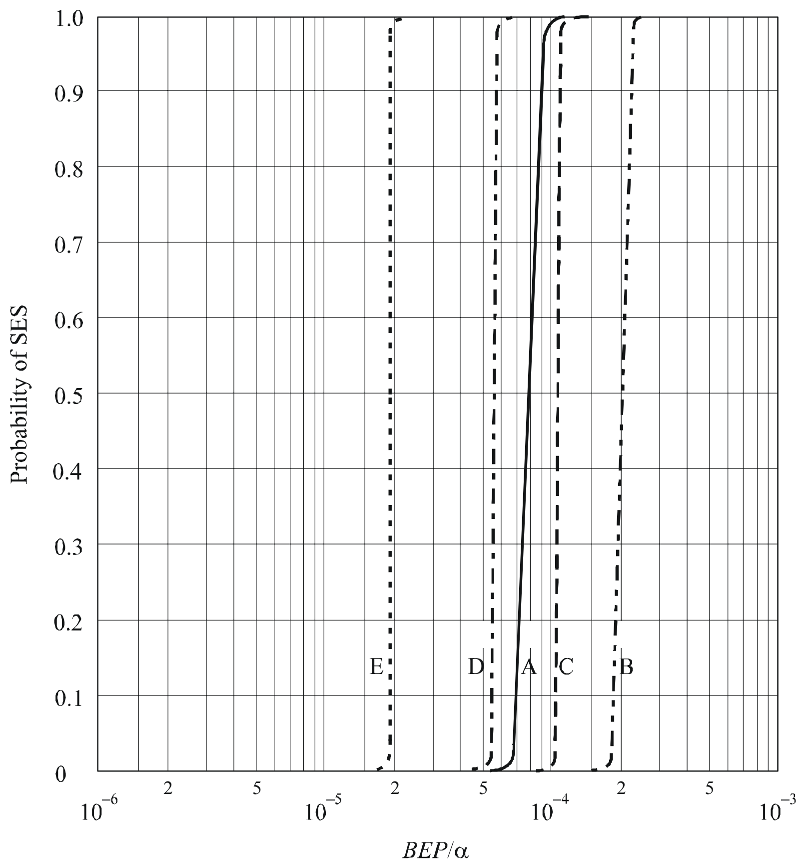


The unavailability threshold, T_{th}, is defined by $P_{SES} = 0.933$. This value corresponds to a probability of ten consecutive SESs of 0.5.

The corresponding values of BEP_{th}/α, at various data rates, are included in Fig. 3 and are also listed in Table 7.

FIGURE 3

P_{SES} versus BEP/α



A: 1.5 Mbit/s
B: 2 Mbit/s
C: 6 Mbit/s
D: 51 Mbit/s
E: 155 Mbit/s

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TABLE 7

Bit rate (Mbit/s)	**BEP_{th}/α**
0.064	3×10^{-3}
1.544	9.00×10^{-5}
2.048	1.90×10^{-4}
6.432	1.17×10^{-4}
51.84	5.68×10^{-5}
155.52	1.89×10^{-5}

In selecting the value of BEP_{th}/α for the generation of the masks, however, attention should be paid to the fact that modems experience loss of synchronization at a certain BEP threshold, denoted here by BEP_{mod}. Based on the above considerations, the value of BEP_{th}/α to be used is given by the formula:

$$BEP_{th}/\alpha = \min\,(BEP_{th}/\alpha \text{ of Table 7; } BEP_{mod}/\alpha)$$

For most modems in operation today, BEP_{mod} is well approximated by the value 1×10^{-3}.

The above method will result in an infinite number of masks meeting the ITU-T Recommendation G.826 performance objectives. Therefore, the following process is used to define a mask and to determine points C, D, E and F of the mask (see Fig. 2).

Step 1 – Set the mask values at 100%, 10%, 2% and 0.2% of the time (points C, D, E and F).

Step 2 – Determine the value BEP_{th}/α.

Step 3 – Choose an unavailability threshold time value, T_{th} ($T_{th} < 0.2\%$).

Step 4 – Assume a straight line between points B and C.

Step 5 – Calculate ESR, SESR and BBER by integrating over the region between 0.9 T_{th} and 100% (see Note 1).

NOTE 1 – Based on results given in Recommendation ITU-R S.579, showing propagation attenuation events which do not result in unavailable time, a "propagation availability factor" of 10% was used for deriving these masks. Therefore, 10% of T_{th} was incorporated into the available time to account for the cases where BEP is worse than BEP_{th} but recovers in less than 10 s.

Step 6 – Select a new value of T_{th} and repeat Steps 4 and 5 until the maximum values for ESR, SESR, and BBER are found for any $T_{th} < 0.2\%$ of the time.

If the objectives for ESR, SESR and BBER in Tables 5 or 6 are satisfied for all $T_{th} < 0.2\%$, then the mask defined by points C, D, E, and F is considered to meet the requirements of this Recommendation. Moreover, the above process ensures that a link unavailability of less than 0.2% of the total time is achieved.

As a consequence of the iterative process in Steps 4, 5 and 6, any straight line between points B and C, where B can be anywhere between 0% and 0.2% of the time, will meet the defined objectives of this Recommendation and the unavailability objectives given in Recommendation ITU-R S.579. Therefore, the general shape of the mask can be further simplified by extending the mask vertically from point C as shown in Fig. 4.

Using the above process with the additional assumptions that:

– BEP/α corresponding to points E and F are the same,

– BEP/α corresponding to points E and D differ by one decade,

an example set of masks for various transmission bit rates was generated and is shown in Fig. 5.

In developing these masks it was assumed that $BEP_{mod} = 1 \times 10^{-3}$. In Fig. 5, the second assumption was modified to achieve a smooth mask. For example, for 1.5 Mbit/s mask the ratio between BEP/α values corresponding to points E and D was changed from 10 to 3.

FIGURE 4

Simplified mask

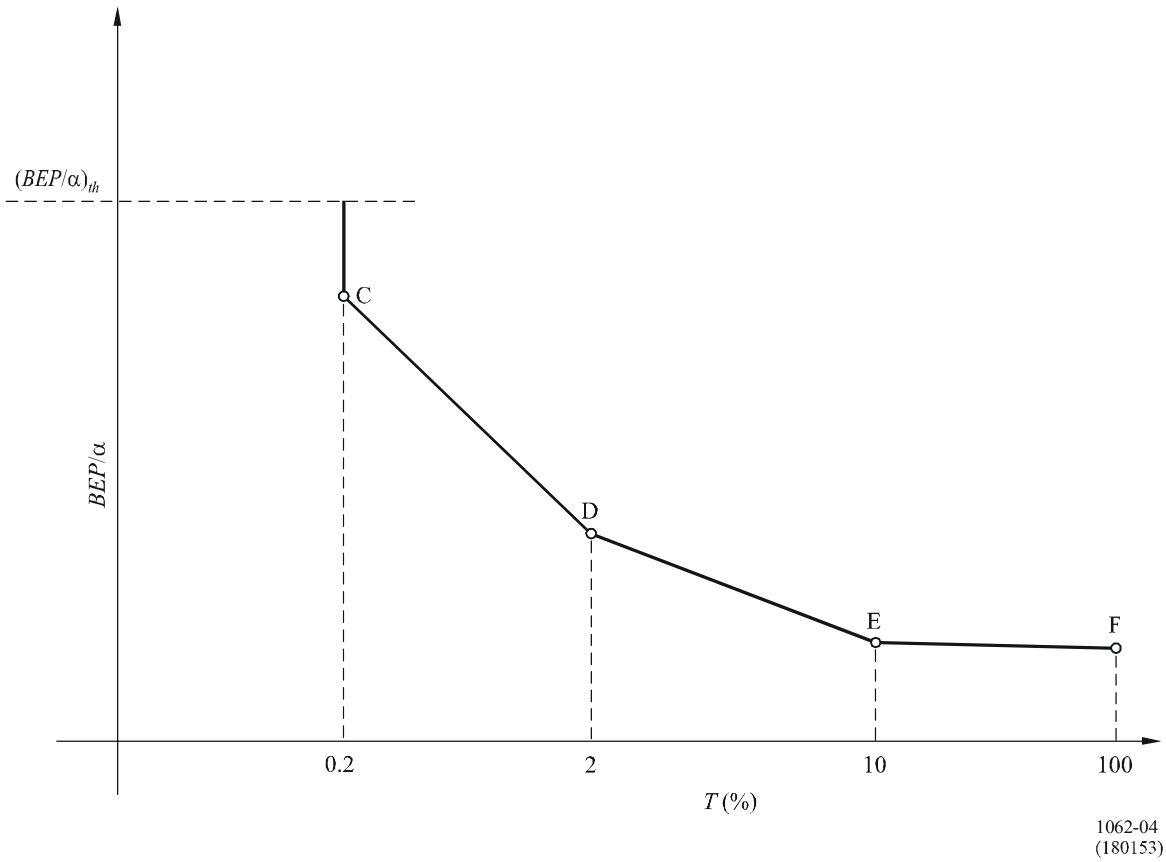
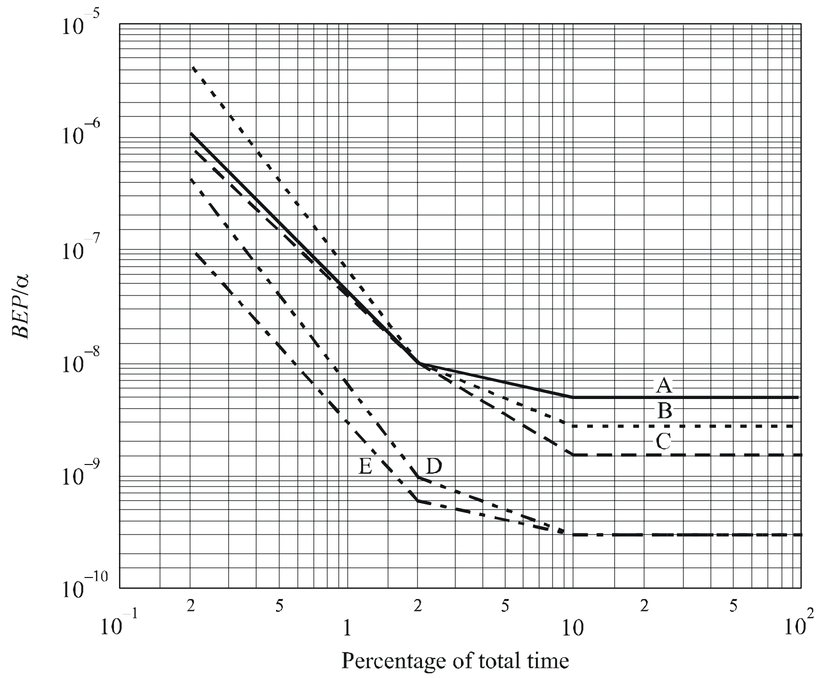


FIGURE 5

Generated masks for satellite hops



A: 1.5 Mbit/s
B: 2 Mbit/s
C: 6 Mbit/s
D: 51 Mbit/s
E: 155 Mbit/s

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3 Relationship between BER and error-event ratio

It is well known that errors on satellite links employing FEC and scrambler schemes tend to occur in clusters. The appearance of the clusters, which can also be called error events, is random following a Poisson distribution. The resulting block error rate is the same as if it were caused by randomly (Poisson distributed) occurring bit errors with a bit-error ratio *BER*/α, where α (used in § 2.1 to account for the burstiness of errors) is the average number of errored bits within a cluster, α also represents the ratio between the BER and error-event ratio. For example, in a random binary error channel without FEC and scrambler α is considered to be one. With higher order modulation schemes, however, α may be larger than one.

In a given FEC scheme, theoretical values of α can be estimated using the weight distribution of the FEC scheme. Background of the theoretical value derivation is given in § 3.1. Statistical properties of the clusters of errors are dependent on the FEC/scrambler scheme used. Computer simulations and measurements of various FEC schemes (without scrambler or differential encoding) were used to determine the factor α. An additive white Gaussian channel is assumed in the simulation. These results are given in § 3.2 to 3.6.

3.1 Derivation of the average number of errored bits in a cluster

Given an (n,k) systematic block code C, its well-known weight enumerating function (WEF) is:
\

$$B^C(H) \triangleq \sum_{i=0}^{n} B_i H^i \tag{18}$$

where:

B_i: (integer) number of codewords with Hamming weight (number of ones) i

H: dummy variable.

The WEF of a code can be used to compute the exact expression of the probability of undetected errors and an upper bound to the word error probability.

The input-redundancy weight enumerating function (IRWEF) of the code can be defined as:

$$A^C(W,Z) \triangleq \sum_{w,j} A_{w,j} W^w Z^j \tag{19}$$

where $A_{w,j}$ denotes the (integer) number of codewords generated by an input information word of Hamming weight w whose parity check bits have Hamming weight j, so that the overall Hamming weight is $w + j$. The IRWEF shows the separate contributions of the information and of the parity check bits to the total Hamming weight of the codewords, and thus provides additional information on the (Hamming) weight profile of the code.

By using the above expression, the BEP, P_b, can be upper-bounded by:

$$P_b \le \sum_{m=d_{min}}^{\infty} D_m P(R_m'' | C_0) \tag{20}$$

where d_{min} is the minimum distance of the code, $P(R_m'' | C_0)$ is the probability of the decoder selecting the codeword of weight m provided the transmitted codeword is all-zero codeword, and:

$$D_m = \sum_{j+w=m} \frac{w}{k} A_{w,j} \tag{21}$$

Therefore, the average number of bits in a cluster α will be the mean value of w, and leading to:

$$\overline{w} = \sum_{m=d_{min}}^{\infty} \sum_{m=w+j} w A_{w,j} P_m \tag{22}$$

where P_m is the probability of error events with m errors in all error events. Because P_m decreases rapidly with m, especially in low BEP values, $\overline{w}$ can be approximated by:

$$\overline{w} \approx \sum_{d_{min}=w+j} w \; A_{w,j} P_{d_{min}} \tag{23}$$

3.2 Factors for binary BCH codes

Using equation (23), α values for systematic BCH codes can be estimated. Table 8 shows weight distribution of the (7,4) BCH code, and the minimum distance of the (7,4) code is 3. Therefore, α for the code can be estimated as follows:

$$\overline{w}_{(7,4)} = \alpha_{(7,4)} \approx 1 \times \frac{3}{7} + 2 \times \frac{3}{7} + 3 \times \frac{1}{7} \cong 1.7 \tag{24}$$

TABLE 8

Weight distribution of the (7,4) BCH code

w	j	$A_{w,j}$
0	0	1
1	2	3
1	3	1
2	1	3
2	2	3
3	0	1
3	1	3
4	3	1

Table 9 shows the estimated α for various systematic BCH codes, and Table 10 compares the simulation results for the (15,11) BCH code to the estimated results. As BER gets lower, the estimated value approximates to the simulation value.

For non-systematic codes, when decoding fails, approximately half of the information word will be in error. In this case, α can be approximated to $k/2$.

TABLE 9

Theoretical α values estimated for various BCH codes

(*n*,*k*) BCH code	**α**	**(*n*,*k*) extended code**	**α**	**(*n*,*k*) expurgated code**	**α**
(15,11)	2.20	(16,11)	2.75	(15,10)	2.67
(31,26)	2.52	(32,26)	3.25	(31,25)	3.23
(31,21)	3.73	(32,21)	4.56	(31,20)	4.53
(63,57)	2.06	(64,57)	2.96	(63,56)	2.96
(63,51)	4.07	(64,51)	4.50		

TABLE 10

Comparison of theoretical and simulated α values for the (15,11) BCH code

BER	**Simulated α**	**Theoretical α**
2.88×10^{-2}	2.60	2.2
4.69×10^{-3}	2.37	
5.57×10^{-4}	2.36	
2.36×10^{-5}	2.33	

3.3 Factors for convolutional codes

A similar approach can be applied to convolutional codes. For known convolutional codes, various studies identified their weight distributions in terms of a_d, the number of codewords of distance d, and c_d, the sum of bit errors (the information error weight) for codewords of distance d. With the same approximation to the binary BCH codes, $\overline{w}\,(=\alpha)$ for the convolutional codes can be approximated to $(c_{d_f})/(a_{d_f})$, where d_f is the free distance of the code.

Table 11 shows weight distributions of popular convolutional codes, and Table 12 compares the theoretically estimated α values and simulated values. As was confirmed in the binary BCH codes, the estimated α values are nearly equal to the simulated values in the low BER ranges.

TABLE 11

Weight distribution of convolutional codes

Code rate R	Constraint length K	Generator (in octal)	d_f	(a_d, $d = d_f$, $d = d_f + 1$, $d = d_f + 2$, …) (c_d, $d = d_f$, $d = d_f + 1$, $d = d_f + 2$, …)
1/2	7	133, 171	10	(11, 0, 38, 0, 193, 0, 1 331, 0, 7 275,···) (36, 0, 211, 0, 1 404, 0, 11 633, ···)
	9	561, 753	12	(11, 0, 50, 0, 286, 0, 1 630, 0, 9 639, ···) (33, 0, 281, 0, 2 179, 0, 15 035, ···)
2/3*	7	133, 171	6	(1, 16, 48, 158, 642, 2 435, 9 174) (3, 70, 285, 1 276, 6 160, 27 128, ···)
7/8*	7	133, 171	3	(2, 42, 468, 4 939, 52 821) (14, 389, 6 792, 97 243, 1 317 944)

* Punctured codes from R 1/2 code with $K = 7$.

3.4 Factors for concatenated codes

For a concatenated code with a Reed-Solomon (RS) outer code and a convolutional inner code, the α value is directly related to the weight distribution of the RS code because the RS code is outer code. The α value for the RS codes can be found using the same rule as used in the binary BCH code, if maximum likelihood decoding is used. In this case, the binary weight distribution of the RS codes should be found.

Table 13 shows simulated α values for the RS codes in the concatenated coding scheme specified in Recommendations ITU-R BO.1724 and ITU-R S.1709. The RS (204,188) code shortened from the original RS (255,239) code is used. The RS (71,55) shortened code is also used for a different packet size.

TABLE 12

Comparison of theoretical and simulated α values for convolutional codes

Code rate *R*	Constraint length *K*	Generator (in octal)	d_f	α (estimated)	BER	α (simulated)
1/2	7	133, 171	10	3.27	1.74×10^{-2}	7.21
					1.91×10^{-3}	5.68
					1.05×10^{-4}	3.74
					5.05×10^{-6}	3.48
					1.07×10^{-7}	3.00
	9	561, 753	12	3.00	1.22×10^{-2}	13.00
					1.77×10^{-3}	11.56
					2.10×10^{-5}	4.38
					4.20×10^{-7}	3.96
2/3	7	133, 171	6	3.00	3.61×10^{-2}	8.00
					7.86×10^{-4}	7.14
					2.96×10^{-6}	5.32
					2.14×10^{-7}	5.67
7/8	7	133, 171	3	7.00	6.24×10^{-2}	9.08
					2.68×10^{-2}	8.85
					9.82×10^{-3}	7.77
					1.77×10^{-5}	7.57
					1.49×10^{-6}	7.29

TABLE 13

Simulated α values for RS codes in the concatenated coding scheme

(N,K) RS code	BER	α	(N,K) RS code	BER	α
(204,188)	7.74×10^{-3}	12.80	(71,55)	6.17×10^{-3}	8.47
	5.19×10^{-4}	9.14		2.03×10^{-4}	7.74
	1.02×10^{-6}	8.58		2.02×10^{-7}	7.32

3.5 Factors for turbo codes

For turbo codes, a similar approach to convolutional codes can be used because they are based on convolutional codes. Table 14 shows weight distributions of turbo codes specified in Recommendations ITU-R BO.1724 and ITU-R S.1709, and Table 15 shows corresponding estimated α values. Table 16 shows the simulated α values for the packet size of 53 bytes. Because the turbo codes use an iterative decoding algorithm, α values and BER depend on the decoding algorithm and the number of iterations. In the simulation, a max-log MAP decoding algorithm was used and α values were estimated at iterations of 6 and 15. Because the theoretical values estimated in Table 15 can be considered as a lower bound, they are smaller than the simulated values in Table 16.

TABLE 14

Weight distribution of turbo codes ($d_f/a_d/c_d$)

Packet size (bytes)	*R* = 1/3	*R* = 1/2	*R* = 2/3	*R* = 3/4	*R* = 6/7
53	31/106/954	18/159/954	11/159/901	7/10/50	4/9/27
	32/265/1643	19/159/1431	12/265/1325	8/85/375	5/194/719
	33/106/901	20/530/3551	13/1802/11342	9/486/2335	6/1228/5371
188	33/3476/3384	19/376/3384	12/188/1316	9/27/171	6/199/826
	35/376/3760	20/376/3008	14/752/5264	10/148/1025	7/1578/7269
	36/752/6392	22/752/6768	15/1504/12220	11/1462/9674	8/9144/49558

TABLE 15

Theoretically approximated α values for turbo codes

Packet size (bytes)	*R* = 1/3	*R* = 1/2	*R* = 2/3	*R* = 3/4	*R* = 6/7
53	9.00	6.00	5.67	5.00	3.00
	6.20	9.00	5.00	4.41	3.70
	8.50	6.70	6.29	4.80	4.37
752	9.00	9.00	7.00	6.33	4.15
	10.00	8.00	7.00	6.93	4.60
	8.50	9.00	8.13	6.62	5.42

TABLE 16

Simulated α values for turbo codes

Iteration number	*R* = 1/3 BER/α	*R* = 2/5 BER/α	*R* = 1/2 BER/α	*R* = 3/4 BER/α	*R* = 6/7 BER/α
6	5.58×10^{-5}/16.8	3.79×10^{-5}/16.6	1.39×10^{-4}/21.5	9.53×10^{-4}/15.9	3.44×10^{-5}/6.8
	9.28×10^{-6}/14.0	5.56×10^{-6}/12.8	2.24×10^{-5}/17.1	3.47×10^{-5}/11.3	2.34×10^{-6}/5.2
	1.42×10^{-6}/10.6	9.68×10^{-7}/10.6	5.69×10^{-7}/9.0	9.89×10^{-7}/7.8	2.53×10^{-7}/4.1
15	2.25×10^{-5}/23.7	1.57×10^{-5}/20.8	6.36×10^{-5}/26.6	6.46×10^{-4}/18.3	2.67×10^{-5}/7.0
	3.28×10^{-6}/16.5	2.41×10^{-6}/14.5	9.30×10^{-6}/18.9	1.89×10^{-5}/12.2	1.74×10^{-6}/4.8
	5.62×10^{-7}/11.6	4.25×10^{-7}/10.8	3.02×10^{-7}/8.9	6.02×10^{-7}/7.9	1.78×10^{-7}/4.3

3.6 Factors for block turbo codes

Block turbo codes (BTCs) are product codes that are decoded iteratively. The minimum distance of a product code is the product of the minimum distances of its constituent codes. For example, the minimum distance of m-dimensional product code with the same constituent code with minimum distance of d_{min} will be $(d_{min})^m$. Using the same principle, the α value for a BTC α_{BTC} can be represented as follows:

$$\alpha_{BTC} = \alpha_{c_1} \cdot \alpha_{c_2} \cdots \alpha_{c_m} \tag{25}$$

where α_{c_i} is the α value for the i-th constituent code. The binary systematic codes demonstrated in § 3.2 are usually used as constituent codes.

Table 17 shows theoretically estimated α_{BTC} using equation (25), where the same constituent codes previously used are assumed in the BTC. Therefore, the α_c in Table 17 is the same values in Table 9. Tables 18 and 19 compare the theoretically estimated values and simulated values for two-dimensional BTCs. As confirmed in § 3.2 and 3.3, the estimated values are nearly equal to the simulated values in low BER ranges.

TABLE 17

Theoretically approximated values for block turbo codes

(*n*,*k*) extended code	d_{min}	α_c	**2-dimensional** α_{BTC}	**3-dimensional** α_{BTC}
(16,11)	4	2.75	7.56	20.80
(32,26)	4	3.25	10.56	34.33
(32,21)	6	4.56	20.79	94.82
(64,57)	4	2.96	8.76	25.93
(64,51)	6	4.50	20.25	91.13

TABLE 18

Comparison of theoretical and simulated α values for the (16,11) × (16,11) BTC

E_b/N_0 **(dB)**	**BER**	α_{BTC}	**Constituent code**	
			BER	α_c
1.0	4.41×10^{-2}	14.50	1.25×10^{-1}	2.82
2.0	3.43×10^{-3}	10.35	7.82×10^{-2}	2.88
2.5	4.24×10^{-4}	7.46	5.97×10^{-2}	2.52
3.0	8.30×10^{-5}	7.25	4.31×10^{-2}	2.82
3.5	8.51×10^{-6}	7.31	2.97×10^{-2}	2.99

TABLE 19

Comparison of theoretical and simulated α values for the (32,26) × (32,26) BTC

E_b/N_0 (dB)	BER	α_{BTC}	Constituent code	
			BER	α_c
2.0	4.19×10^{-3}	31.57	5.96×10^{-2}	3.88
3.0	7.80×10^{-6}	11.21	3.10×10^{-2}	3.33
3.3	2.10×10^{-6}	9.76	2.35×10^{-2}	3.15

3.7 Other measurement results and summary

Laboratory measurements of the INTELSAT IDR type digital transmissions (FEC $R = 3/4$ plus scrambler) led to an $\alpha = 10$ over the range of BER 1×10^{-4} to 1×10^{-11}. An $\alpha = 5$ was determined in the same measurements for the INTELSAT IBS-type digital transmissions (FEC $R = 1/2$ plus scrambler).

From the results investigated, it is shown that α is the function of the weight distribution of the FEC scheme and the BEP. The impact of parameter α on the performance model could be assessed as follows.

The masks in Figs. 2 and 3 were generated using $\alpha = 10$. If, for example, no FEC/scrambler ($\alpha = 1$) were used, the models would be shifted by one decade and the BER requirements would be more stringent (by one decade).

4 Conclusions

The results of studies have shown that the masks required for meeting the objectives specified in this Recommendation that were derived from ITU-T Recommendation G.826 are transmission rate dependent. The design masks are also dependent on the error distribution which in turn are influenced by the FEC/scrambler scheme employed.

Service requirements need also to be taken into account in deriving the allowable error design masks.

5 List of acronyms/abbreviations

BBE	Background block error
BBER	Background block error ratio
BCH	Bose, Ray-Chaudhuri, Hocquenghem
BEP	Bit error probability
BER	Bit-error rate
BTC	Block turbo code
EB	Errored block
ES	Errored second
ESR	Errored second ratio
FEC	Forward error-correction
FSS	Fixed-satellite service

GSO	Geostationary orbit
HRDP	Hypothetical reference digital path
HRX	Hypothetical reference connections
IBS	INTELSAT business service
IDR	Intermediate data rate
IG	International gateway
INTELSAT	International Telecommunication Satellite Organization
IRWEF	Input-redundancy weight enumerating function
MAP	Maximum *a posteriori*
MSS	Mobile-satellite service
PDH	Plesiochronous digital hierarchy
RS	Reed-Solomon
SDH	Synchronous digital hierarchy
SES	Severely errored second
SLA	Service level agreement
SESR	Severely errored seconds ratio
VC	Virtual container
WEF	Weight enumeration function