LEARNING TO DETECT:

ON SITE-SPECIFIC CHANNEL ESTIMATION WITH HYBRID MIMO ARCHITECTURES

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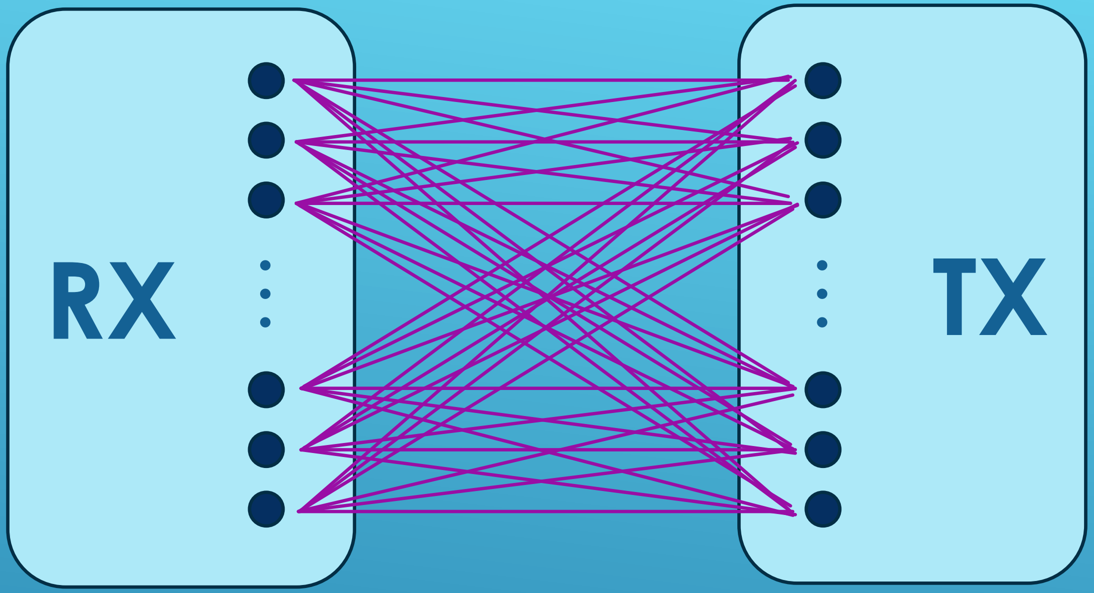


Channel per frequency f:

$$\mathbf{H}_f$$

Measurement m per frequency f:

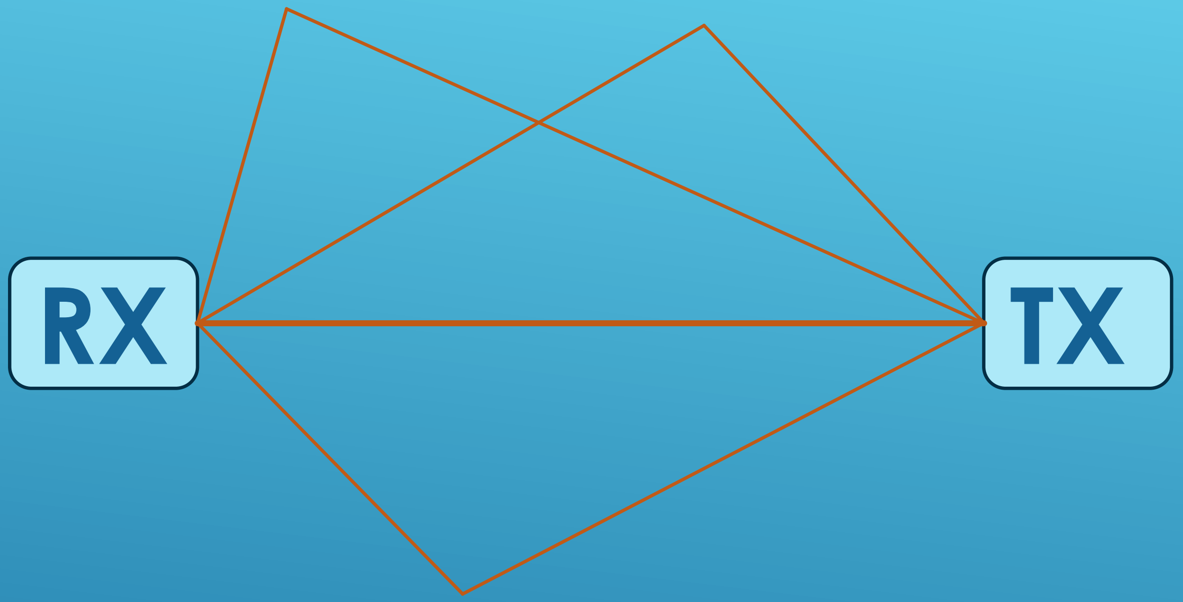
$$y_{m,f} = \mathbf{W}_m^{\mathrm{H}} \mathbf{H}_f \mathbf{F}_m + \mathbf{W}_m^{\mathrm{H}} n_{m,f}$$

Objective:

$$\{y_{m,f}\}_{m,f} \to \{\mathbf{H}_f\}_f$$

THE PROBLEM

Geometric channel



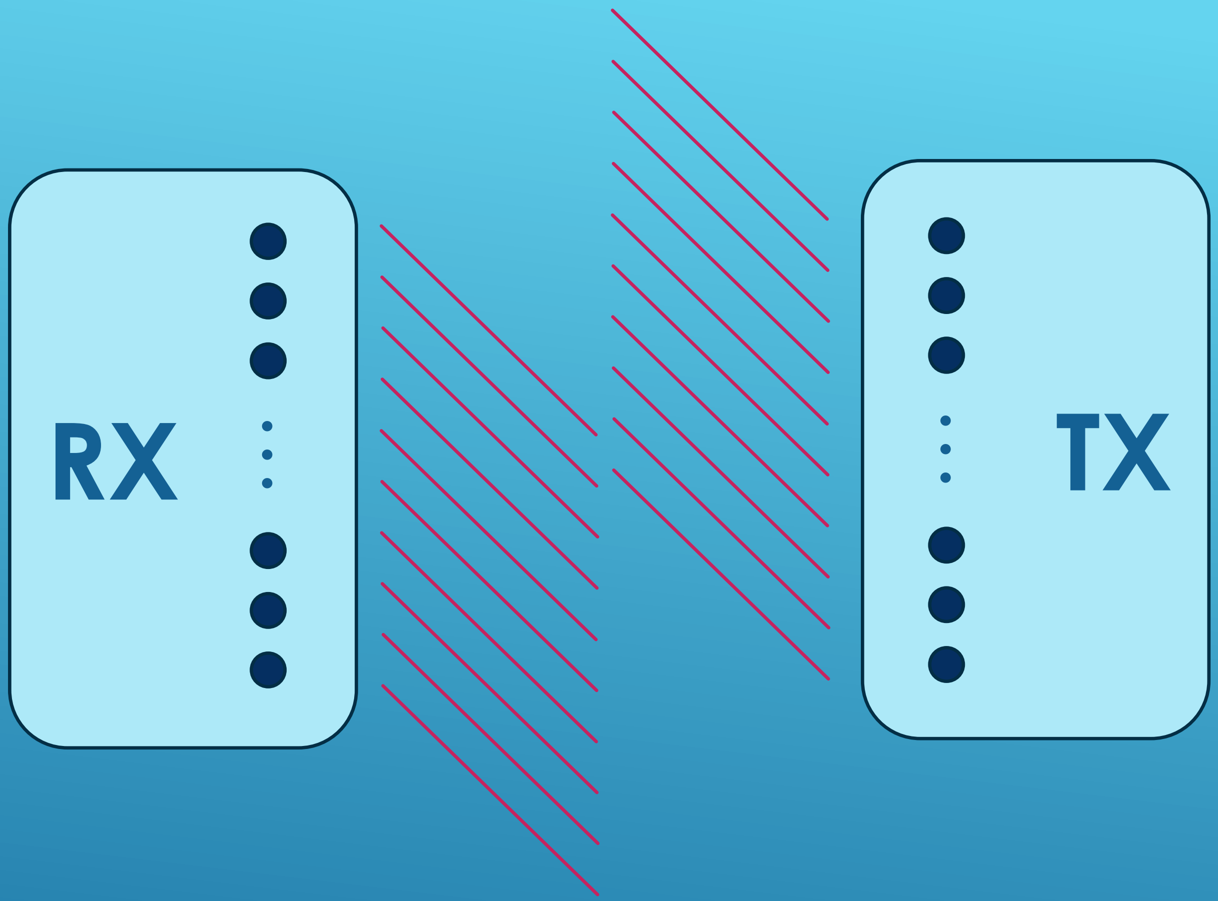
Channel contribution per path l and frequency f:

$$\mathbf{H}_f^{(l)}$$

Channel per frequency as interaction of paths:

$$\mathbf{H}_f = \sum_l \mathbf{H}_f^{(l)}$$

THE CHANNEL



Path properties:

- θ_l' Angle of departure
- ϕ_l' Angle of arrival
- τ_l Time of arrival
- α_l Complex gain

$$\phi_l = \pi \sin \phi_l'$$

$$\theta_l = \pi \sin \theta_l'$$

Steering vectors:

$$[\mathbf{a}_{\mathrm{RX}}(\phi_l)]_n = e^{-in\phi_l}$$

$$[\mathbf{a}_{\mathrm{TX}}(\theta_l)]_n = e^{-in\theta_l}$$

THE PATH

Path l contribution for frequency f:

$$\mathbf{H}_f^{(l)} = \alpha_l \mathbf{a}_{\mathrm{RX}}(\phi_l')\, \mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}(\theta_l') e^{-i2\pi f \tau_l}$$

$$y_{m,f} = \mathbf{W}_m^{\mathrm{H}} \mathbf{H}_f \mathbf{F}_m + \mathbf{W}_m^{\mathrm{H}} n_{m,f}$$

$$\mathbf{H}_f = \sum_l \alpha_l \mathbf{a}_{\mathrm{RX}}(\phi_l') \, \mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}(\theta_l') e^{-i2\pi f \tau_l}$$

FORMULATION

$$\mathbf{M} = \Phi^{\mathrm{H}}\hat{\mathbf{H}} + \mathbf{N}$$

$$\hat{\mathbf{H}} = \sum_{l} \alpha_l \mathbf{a}_{\mathrm{RX-TX}}(\phi_l, \theta_l)\mathbf{a}_{\mathrm{F}}^{\mathrm{T}}(\tau_l)$$

- $\mathbf{M}$: Measurements
- Φ : Measurement matrix
- $\mathbf{N}$: Noise

Objective:

$$\mathbf{Y} \rightarrow \hat{\mathbf{H}}$$

FORMULATION

$$\min_{\widehat{\mathbf{H}}} ||\mathbf{D}_{\mathrm{W}}\mathbf{M} - \mathbf{D}_{\mathrm{W}}\Phi^{\mathrm{H}}\widehat{\mathbf{H}}||_{\mathrm{F}}$$

→ Equivalent to Best Matching Projection Problem

→ Solve by a custom Matching Pursuit algorithm

→ Best projection problem

$$\max_{\phi,\theta,\tau}\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right|$$

APPROACH

$\mathbf{D}_{\mathrm{W}}$: noise whitening matrix

Initialize $\mathbf{M} \rightarrow \mathbf{M}_\mathrm{r}$
while path in $\mathbf{M}_\mathrm{r}$ detected:
    Extract new path components ϕ, θ, τ
    Compute all paths' α values by MMSE
    Subtract all estimated paths contributions to $\mathbf{M}$ to update $\mathbf{M}_\mathrm{r}$
Use all paths components with all α to reconstruct $\widehat{\mathbf{H}}$

Detection: Null hypothesis testing
Extraction: Cutting plane optimization

SOLUTION

Null hypothesis: $\mathbf{M} = \mathbf{N}$ with confidence γ

$$\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right| \sim Rayleigh(\sigma)$$

Median: $\sigma\sqrt{2\ln 2}$

$\max\limits_{\phi,\theta,\tau}\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right|$ has a γ percentile $\sigma\sqrt{-2\ln\left(1-\gamma^{\frac{1}{R}}\right)}$

$$\frac{\max\limits_{\phi,\theta,\tau}\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right|}{\operatorname*{median}\limits_{\phi,\theta,\tau}\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right|} < \sqrt{-\ln_2\left(1-\gamma^{\frac{1}{R}}\right)}$$

DETECTION

R : Number of ϕ,θ,τ combinations

$$\max_{\phi,\theta,\tau} \left| \mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau) \right|$$

Retrieve ϕ, θ, τ from the detection
Sequentially estimate ϕ, θ
Estimate τ
Iterate:
    Sequentially estimate ϕ, θ
    Estimate τ

EXTRACTION

- All estimations are done by a well-designed grid-search.
- When we sequentially estimate the angles, we prioritize the one in the device with the larger amount of antenna elements
- The estimation resolution for the detection is much lower than the desired one. Due to this, the first algorithm iteration is slightly different.

Solution is an iterative algorithm in which γ only matters for the detection → Error function respect to γ is a step function

Run the solution using a lower bound $\gamma_{\min}$ and take note of the required γ for the threshold to be triggered and the resulting error

Reconstruct the error step function

Merge the error step functions of different runs

Select γ minimizing the error

Null hypothesis condition

$$\gamma > \left(1 - 2^{-\left(\frac{\max\limits_{\phi,\theta,\tau}\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right|}{\operatorname{median}\limits_{\phi,\theta,\tau}\left|\mathbf{a}_{\mathrm{RX-TX}}^{\mathrm{H}}(\phi,\theta)\Phi\mathbf{D}_{\mathrm{W}}^{\mathrm{H}}\mathbf{D}_{\mathrm{W}}\mathbf{M}\mathbf{a}_{\mathrm{F}}^{*}(\tau)\right|}\right)^{2}}\right)^{R}$$

LEARNING

TRAINING

Training set consisting of 50 samples out of the 10000 available. Training on an 8th gen Intel core i7 CPU, no GPU required.

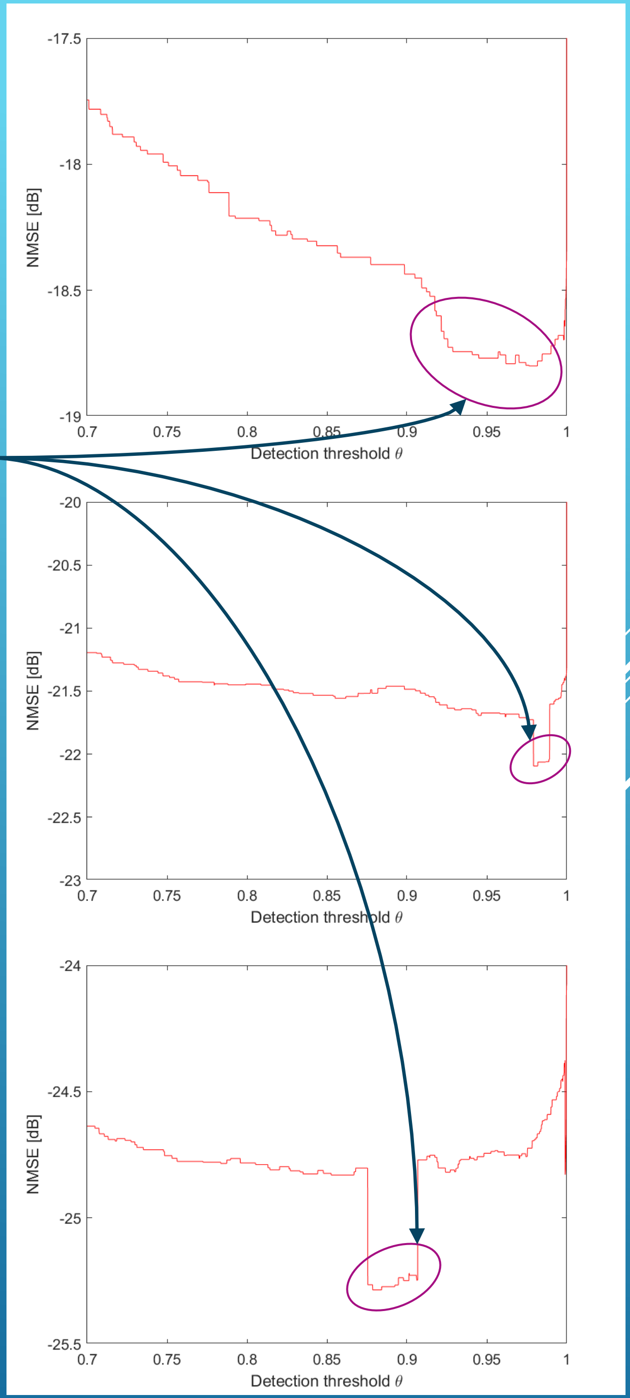
Training is analytical instead of iterative, leading always to the optimal solution for γ.

Dataset 1
102s

Dataset 2
145s

Dataset 3
186s

Typical scenario's detection threshold



By having a proper analysis of the problem, we:

- Improve the model's accuracy
- Improve the model's robustness
- Reduce the model's complexity
- Improve the training method
- Increase the training accuracy
- Reduce the training time

CONCLUSIONS