**Alternative Routing Networks**

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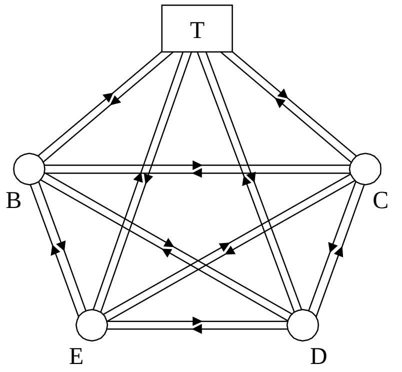
Example:

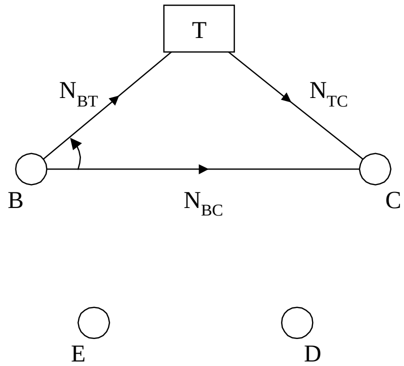
A small network.
4 terminal exchanges.
1 tandem.




| | B | C | D | E | T |
|---|---|---|---|---|---|
| B | | | | | |
| C | | | | | |
| D | | | | | |
| E | | | | | |
| T | | | | | |

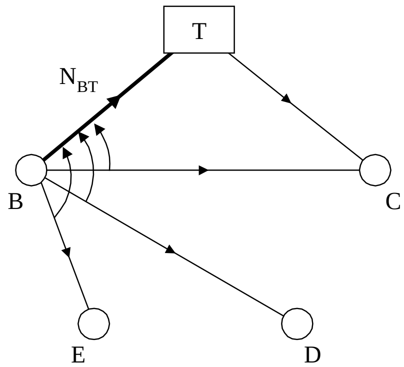
Consider the traffic case
B→C!

Tasks:
1) Optimise $N_{BC}$
2) Dimension $N_{BT}$ and $N_{TC}$



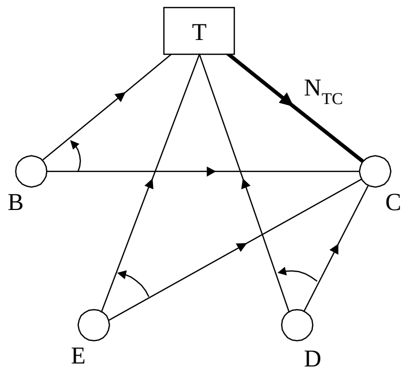

| | B | C | D | E | T |
|---|---|---|---|---|---|
| B | | X | | | X |
| C | | | | | |
| D | | | | | |
| E | | | | | |
| T | | X | | | |

But $N_{BT}$ carries also overflow traffic (background traffic) from traffic cases
B→D
and
B→E!

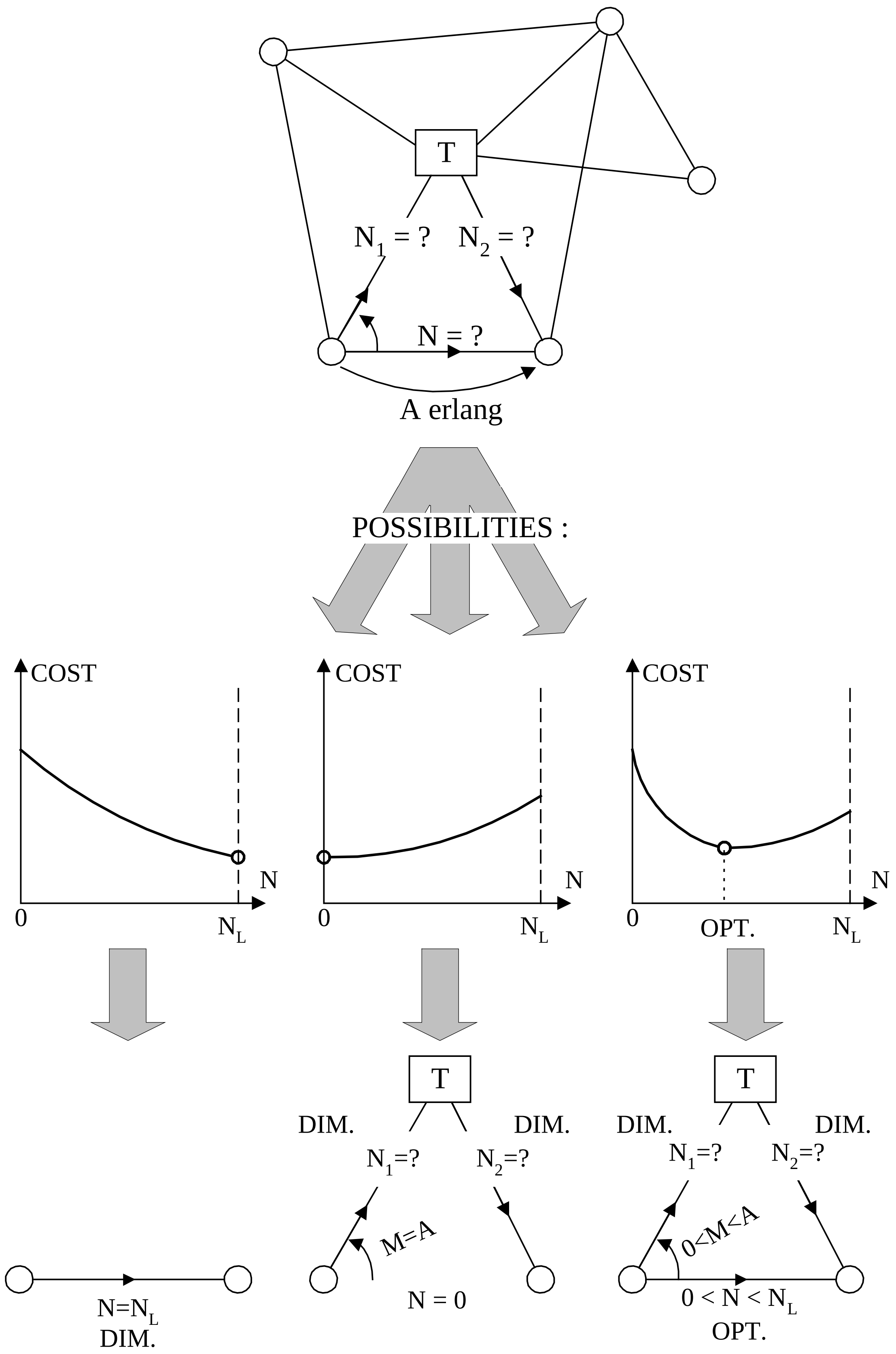


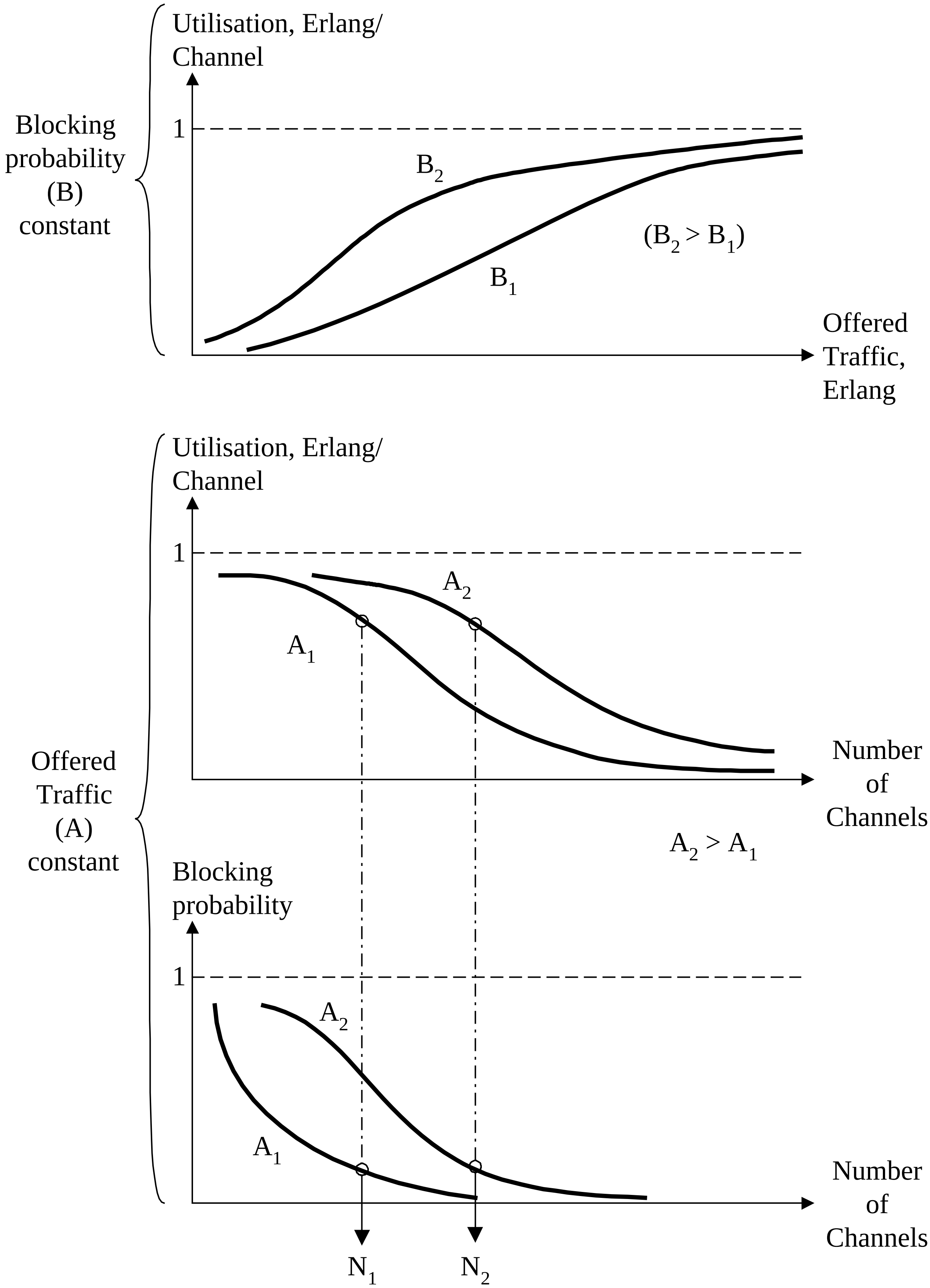

| | B | C | D | E | T |
|---|---|---|---|---|---|
| B | | X | X | X | X |
| C | | | | | |
| D | | | | | |
| E | | | | | |
| T | | | | | |

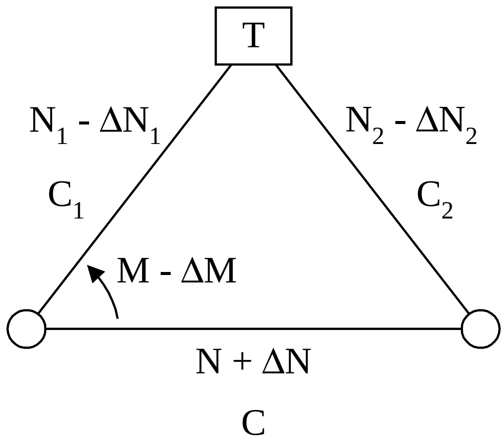
And $N_{TC}$ carries also overflow traffic (background traffic) from traffic cases
D→C
and
E→C




| | B | C | D | E | T |
|---|---|---|---|---|---|
| B | | X | | | |
| C | | | | | |
| D | | X | | | |
| E | | X | | | |
| T | | X | | | |


T
$N_1$ = ?
$N_2$ = ?
N = ?
A erlang
POSSIBILITIES :
COST
N
0
$N_L$
COST
N
0
$N_L$
COST
N
0
OPT.
$N_L$
T
DIM.
DIM.
$N_1$=?
$N_2$=?
M=A
N = 0
T
DIM.
DIM.
$N_1$=?
$N_2$=?
0<M<A
0 < N < $N_L$
OPT.
N=$N_L$
DIM.


Blocking probability (B) constant
Utilisation, Erlang/ Channel
1
B2
B1
(B2 > B1)
Offered Traffic, Erlang
Offered Traffic (A) constant
Utilisation, Erlang/ Channel
1
A2
A1
Number of Channels
A2 > A1
Blocking probability
1
A2
A1
Number of Channels
N1
N2




$$N + \Delta N \quad \Rightarrow \quad C_{TOT} + \Delta C_{TOT}$$

$$\Delta C_{TOT} = C \cdot \Delta N - C_1 \cdot \Delta N_1 - C_2 \cdot \Delta N_2$$

$$\Delta C_{TOT} = 0 \text{ when :}$$

$$C \cdot \Delta N = C_1 \cdot \Delta N_1 + C_2 \cdot \Delta N_2$$

Divide by $\Delta M$ :

$$C \cdot \frac{\Delta N}{\Delta M} = C_1 \cdot \frac{\Delta N_1}{\Delta M} + C_2 \cdot \frac{\Delta N_2}{\Delta M}$$

or:

$$\frac{\Delta M}{\Delta N} = \frac{C}{C_1 \cdot \frac{\Delta N_1}{\Delta M} + C_2 \cdot \frac{\Delta N_2}{\Delta M}}$$

$$\frac{\Delta M}{\Delta N} = \frac{C}{C_1 \cdot \frac{\Delta N_1}{\Delta M} + C_2 \cdot \frac{\Delta N_2}{\Delta M}}$$

If $\Delta N = 1$ then :

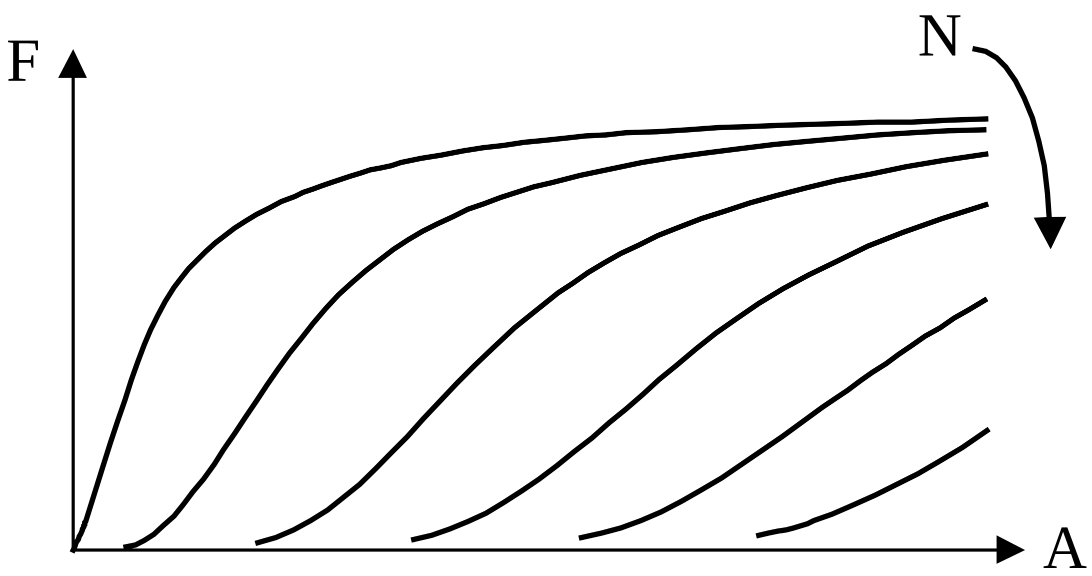
$$\frac{\Delta M}{\Delta N} = F = \text{The Improvement Factor}$$

*F* is calculated as

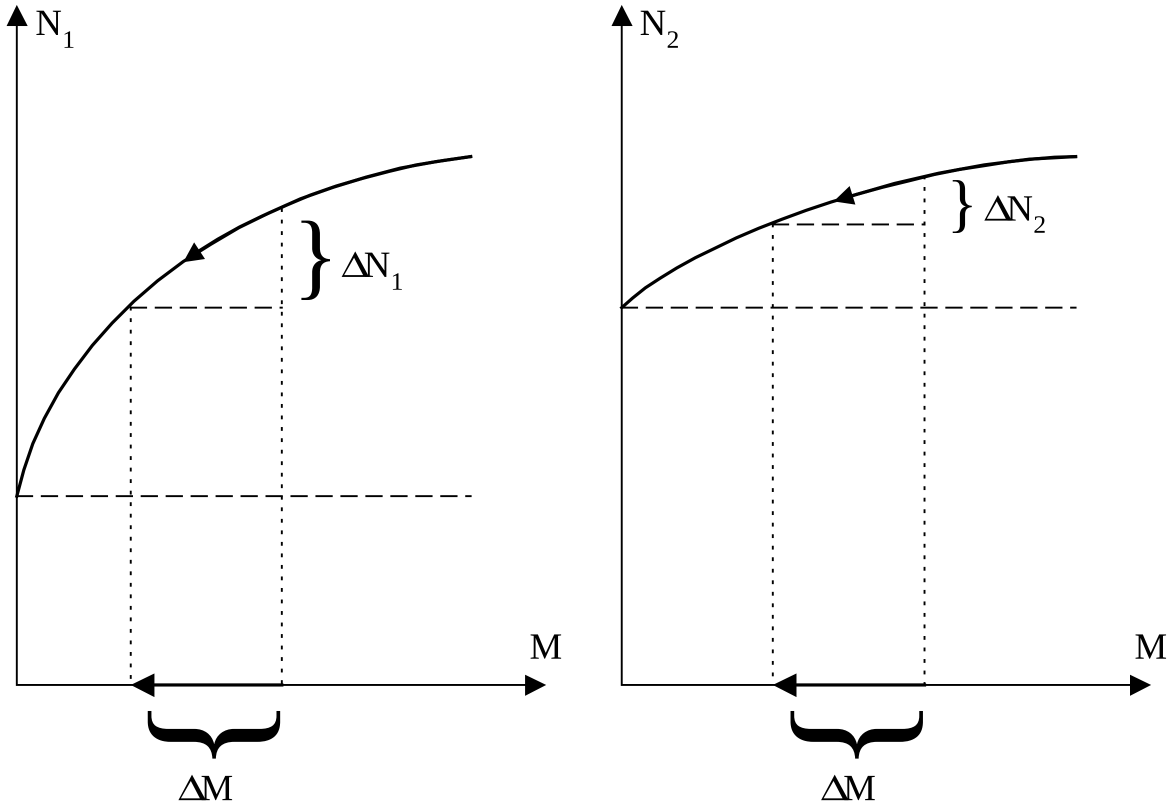
$$F = A \cdot \left[B_N(A) - B_{N+1}(A)\right]$$

where $B_N(A)$ is a general expression for the congestion in a trunk group with *N* trunks and *A* erl. offered.

Diagram for F:



$$\frac{\Delta M}{\Delta N} = F = \frac{C}{C_1 \cdot \frac{\Delta N_1}{\Delta M} + C_2 \cdot \frac{\Delta N_2}{\Delta M}}$$




Rapp's Approximation:

$$F = \varepsilon \cdot \left[0.7 + 0.3 \cdot \varepsilon^2\right]$$

where

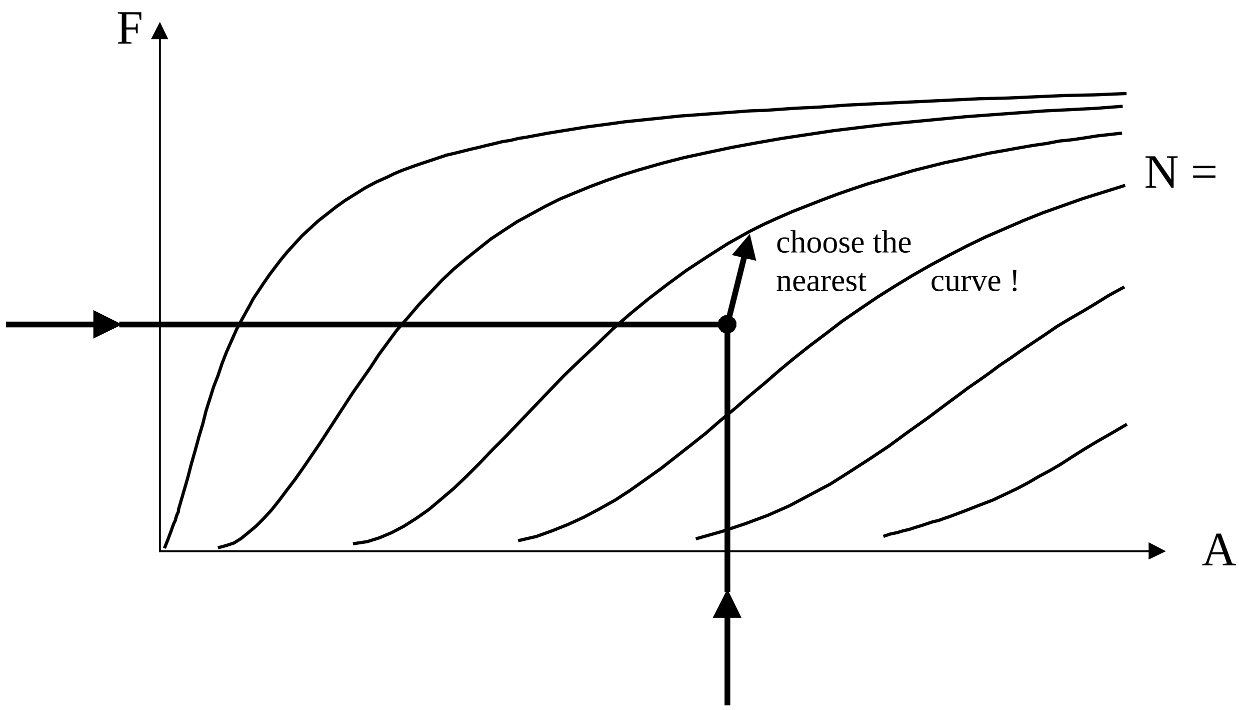
$$\varepsilon = \frac{C}{C_1 + C_2}$$

Optimisation Procedure:

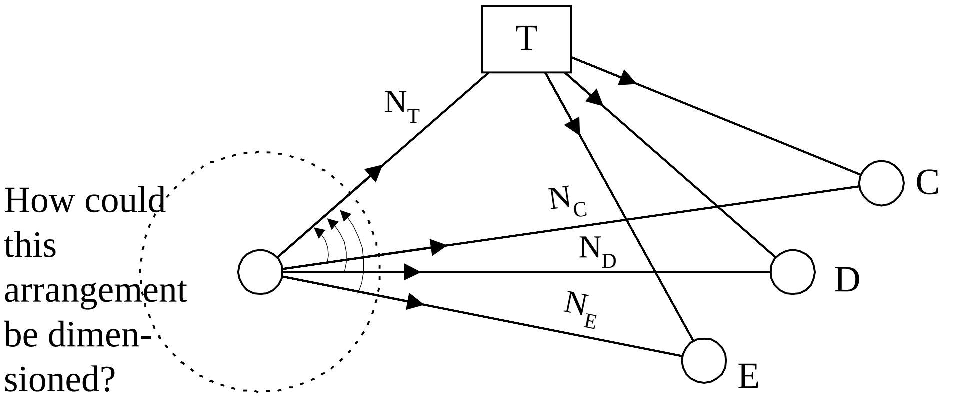
1) Calculate F e.g. using Rapp's approximation.

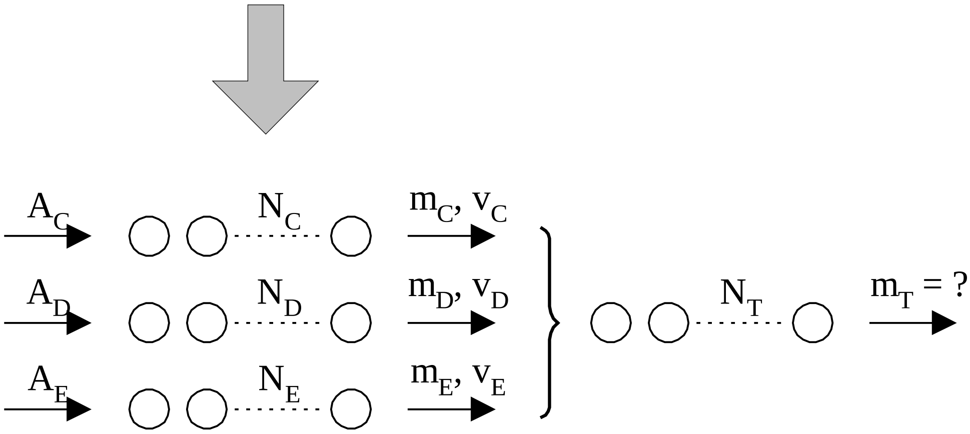
2) Using the calculated F-value and the total traffic A, enter the appropriate F-diagram and read:

N = optimal number of trunks in the high-usage route.

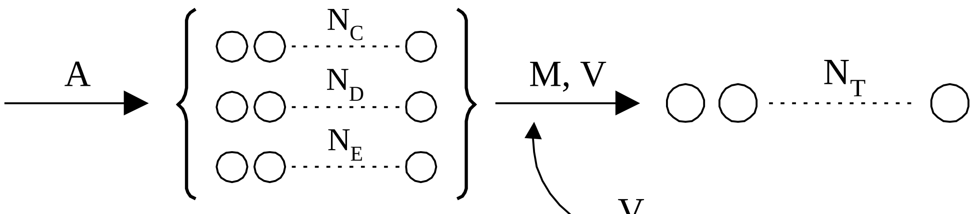


## Wilkinson's Method:



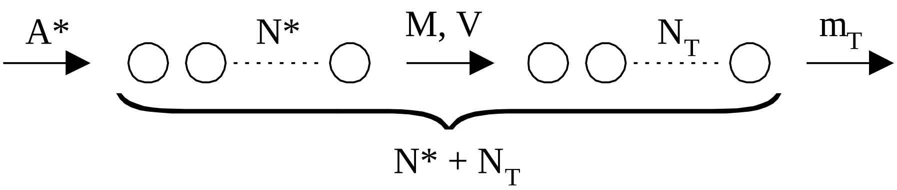




## We sum up:




$\frac{V}{M} > 1$ : Erlang's formula is not valid!

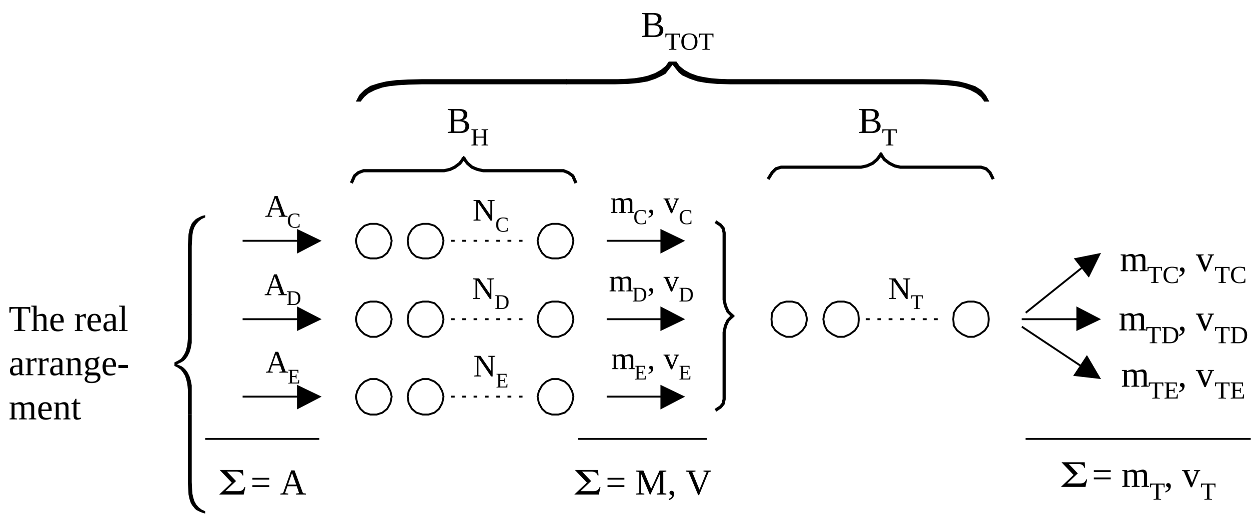
Solution: find fictitious traffic A*, offered to a fictitious trunk group N*, so that the mean and the variance of the rejected traffic is exactly equal to M resp. V !

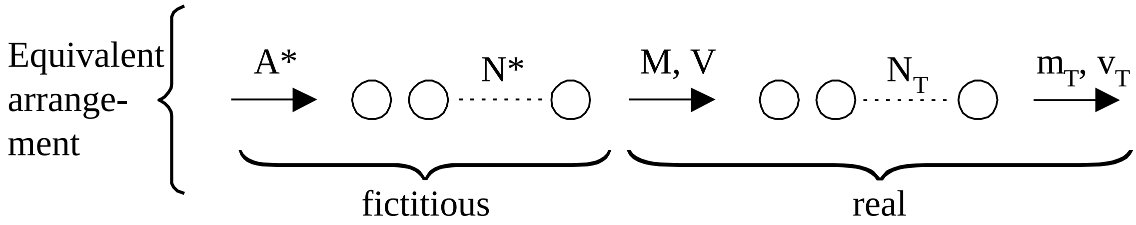



Erlang's formula is now valid:

$$m_T = A^* \cdot E_{N^* + N_T}(A^*)$$

# Blocking Probabilities:

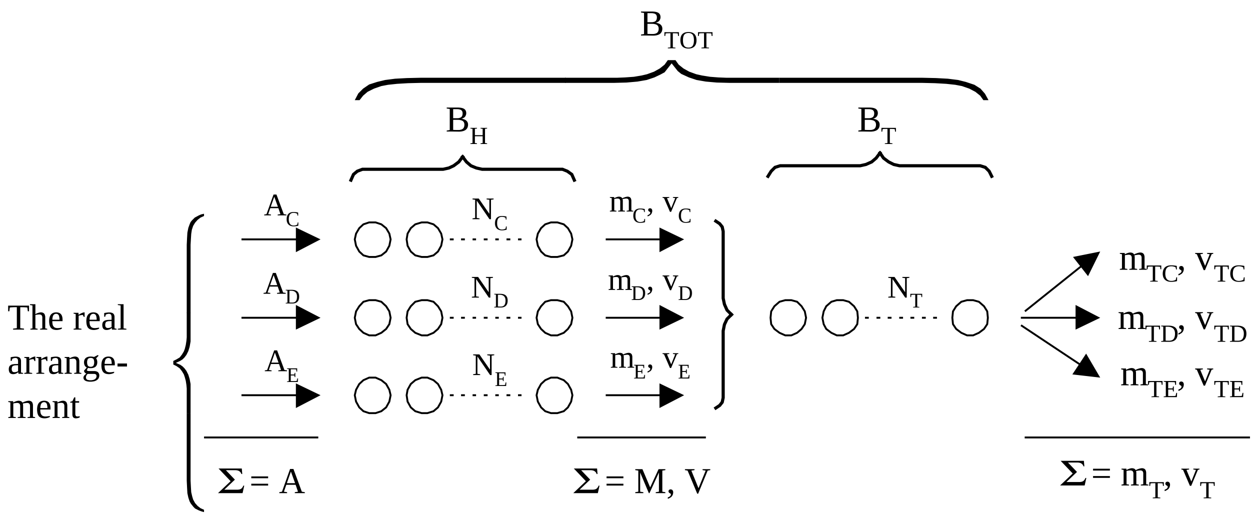


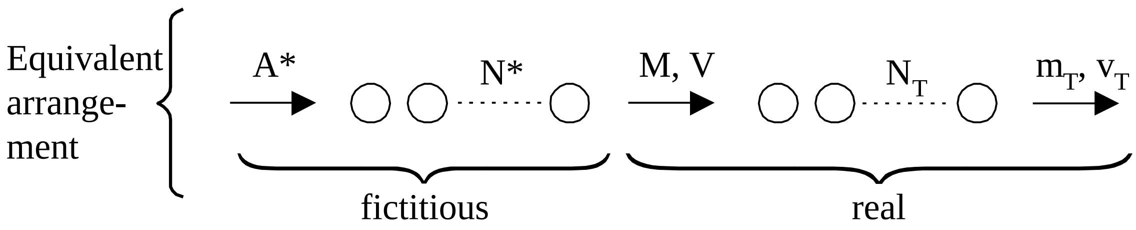




1. $$\overline{B}_H = \frac{m_C + m_D + m_E}{A_C + A_D + A_E} = \frac{A_C \cdot E_{N_C}(A_C) + A_D \cdot E_{N_D}(A_D) + A_E \cdot E_{N_E}(A_E)}{A_C + A_D + A_E} = \frac{M}{A}$$

2. $$\overline{B}_T = \frac{m_{TC} + m_{TD} + m_{TE}}{m_C + m_D + M_E} = \frac{m_T}{M} = \frac{A^* \cdot E_{N^*+N_T}(A^*)}{M} = \frac{E_{N^*+N_T}(A^*)}{E_{N^*}(A^*)}$$

3. $$\overline{B}_{TOT} = \frac{m_T}{A_C + A_D + A_E} = \frac{A^* \cdot E_{N^*+N_T}(A^*)}{A}$$




4. $B_{HC} = \dfrac{m_C}{A_C} = E_{N_C}\left(A_C\right)$

5. $B_{TC} = \overline{B}_T = \dfrac{E_{N^*+N_T}(A^*)}{E_{N^*}(A^*)}$

6. $B_{TOTC} = \dfrac{m_{TC}}{A_C} = \dfrac{m_C \cdot \overline{B}_T}{A_C} = \dfrac{E_{N_C}(A_C) \cdot E_{N^*+N_T}(A^*)}{E_{N^*}(A^*)}$

7. $B'_{TC} = \dfrac{v_C \cdot M}{V \cdot m_C} \cdot \overline{B}_T$

8. $B'_{TOTC} = \dfrac{m'_{TE}}{A_C} = \dfrac{m_C \cdot B'_{TC}}{A_C} = \dfrac{v_C \cdot M}{V \cdot m_C} \cdot B_{TOTC} =$

$$= \frac{v_C \cdot M}{V \cdot m_C} \cdot \frac{E_{N_C}(A_C) \cdot E_{N^*+N_T}(A^*)}{E_{N^*}(A^*)}$$

1. (Approx.) $n_\nu$ from

$$\begin{cases} F(n_\nu, A_\nu) \approx \varepsilon \cdot \left[1 - 0.3 \cdot \left(1 - \varepsilon^2\right)\right] \\ \varepsilon = C_{ij} \Big/ \left(C_{it} + C_{Tj}\right) \\ F(n, A) = A \cdot \left[E(n, A) - E(n+1, A)\right] \quad (\text{exact}) \end{cases}$$

2. (Exact)

$$m_\nu = A_\nu \cdot E_{n_\nu}\left(A_\nu\right)$$

$$v_\nu = m_\nu \cdot \left(1 - m_\nu + \frac{A_\nu}{1 + n_\nu + m_\nu - A_\nu}\right)$$

3. (Exact)

$$M = \sum_\nu m_\nu \qquad V = \sum_\nu v_\nu$$

4. (Exact)

$A^*$ and $n^*$ from

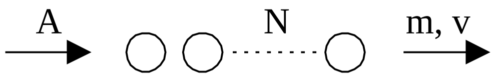
$$\begin{cases} M = A^* \cdot E_{n^*}\left(A^*\right) \\ V = M \cdot \left(1 - M + \dfrac{A^*}{1 + n^* + M - A^*}\right) \end{cases}$$

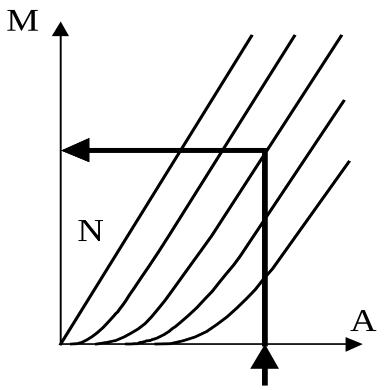
4. (Approx.)

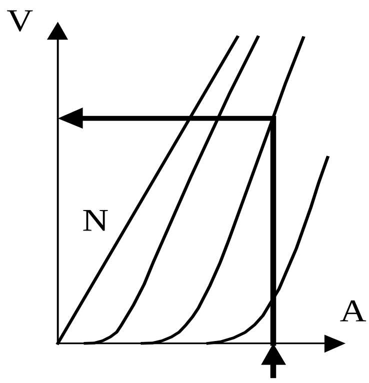
$$A^* \approx V + 3 \cdot \frac{V}{M} \cdot \left(\frac{V}{M} - 1\right)$$

$$n^* \approx \frac{A^*}{1 - \dfrac{1}{M + \dfrac{V}{M}}} - M - 1$$

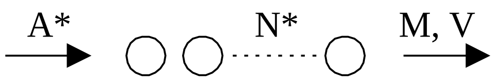
There are diagrams for calculation of *m* and *v* from high-usage trunk groups...

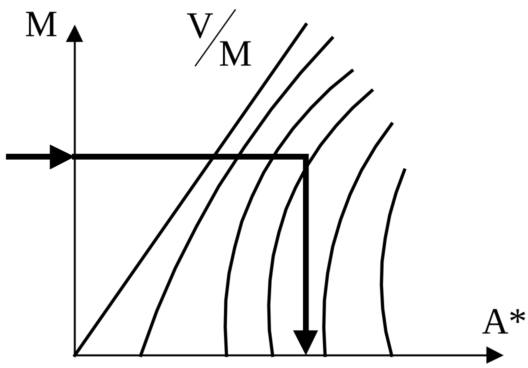


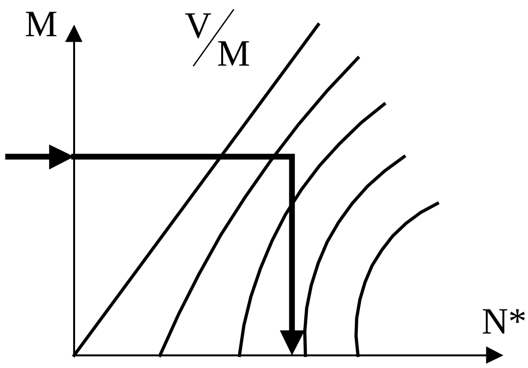



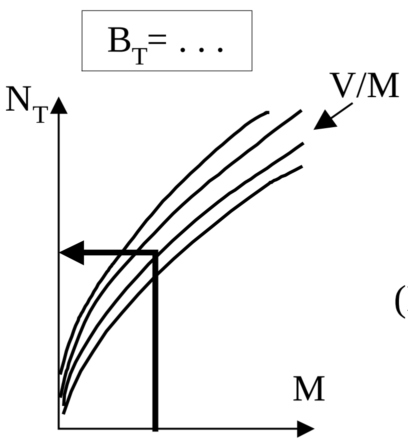

and other diagrams for calculation of fictitious traffic and fictitious trunk group:




If the tandem route is to be dimensioned for a <u>fixed, standard congestion value</u>, these <u>diagrams</u> may be used (instead of calculations) :




(In that case, A* and N* are not needed!)