Theory for Full Availability Group,

Delay System

(Solution to Exercises)

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TXB 1

$n = 30; \quad \lambda = 700 \text{ calls / hour} = \frac{700}{3600} \text{ calls / sec;} \quad \tau = 108 \text{ sec.}$

$\underline{\underline{A}} = \lambda \cdot \tau = \frac{700}{3600} \cdot 108 = \underline{\underline{21 \text{ erl.}}}$ a

Probability of delay = Call congestion = Time congestion = $E_{2,n}(A) = D_n(A)$ b

$$D_n(A) = \frac{n \cdot E_n(A)}{n - A + A \cdot E_n(A)} = \frac{30 \cdot E_{30}(21)}{30 - 21 + 21 \cdot E_{30}(21)}$$

$E_{30}(21) = 0.013594$ *(The Erlang Table)*

We find: Probability of delay = 0.044

$\underline{\underline{A'}} = A = \underline{\underline{21 \text{ erl.}}}$ c

a = mean of the traffic handled by the different devices = mean load on the devices = d

$$= \frac{A}{n} = \frac{21}{30} = \underline{\underline{0.7 \text{ erl.}}}$$

U *= mean waiting time for all calls =* $D_n(A) \cdot \frac{\tau}{n - A} = 0.044 \cdot \frac{108}{30 - 21} = \underline{\underline{0.53 \text{ sec.}}}$ e

u *= mean waiting time for calls which have to wait =* $\frac{\tau}{n - A} = \frac{108}{30 - 21} = \underline{\underline{12 \text{ sec}}}$ f

TXB 2

$\underline{A = 21\ erl.}$; $\underline{n = 30}$; $\underline{s = 108\ sec}$.

Ordered queueing (first come - first served)

No calls give up $(\Theta = 0)$

$P_v(> t_0) =$ *Probability that a* *waiting call* *must wait longer than* $t_0 = e^{-\frac{n-A}{s}\cdot t_0}$ *b*

(s and t_0 are expressed in the same unit of time)

$$\underline{\underline{P_v(> 3\,sec)}} = e^{-\frac{30-21}{108}\cdot 3} = e^{-0.25} = \underline{\underline{0.779}}$$

$$\underline{\underline{P_v(> 6\,sec)}} = e^{-\frac{30-21}{108}\cdot 6} = e^{-0.5} = \underline{\underline{0.607}}$$

$$\underline{\underline{P_v(> 12\,sec)}} = e^{-\frac{30-21}{108}\cdot 12} = e^{-1} = \underline{\underline{0.368}}$$

$$\underline{\underline{P_v(> 24\,sec)}} = e^{-\frac{30-21}{108}\cdot 24} = e^{-2} = \underline{\underline{0.135}}$$

We consider *all calls* *a*

$$P(> t_0) = D_n(A)\cdot e^{-\frac{n-A}{s}\cdot t_0}$$

$$\underline{\underline{P(> 3\,sec)}} = 0.044\cdot 0.779 = \underline{\underline{0.034}}$$

$$\underline{\underline{P(> 6\,sec)}} = 0.044\cdot 0.607 = \underline{\underline{0.027}}$$

$$\underline{\underline{P(> 12\,sec)}} = 0.044\cdot 0.368 = \underline{\underline{0.016}}$$

$$\underline{\underline{P(> 24\,sec)}} = 0.044\cdot 0.135 = \underline{\underline{0.006}}$$

TXB 3

Erlangs distribution for delay systems. Ordered queueing. No calls give up $(\Theta = 0)$

$\lambda = 4320 \; calls / hour; \quad n = 1; \quad s = mean\ holding\ time\ in\ sec.$

$$A = \lambda \cdot s = \frac{4320}{3600} \cdot s = \frac{6 \cdot s}{5} \; erl.$$

For an equilibrium distribution to exist,

A must be < 1, i.e. $\frac{6 \cdot s}{5} < 1, \quad i.e. \quad s < \frac{5}{6} \; sec = 0.833 \; sec.$

$$D_n(A) = D_1(A) = A = \frac{6 \cdot s}{5}$$

$$P(> t_0) = D_n(a) \cdot e^{-\frac{n-A}{s} \cdot t_0}$$

Thus,

$$\frac{6 \cdot s}{5} \cdot e^{-\frac{1 - \frac{6 \cdot s}{5}}{s} \cdot 3} \leq 0.01$$

$$6 \cdot s \cdot e^{-\frac{3}{s}} \cdot e^{\frac{18}{5}} \leq 0.05$$

$$6 \cdot s \cdot e^{-\frac{3}{s}} \leq 0.05 \cdot e^{-3.6}$$

$$6 \cdot s \cdot e^{-\frac{3}{s}} \leq 0.001366$$

L.H.S. = ***L**eft **h**and **s**ide* = $6 \cdot s \cdot e^{-\frac{3}{s}}$

R.H.S. = ***R**ight **h**and **s**ide* = 0.001366

L.H.S. is monotonously increasing from 0 to + ∞ when *s* increases from 0 to + ∞.

Trial and error:

$S = 0: \quad L.H.S. = 0 < R.H.S.$

$S = 0.1: \quad L.H.S. = 0.6 \cdot e^{-3} < R.H.S.$

$S = 0.3: \quad L.H.S. = 1.8 \cdot e^{-1} < R.H.S.$

$S = 0.4: \quad L.H.S. = 2.4 \cdot e^{-7.5} = 0.001320 < R.H.S.$

$S = 0.5: \quad L.H.S. = 3 \cdot e^{-6} = 0.00744 > R.H.S.$

With 1 decimal, we see that *s* shall be chosen = *0.4 sec.*

(S < 0.833 sec., i.e. an equil. distr. Exists)

$\underline{\underline{s \approx 0.4 \; sec.}}$

TXB 4

Erlangs distribution for delay systems.

n = 2 devices; calling rate = λ calls / sec.; τ = 0.4 sec.; A = 0.4 λ erl.

$$A < n \quad \Rightarrow \quad 0.4 \cdot \lambda < 2 \quad \Rightarrow \quad \lambda < 5 \text{ calls / sec} = 18000 \text{ calls / hour}$$

Mean waiting time for all calls:

$$U = D_n(A) \cdot \frac{\tau}{n - A}$$

$$D_n(A) = D_2(A) = \frac{\frac{A^2}{2} \cdot \frac{2}{2-A}}{1 + A + \frac{A^2}{2} \cdot \frac{2}{2-A}} = \frac{A^2}{2+A} = \frac{0.4^2 \cdot \lambda^2}{2 + 0.4 \cdot \lambda}$$

$$U = \frac{0.4^2 \cdot \lambda^2}{2 + 0.4 \cdot \lambda} \cdot \frac{0.4}{2 - 0.4 \cdot \lambda} \le 2$$

$$0.4^3 \cdot \lambda^2 \le 8 - 2 \cdot 0.4^2 \cdot \lambda^2 \quad \Rightarrow \quad \lambda^2 \le \frac{125}{6}$$

$$\lambda \le \frac{5}{6} \cdot \sqrt{30} \text{ calls / sec} = 3000 \cdot \sqrt{30} \text{ calls / hour}$$

$$\lambda \approx 16400 \text{ calls / hour}$$

TXB 5

We use the relation

$$E_{2,n}(A) = \frac{n \cdot E_{1,n}(A)}{n - A \cdot [1 - E_{1,n}(A)]}$$

$$0 < n - A[1 - E_{1,n}(A)] < n$$

Thus

$$\frac{n}{n - A[1 - E_{1,n}(A)]} > 1$$

so that

$$E_{2,n}(A) > E_{1,n}(A)$$

TXB 6

Note that the value of A (8 erl.) is not relevant for the solution of the problem. We are asking for a conditional probability, where the condition is that the arriving call finds one other call in the queue.

The waiting time of the call consists of 2 parts, viz:

1. The time spent in the queue as No. 2, and

2. The time spent in the queue as No. 1.

Each of these parts are exponentially distributed with the mean *5/10 = 0.5 min.*

The sum of the two parts has an Erlang 2-phase distribution, i.e.

$$P\{W > t\} = e^{-\frac{t}{0.5}} + \frac{t}{0.5} \cdot e^{-\frac{t}{0.5}}$$

where W is the waiting time. In this example, t is *0.5 min.*, so

$$P\{W > 0.5\} = e^{-\frac{0.5}{0.5}} + 1 \cdot e^{-\frac{0.5}{0.5}} = 2 \cdot e^{-1} = 2 \cdot 0.368 = \underline{\underline{0.736}}$$

TXB 7

In the general case with offered traffic *A erl.* and n trunks, we have

$$P_{n+r} = \frac{\left(\frac{A}{n}\right)^r \cdot \frac{A^n}{n!}}{M}$$

where M is the denominator of $E_{2,n}(A)$,

i.e. $M = \sum_{k=0}^{n-1} \frac{A^k}{k!} + \frac{n}{n-A} \cdot \frac{A^n}{n!}$

The proportion of time, when there is at least one call waiting, is

$$P = \sum_{r=1}^{\infty} P_{n+r} = \frac{A}{n} \cdot \sum_{r=1}^{\infty} \frac{\left(\frac{A}{n}\right)^{r-1} \cdot \frac{A^n}{n!}}{M} =$$

$$= \frac{A}{n} \cdot \sum_{r=0}^{\infty} \frac{\left(\frac{A}{n}\right)^{r} \cdot \frac{A^n}{n!}}{M} = \frac{A}{n} \cdot \sum_{r=0}^{\infty} P_{n+r} =$$

$$= \frac{A}{n} \cdot E_{2,n}(A) = \frac{A}{n} \cdot \frac{n \cdot E_{1,n}(A)}{n - A[1 - E_{1,n}(A)]} =$$

$$= \frac{4 \cdot 0.0053}{10 - 4 \cdot 0.9947} = \underline{\underline{0.035}}$$

TXB 8

Mean holding time $\tau = 0.2 \cdot \frac{1}{3} + 0.4 \cdot \frac{2}{3} = \frac{1}{3}$ *sec.*

$\lambda = 4320$ *calls / hour*

$$A = \lambda \cdot \tau = \frac{4320}{3600} \cdot \frac{1}{3} = 0.4 \text{ erl.}$$

Probability of delay $= P(>0) = a = A = \underline{\underline{0.4}}$ *a*

Use Pollaczek - Khintchine's formula *c*

$$U = \left(1 - \frac{\sigma^2}{\tau^2}\right) \cdot U_{const} + \frac{\sigma^2}{\tau^2} \cdot U_{exp}$$

where

$$\sigma^2 = \text{the variance} = (0.2)^2 \cdot \frac{1}{3} + (0.4)^2 \cdot \frac{2}{3} - \left(\frac{1}{3}\right)^2 = \frac{2}{225}$$

$$U_{const} = \frac{\tau}{2} \cdot \frac{a}{1-a} \qquad U_{exp} = D_n(A) \cdot \frac{\tau}{n-A}$$

$$(a = A) \qquad D_n(A) = P(>0)$$

$$U = \left(1 - \frac{2}{225} \cdot 3^2\right) \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{0.4}{0.6} + \frac{2}{225} \cdot 3^2 \cdot 0.4 \cdot \frac{\frac{1}{3}}{1-0.4} = \underline{\underline{0.12 \text{ sec.}}}$$

$$u = \frac{U}{D_n(A)} = \frac{0.12}{0.4} = \underline{\underline{0.3 \text{ sec.}}}$$ *b*

TXB 9

$N = \infty; \qquad n = 1; \qquad \lambda = 2880 \text{ calls / hour}.$

$$\tau = \int_0^\infty t \cdot f(t)\, dt = \int_0^\infty 16 \cdot t^2 \cdot e^{-4\cdot t}\ dt = \frac{1}{2} \cdot \int_0^\infty 4 \cdot \frac{(4 \cdot t)^2}{2!} \cdot e^{-4\cdot t}\ dt = 0.5 \text{ sec.}$$

$$\sigma^2 = \int_0^\infty t^2 \cdot f(t)\, dt - \tau^2 = \int_0^\infty 16 \cdot t^3 \cdot e^{-4\cdot t}\ dt - 0.5^2 = \frac{3}{8} \cdot \int_0^\infty 4 \cdot \frac{(4 \cdot t)^3}{3!} \cdot e^{-4\cdot t}\ dt - 0.25 =$$

$$= \frac{3}{8} - 0.25 = 0.375 - 0.25 = 0.125$$

$$a = A' = A = \lambda \cdot \tau = \frac{2880}{3600} \cdot 0.5 = \underline{\underline{0.4 \text{ erl.}}}$$

$$\text{Prob. of delay} = D_n(A) = a = A = \underline{\underline{0.4}} \qquad a$$

$$U \approx \left(1 - \frac{\sigma^2}{\tau^2}\right) \cdot \frac{\tau}{2} \cdot \frac{a}{1-a} + D_n(A) \cdot \frac{\tau}{n-A} =$$

$$= \left(1 - \frac{0.125}{0.5^2}\right) \cdot \frac{0.5}{2} \cdot \frac{0.4}{1-0.4} + \frac{0.125}{0.5^2} \cdot 0.4 \cdot \frac{0.5}{1-0.4} = \underline{\underline{0.25 \text{ sec.}}} \qquad c$$

$$u = \frac{U}{D_n(A)} = \frac{0.25}{0.4} = \underline{\underline{0.625 \text{ sec.}}} \qquad b$$