Economic Period of Provisioning

Planning of Fiber Optics Cable

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1. General

Telecommunications plants must be expanded at regular time intervals, known as provisioning periods as long as the demand for telecommunications services continue to grow. The provisioning periods and the expansion steps, at each addition, can be chosen in different ways dependent on a number of factors.

The need for expansion is dictated by forecast, and the size of the expansion can be different for different types of equipment. For details, see Reference [2].

2. Provisioning Period

The demand for circuits in a route is assumed to increase linearly for an infinite period at a growth of circuits/year. At time *t*, the demand will be:

$$D(t) = \lambda \cdot t \qquad (1)$$

The cost of an expansion sufficiently large to cater for the demand over *t* years is:

$$C(S) = A + B \cdot S \qquad (2)$$

where *A* and *B* are the basic and incremental cost respectively and *S* the size in pairs of the added plant. At time *t*, when the whole plant is exhausted, the demand *D* should be equal to the size *S* of the plant. Figure 1 illustrates the demand and the expansion pattern. The present worth of all expansion during an unlimited period of time is

$$PW = (A + B\lambda \cdot t) + (A + B\lambda \cdot t)(1+i)^{-t} + (A + B\lambda \cdot t)(1+i)^{-2t} + \ldots$$

$$= (A + B\lambda \cdot t)\sum_{n=o}^{\infty}(1+i)^{-nt} = (A + B\lambda \cdot t)\sum_{n=o}^{\infty} e^{-rnt}$$

then $$PW = \frac{A + B\lambda \cdot t}{1 - e^{-rt}} \qquad (3)$$

where $r = \lambda n(1+i)$

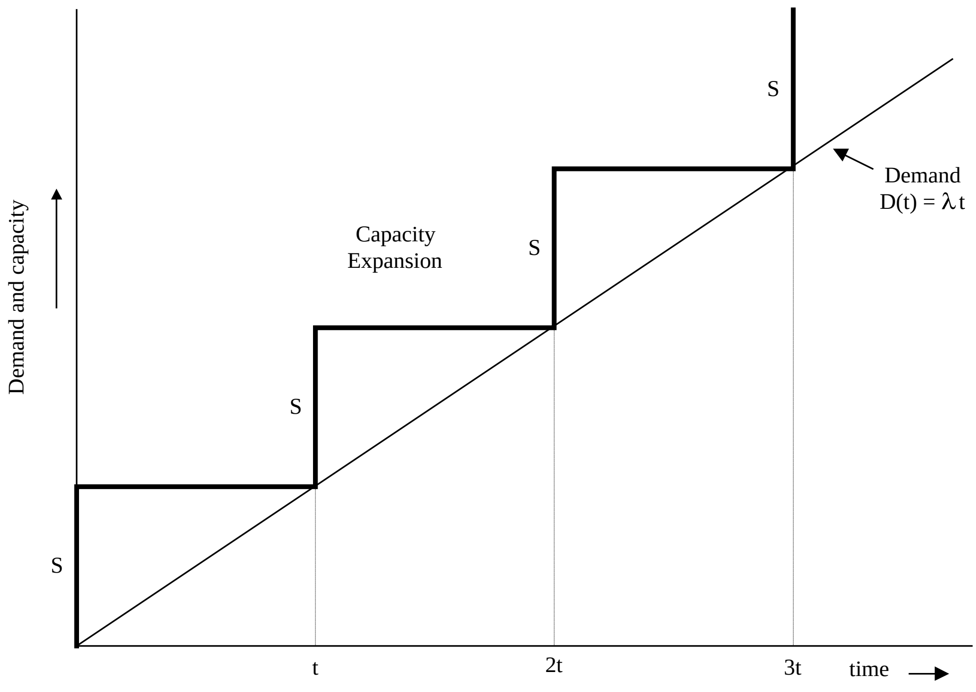


Figure 1
Capacity expansions to meet linearly growing demand

Figure 2 illustrates the variation of PW as a function of t according to the Eq (3).

$$PW = \frac{A + B\lambda \cdot t}{1 - e^{-rt}} = \frac{A + BS}{1 - e^{-rS/\lambda}} \qquad (3A)$$

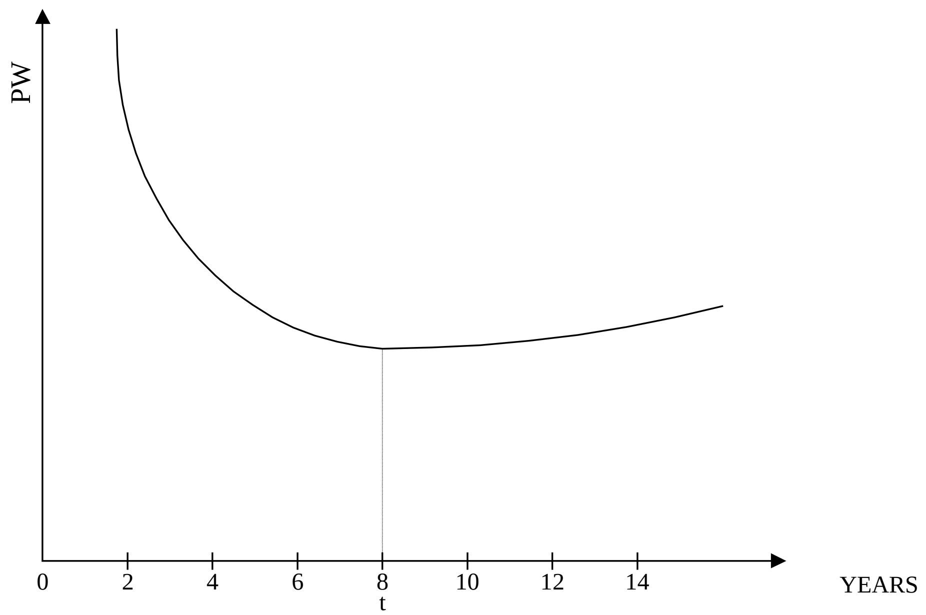


Figure 2

For PW, there is time t for which a minimum occurs. This time t is defined as the economic period of provision. The minimum of PW is determined by equating the first derivative to zero. Thus, we get:

$$e^{rt} - 1 = (t + t_o)r \qquad (4)$$

where $r = ln(1 + i)$ and $t_o = A / (B\lambda)$.

An approximate solution of the above-mentioned equation is given by

$$t = \frac{1}{r} \lambda n\left(1 + P + \sqrt{2P}\right) \qquad (5)$$

where $P = Ar / B\lambda$

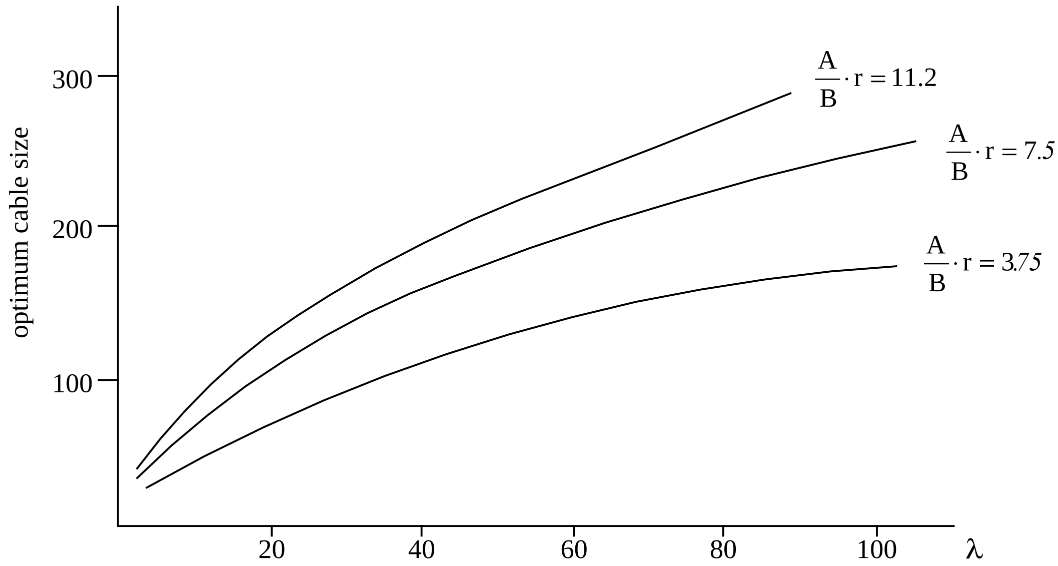
The optimum size of the expansion cable is

$$S = \frac{\lambda}{r} \lambda n\left(1 + P + \sqrt{2P}\right) \qquad (6)$$

The present worth of the plant at optimum size is found to be:

$$PW = \frac{B\lambda}{r} e^{rt} = \frac{B\lambda}{r} e^{rs/\lambda} \qquad (7)$$

Figure 3 illustrates the variation of the optimum size as a function of demand growth.



Example 1

The demand growth for telephone lines is 70 subs./year. The existing facilities have been completely used up. We must place a new cable to meet the demand. What is the optimum cable size, the provisioning time and the present worth of expenditures at optimum size?

The following data are given:

- Basic cost of cable $a_c = 70\,MU$
- Incremental cost of cable $b_c = 1 \cdot 6\ MU\ /\ pair$
- Digging cost $a_d = 500\,MU$
- Jointing cost $a_j = 50\,MU$
- Interest rate $i = 10\%$
- *pvf* for cable 1.223
- service life $30 \quad years$

The plant cost is given as

$$C(S) = A + BS$$

We have:

$$A = a_c\mu + \left(a_d + a_j\right)\left[1 + \frac{1}{\left((1+i)^T - 1\right)}\right] = 85.6 + 583 = 669\,MU$$

$$B = b_c\mu = 1.957\ MU\ /\ pair$$

$$r = \lambda n(1+i)$$

The optimum time for λ = 70 subs./year is evaluated as follows:

$$P = \frac{A \cdot r}{B\lambda} = \frac{669 \cdot 0.095}{1.957 \cdot 70} = 0.464$$

$$t = \frac{1}{r} \lambda n\left(1 + P + \sqrt{2P}\right) = 9.33 \quad years$$

The optimum size is found by:

$$S = \lambda\, t = 70 \cdot 9.33 = 643 \approx 700 \quad pairs$$

The present worth of expenditures of the plant for optimum size is given by:

$$PW = \frac{A + BS}{1 - e^{-rS/\lambda}}$$

We do not use Eq (7) by giving the present worth at optimum time because the calculation of optimum time was performed approximately. Eq (7) is sensitive to time, whereas Eq (3) is not. Thus we get:

$$PW = \frac{669 + 1.957 \cdot 700}{1 - e^{-0.095 \cdot 700/70}} = 3324\, MU$$

Consider now, in our example, that we pick an incorrect time double the provisioning period. In other words, the size of the cable is doubled. The present worth of the plant for doubled size is:

$$PW = \frac{A + B2S}{1 - e^{-r2S/\lambda}} = 4008\, MU$$

The percentage variation in PW, with respect to optimum size, is:

$$var\,iation = \frac{4008 - 3324}{3324} \cdot 100 = 20.5\%$$

The penalty is only 5 % for double size plant. This happens because the curve at optimum size is flat (see Figure 2). The smaller the coefficient *b*, the flatter the curve and the smaller the variation in *PW*. From the above reasoning, we come to the conclusion that the exact choice of optimum time is not critical.

Close to minimum value, however, the percentage of present worth may not always be an appropriate measure of the penalty for incorrect decisions. It may be more appropriate to first substract obviously “uncontrollable” components from the total. One such component is *b*-cost. Whatever replacement time is adopted, the cost pairs is unavoidable. This cost consists of infinite annuity with $\lambda \cdot B$ MU/year.

The present worth of this infinite annuity is given by:

$$PW_b = \lambda\, B / i$$

We also assume that there is some initial shortage implying that we must incur at least one basic cost:

$$PW_a = A$$

The uncontrollable cost is:

$$PW_a + PW_b = A + \lambda\, B / i = 2039$$

The cost that is subject to optimization is:

- for optimum time:

$$3324 - 2039 = 1285$$

- for double optimum time:

$$4008 - 2039 = 1969$$

The percentage variation in PW is now:

$$variation = \frac{1969 - 1285}{1285} \cdot 100 = 53\%$$

This percentage is considerable. Thus the cable sizing problem incurs economics.

Example 2

The demand growth is λ = 10 subs./year and the existing facilities are exhausted. There are two alternatives to meet the demand, either to lay a buried cable or to place an aerial cable. We have for:

buried cable

• service life	40 years
• maintenance plus operating cost	2 %
• basic cost	70
• digging + installation	550
• incremental cost	1.6 *MU/pair*

aerial cable

• service life	10 years
• maintenance plus operating cost	10 %
• basic cost	20 *MU*
• installation	280 *MU*
• incremental cost	2.0 *MU/pair*

An average interest rate of 10 % is accepted. Which alternative should be adopted? We evaluate the present worth of expenditures for each alternative.

Buried cable

- *Basic cost* A_B

  Provisioning cost $a_b = 70\,MU$

  Digging + installation $a_{di} = 550\,MU$

  Present value factor

$$\mu_B = 1 + \frac{1}{(1+0.1)^{40} - 1} + \frac{0.02}{0.1} = 1.223$$

$$A_B = \mu_B a_b + a_{di} \cdot \left(1 + \frac{1}{(1+i)^T - 1}\right) = 85.6 + 562.4 = 648\,MU$$

- *Incremental cost* B_B

$$B_B = \mu_B b = 1.223 \cdot 1.6 = 1.957\,MU\,/\,pair$$

- *Evaluation of provisioning period* t_B

$$P = \frac{A_B r}{B_B \lambda} = \frac{648 \cdot 0.095}{1.957 \cdot 10} = 3.15$$

$$t_B = \frac{1}{r} \lambda n\left(1 + P + \sqrt{2P}\right) = 19.95 \approx 20 \quad years$$

- *Evaluation of optimum capacity size*

$$S = \lambda \; t_B = 10 \cdot 20 = 200 \quad pairs$$

- *Evaluation of present worth*

$$PW_B = \frac{A_B + B_B S}{1 - e^{-rs/\lambda}} = \frac{648 + 1.96 \cdot 200}{1 - e^{-0.095 \cdot 200/10}} = 1223\,MU$$

<u>Aerial Cable</u>

- *Basic cost* A_A

Provisioning cost $a_a = 20\,MU$

Installation $a_i = 280\,MU$

Present value factor

$$\mu_a = 1 + \frac{1}{(1.1)^{10} - 1} + \frac{0.1}{0.1} = 2.627$$

$$A_A = a_a \mu_a + a_i \left(1 + \frac{1}{1.1^{10} - 1}\right) = 52.5 + 455.5 = 508\,MU$$

- *Incremental cost*

$$B_A = \mu_a b_a = 2.627 \cdot 2 = 5.25\,MU\,/\,pair$$

- *Evaluation of provisioning period* t_A

$$p = \frac{A_A \cdot r}{B_A \cdot \lambda} = \frac{508 \cdot 0.095}{5.25 \cdot 10} = 0.\;919$$

$$t_A = \frac{1}{r} \lambda n\left(1 + P + \sqrt{2P}\right) = 12.\;5 \quad years$$

- *Evaluation of optimum capacity expansion*

$$S_A = \lambda\, t_A = 10 \cdot 12.5 = 125 \Rightarrow 150 \quad pairs$$

- *Evaluation of present worth*

$$PW_A = \frac{A_A + B_A S_A}{1 - e^{-r s_A/\lambda}} = \frac{505 + 5.25 \cdot 150}{1 - e^{-0.095 \cdot 150/10}} = 1700\,MU$$

Comparing the *PW* of both alternatives, we easily find out that buried cable for the data used is more economical. The reason for this is that maintenance and service life are favourable for buried cables.

3. Sizing under an initial demand

We allow an initial jump in demand as shown in Figure 3 and Figure 4

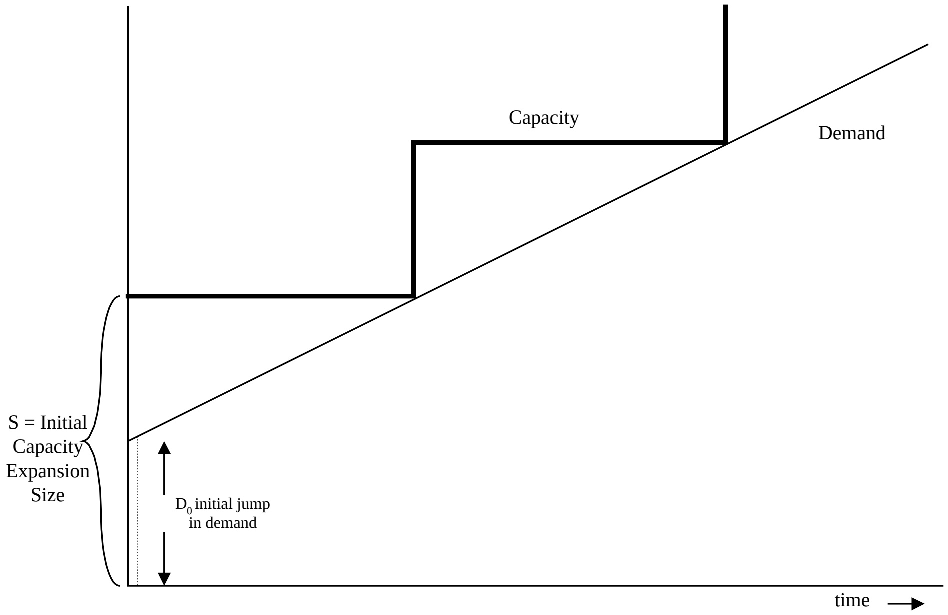


Figure 3
Positive initial demand

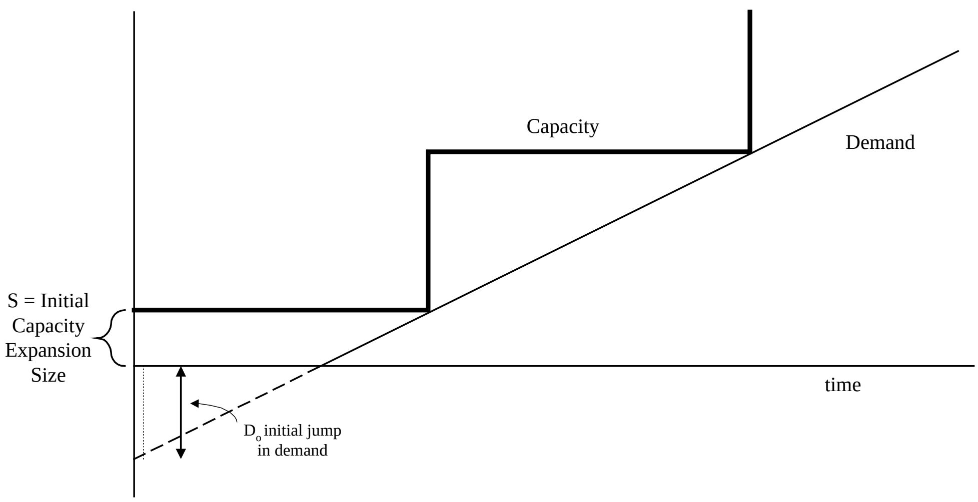


Figure 4
Negative initial demand

A positive value D_o might correspond to the initial jump in demand while a negative value reflects an early expansion forced by external factors, such as coordination with some another construction project. That is, in Figure 3, the expansion at 0 time actually should have taken place D_o / λ time units earlier, while in Figure 4 the expansion is assumed to be undertaken at time 0, although within the context of the model it really is not needed until $-D_o / \lambda$ time units later.

The demand can be written down as

$$D = D_o + \lambda t \qquad (8)$$

The expansion from the second one onwards can be considered to take place when demand reaches the capacity of facilities. Let W_F be the cost of an unlimited sequence of expansions. Figure 5 shows the cash-flow with the initial jump. W_F is equal to the present worth in the event of linear demand with 0 initial demand.

$$W_F = \frac{A + BS}{1 - e^{-rs/\lambda}} \qquad (?)$$

S is the capacity of the cable of the second expansion onwards:

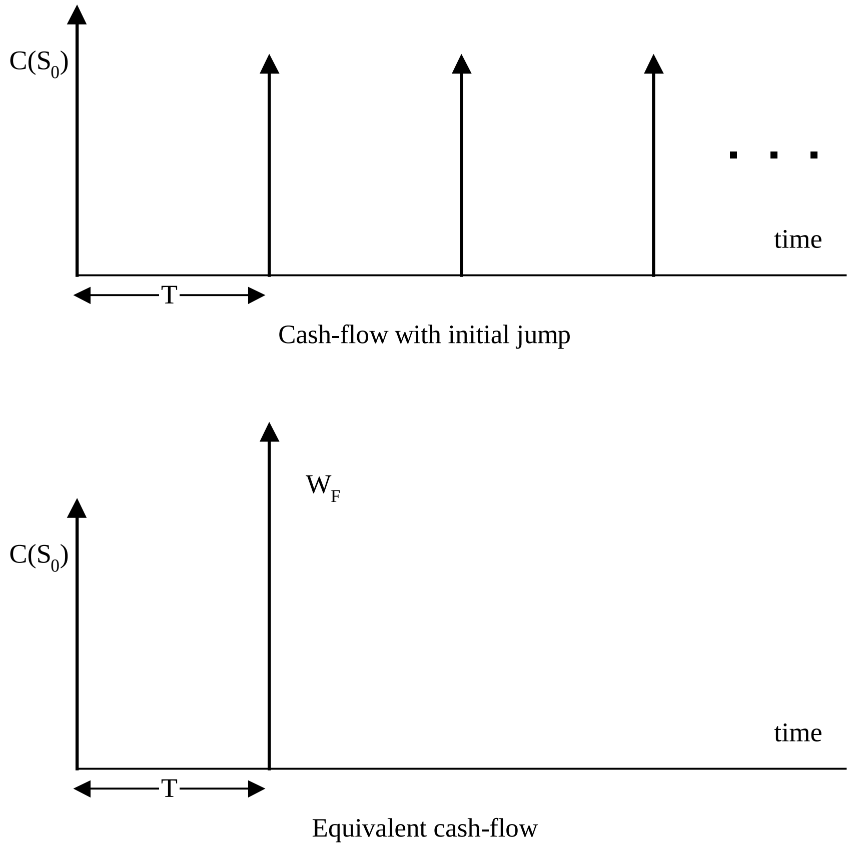


Figure 5

Looking at Figure 5, the initial capacity S_o is given as a function of the next expansion T

$$S_o = D_o + \lambda T \qquad (9)$$

The present worth W of the above-mentioned expansions is

$$W = C(S_o) + W_F\, e^{-rT} = C(S_o) + W_F\, e^{-r(S_o - D_o)/\lambda} \qquad (10)$$

$C(S_o)$ is the cost of the first expansion. In the event of $C(S_o)$ being a linear function of S_o,

$$C(S_o) = A + BS_o \qquad (11)$$

Eq (10) can be written down as

$$W = A + BS_o + W_F \; e^{-r(S_o - D_o)/\lambda}$$

The minimum of the above-mentioned expression, with respect to *y*, is easily calculated

$$W = B + b(Y + D_o) + W_F \; e^{-ry/\lambda} \qquad (12)$$

Since W_F represents the unlimited expansions at 0 initial demand, it gives the optimal capacity *S* when the initial demand is zero. So the initial capacity is:

$$Y = \frac{\lambda}{r} \lambda n \frac{rW_F}{b\lambda} \qquad (13)$$

The optimal size is just what the size would have been without the jump plus sufficient capacity to satisfy the jump. Of course, if D is negative (Figure 4), Eq (9) may yield a negative amount of capacity. In this case it is not economical to install capacity at time 0, even if the only cost of that capacity is the incremental or *B* cost.

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