Exchange Locations and Boundaries

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1 ASSUMPTIONS

As stated in Chapter 0, the problem to be solved is how to extend a given network, over a certain period of time, for specified demands regarding subscriber and traffic development, using certain types of exchange and transmission equipment, observing specifications concerning quality of service, in the most economic way.

Real telecommunication networks are rather complex, and it would be very difficult to use mathematical methods for finding exact solutions to the various planning tasks involved. Also, it is essential to find methods to deal with any kind of network, rather than with a particular one. To make this possible, one has to make a *model network*, an abstraction of the real network, expressing the relations between the various entities in mathematical terms.

In designing a model, the question arises how close to reality such a model should be. Simpler models will usually lead to simpler, and therefore faster, solutions, but also to a certain loss of accuracy in the results. A reasonable compromise has thus to be found between accuracy of results, and speed of calculation. It should also be remembered, that for different types of networks the impact of the complexity of the model on the accuracy of the results can vary considerably.

For all sorts of *equipment* this task is relatively simple. The cost structure of any particular part of an exchange or transmission system, and the technical properties of same are available from administration or manufacturer. The structure and properties of the various types of equipment are, moreover, independent of the network under investigation, although the actual values for costs involved may vary considerably from one network to another.

The same is valid for the various quality of service considerations.

More problematic are the models concerning the *subscriber distribution*, and the *traffic interests* in the network.

1.1 SUBSCRIBERS' NETWORK

For larger networks it is obviously impractical to define the location of every subscriber individually. Although the locations of the existing subscribers are known, the forecasts, being made for the entire population of a city, or for subsets of that population, would be meaningless for defining the location of individual subscribers.

The subscriber distribution can therefore be defined in one of the following ways :

1 Nodes

Here subscriber density is defined in *discrete points*, usually corresponding to DP's or cabinets. This approach is often used in sparsely populated areas, such as *rural areas*, or the outskirts of metropolitan areas. Each node is defined by it's coordinates, and the subscriber forecasts for the points of time to be considered.

2 Rectangular grid

For more densely populated areas, a rectangular grid is placed over a map of the area under consideration, and the forecasts then define the number of subscribers in each grid element. Within a grid element, the subscribers are then assumed to be *evenly distributed.*

The size of the grid element should be chosen according to local conditions, typically ranging from 100 - 500 meters.

3 Arbitrary areas

Instead of a grid, forecasts can be defined for arbitrary polygons, i.e. areas enclosed by a sequence of straight lines. These areas correspond usually to blocks of houses, cabinet areas, industrial complexes, etc. Again, the subscribers are assumed to be evenly distributed within each such area.

Which of the above methods to choose depends on the type of network, and the data available; it is also possible to use a *combination* of these for any given network.

As regards the exchange area boundaries, the way of defining the subscriber distribution has the following effects:

Nodes : each node is assigned to one exchange;

Grid : any grid element is assigned to one exchange;

Areas : any area is assigned to one exchange.

For "grid" and "areas" it is also possible to split an entity between a number of exchanges; this cannot be done for "nodes".

1.2 EXCHANGES

The model concerning exchanges has three main aspects, i.e.

- *exchange configuration*, depending on subscribers, circuits, and traffics;
- *floor space requirements* for a given configuration;
- *line terminal equipment*, depending on the transmission medium used, and the type of exchange on either end of the line.

It is possible to define various exchange types to be investigated in the course of optimising and/or dimensioning the network. What has to be defined are capacities and costs of equipment, and the possibilities of interacting with other equipment, switching or transmission.

Exchange configuration

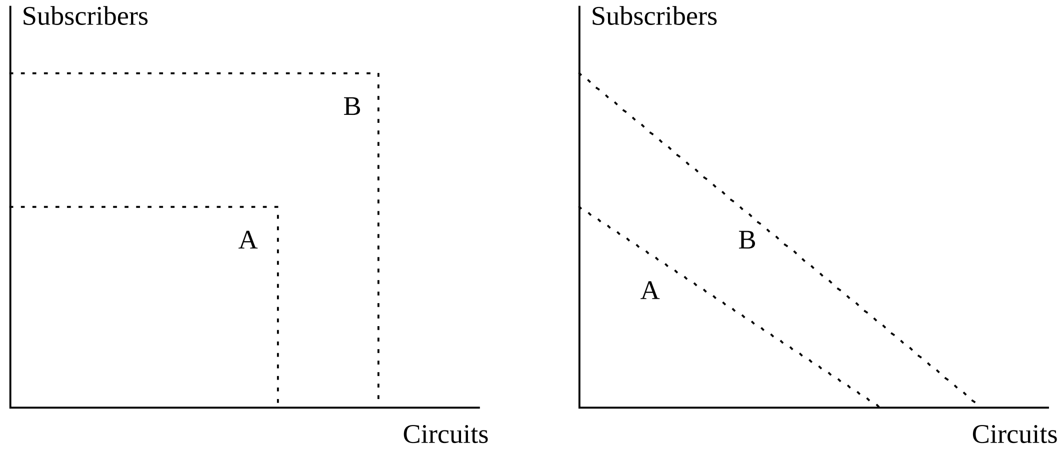
An exchange has a certain *maximum capacity*, expressed in

- number of *subscribers* that can be connected;
- number of *circuits* terminating;
- amount of *traffic* that can be handled;
- number of *call attempts* per hour;

or a combination of these.

At present the program takes into account the first two of these only; as the number of terminating circuits is directly related to the traffic, the third point is indirectly taken into consideration. The call attempts pose a more difficult problem, as there is a strong dependence on *actual network performance*, and are therefore not considered for the time being.

For a given exchange type, 2 ways of defining these capacities are recognized by the program, i.e. independent and dependent subscriber and circuit capacities :



A and B refer to different sizes of a given exchange type.

The cost for a given exchange configuration is defined in the following terms :

- cost of equipment for *one subscriber*;
- cost for equipment common for a *group of subscribers*;
- the size of that group;
- cost of equipment common for *complete exchange unit*.

Line terminal equipment

A circuit connecting 2 exchanges consists of the line itself, and terminal equipment in both end points. The type and cost of this terminal equipment depends on the type of exchange at either end, and on the transmission medium used. It has been found convenient to describe these costs in the form of tables, one table defining the terminal equipment costs between all types of exchanges, for a given group of transmission media.

Example :

Exchange types	Transmission media groups
S x S	.6 .8 .9 .9L cable
X - bar	PCM
Digital	

The line terminal equipment cost specification would then consist of two tables, one for cable and one for PCM, each table having 3 x 3 cost values.

The terminal equipment at each side comprises usually several items. Sometimes it is not easy to draw a clear demarcation line between exchange and transmission equipment, especially for electronic exchanges. This does not actually make any difference to the task of optimising and dimensioning the network, provided all cost items involved are included somewhere.

1.3 TRAFFICS

As in the case of the subscriber distribution, it makes no sense, from a statistical point of view, to make assumptions regarding traffic volume and dispersion for *individual* subscribers. On the other hand, the model should recognize differences in traffic behaviour for the various categories, such as residential, business, PBXs, etc. The traffic forecasts are usually easier to make for such categories than for the whole population, as categories will react differently to such environmental changes as provision of new services, or alterations in the tariff policy.

Local networks

For the cases of rural, urban and metropolitan areas it has been found convenient to subdivide the total network area into a number of so-called *traffic areas*. The traffic properties for all subscribers in such an area are assumed to be uniform; in defining such areas attention has to be paid to the present and future “mix” of categories, and the possibilities of making traffic measurements to obtain the necessary “raw data” for the traffic forecasting process.

The traffics between such traffic areas can then be described in matrix form. As the exchange area boundaries will usually not coincide with the traffic area boundaries, traffic between exchanges will then be found by simple calculations involving the subscribers per exchange per traffic area.

Traffic zones and subscribers

For local networks, traffics calculations are based on subscribers/exchange, subscriber’s categories and traffic zones

Assumptions :

- the area under consideration has been divided into traffic zones; the subscribers belonging to such a zone are assumed to have uniform traffic properties, such as traffic originated and terminated per subscriber, and traffic dispersion to other zones;

- the number of subscribers of any such zone, *T*, are known for any given exchange, *E*; they have been defined in the input data, or calculated in the previous boundary optimization: *NSUB(E, T)*;

- the total number of subscribers belonging to any traffic zone, *T*, is known; this has been calculated after reading the input data concerning *zone definition* and *subscriber distribution* : $SUBTZ_T$

- the total traffic from any traffic zone, *T*, to any other traffic zone, *U*, is known from input data : A_{TU}

The specific traffic interest between one subscriber in traffic zone *T* and one subscriber in traffic zone *U* can then be expressed as

$$a(T,U) = \frac{A_{TU}}{SUBTZ_T \cdot SUBTZ_U}$$

Finally, the traffic from any exchange, E, to any other exchange, F, can now be written as

$$Traffic(E,F) = \sum_{T,U} NSUB(E,T) \cdot NSUB(F,U) \cdot a(T,U)$$

1.4 COST STRUCTURE

Cost subscriber - exchange

The cost of connecting a subscriber to an exchange can be represented

$$D_E \cdot C_s(D_E) + C_f$$

where

D_E is distance subscriber - exchange,
Cs is distance - dependent transmission media cost,
Cf is distance - independent transmission media cost.

Cost exchange - exchange

The cost of a circuit exchange to exchange can be represented as

$$D_{EF} \cdot C_c(D_E) + C_d$$

where

D_{EF} is distance from exchange E to exchange F,
Cc is distance - dependent transmission media cost,
Cd is distance - independent transmission media cost.

When using different transmission systems, the choice is made on the number of circuits between E and F. In this case a separate optimization is carried out.

Additional parameter of a transmission system is the capacity, i.e. the maximum number of circuits which could be carried over.

EXCHANGE COST

Exchange cost consists of two components:

- cost of exchange equipment
- cost of building

Both components are assumed to be a function of the subscribers, incoming and outgoing circuits.

For a given exchange, E, we denote the exchange equipment cost with $Ca(E)$ and the building cost with $Cb(E)$.

Thus, the *total network cost* function, C, could be expressed as

$$C = \sum_{E=1}^{NEX} \sum_{(i,j)\in E} sub(i,j) \cdot \left[C_s(D_E) \cdot D_E + C_f \right] + \sum_{E=1}^{NEX} \left[C_a(E) + C_b(E) \right] +$$
$$+ \sum_{E=1}^{NEX} \sum_{F=1}^{NEX} N_{EF} \cdot \left[C_c(D_{EF}) \cdot D_{EF} + C_d \right]$$

where

NEX is the number of exchanges,

N_{EF} is the number of circuits from exchange E to exchange F.

2 BOUNDARY OPTIMIZATION

Boundary optimization, i.e. finding exchange area boundaries in such a way that total network costs are minimized, is based on the following assumptions :

- exchange locations are fixed (temporarily) ;

- the junction network cost of any subscriber, of a given traffic zone, *K*, belonging to a given exchange, *E*, is known (it will have been calculated in the previous iteration) : $C_j(K,E)$

- the average cost, per subscriber, of exchange and building, is known for any given exchange, *E* : $C_b(E)$

- the cost of connecting a subscriber to any exchange can be calculated from
  - the distance subscriber to exchange, D_E
  - the transmission plan
  - the available transmission media costs,

  and can be written as $D \quad \cdot C \;(D \quad) + C$

The cost of connecting a subscriber at location *(x,y)*, belonging to traffic zone *K*, to an exchange *E* at $\left(X_E, Y_E\right)$ can thus be expressed as

$$C(E) = C_j(K,E) + C_b(E) + D_E \cdot C_s(D_E) + C_f \qquad (2.2.1)$$

where $D_E = D(x, y, X_E, Y_E)$

We now have 2 possibilities of finding the optimal exchange boundaries, depending on one more assumption, i.e. whether a grid element can be split between exchanges or not.

- **The grid element *can not* be split :**

In this case the entire grid element in question will be assigned to one exchange. The decision to which exchange, *E*, a given subscriber grid element should belong can be made simply by comparison. *E* should be chosen so that *C(E)* is minimized.

So, for every grid element *(i,j)* the value *C(E)* is calculated for every exchange *E* , and the lowest *C(E)* then determines *E*. The only remaining problem is to find the distance from the exchange in $\left(X_E, Y_E\right)$ to the grid element *(i,j)*. See Chapter 2.1 “Distance Calculation” for details.

NOTE : For this method a careful decision has to be made regarding the size of the grid elements. Too large, and the accuracy of the boundaries will suffer; too small, and there may be problems with storage space and calculation time.

- **The grid element *can* be split :**

In this case any grid element may belong entirely to one exchange, or it may be split between to or more exchanges. Before discussing the way of finding the optimal boundaries for the whole network area under consideration, let us consider the boundary between to exchanges, *E* and *F*. Obviously the boundary should be devised so that along the boundary the values of *C(E)* and *C(F)* are equal.

Using formula (2.2.1) we find that

$$C_j(K,E)+C_b(E)+D_E \cdot C_s(D_E)+C_f(E)=C_j(K,F)+C_b(F)+D_F \cdot C_s(D_F)+C_f(F) \qquad (2.2.2)$$

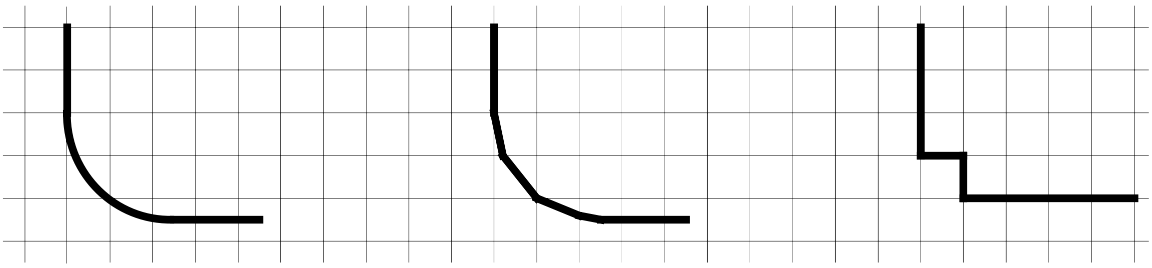
or, simplified,

$$b(E)+D_E \cdot c(E)=b(F)+D_F \cdot c(F)$$

Now, depending on the relation between the values *b(E)* and *b(F)*, and *c(E)* and *c(F)*, the boundary between *E* and *F* is one of the following geometrical figures :

$c(E)=c(F)$	$b(E)=b(F)$	: straight line
	$b(E)\neq b(F)$	: hyperbola
$c(E)\neq c(F)$	$b(E)=b(F)$	: circle
	$b(E)\neq b(F)$	: closed figure of 4^{th} order

These boundary functions are, except for the first one, not very suitable for subsequent calculations, i.e. optimization of exchange locations and boundaries. Therefore any boundary will be represented as a sequence of straight line segments, approximating the exact boundary. The figure below shows a comparison between the exact boundary, and the two methods shown above :



Exact figure Straight lines Grid elements

Comparison between boundary definitions

As the traffic zones and subscriber distribution are still defined on the grid elements, a reasonable approach is to find the boundaries for every row of the grid, i.e. finding the intersection of boundary functions and the upper and lower lines separating this row from its neighbours.

For any pair of exchanges, *E* and *F*, and a given line $y = y_0$, formula (2.2.2) can be used to find the boundary point with any desired accuracy. Corresponding points on successive rows can then be connected to form the boundary sequence.

Simplified methods for boundary optimization

If no information about the distribution of the subscribers by exchanges and the junction network is available (when initial boundaries are defined) or we want to speed up the calculations (when tentative exchanges are investigated), a suitable method is to disregard the influence of exchange, building and junction network costs, i.e. to disregard the first two terms in (2.1).

So, we obtain

$$C(E)=D_E \cdot C_s(D_E)+C_f \qquad (2.3)$$

Moreover, we can disregard the different transmission media costs in the subscriber network, i.e. to make a purely geographical division into exchange areas. In this case the boundaries will be placed equidistant to the adjacent exchanges.

Then, if subscriber distribution on grid is used and the grid element cannot be split, the boundary lines are rounded to the nearest grid element.

Besides, we have to treat fixed exchange boundaries, i.e. predefined exchange areas which cannot be changed and have to be excluded from the optimization process.

Transit exchange boundaries

A special case which could be considered as exchange boundary optimization, is to define the exchanges connected to a higher level exchange, i.e. to define the superior transit or tandem exchange for every exchange.

The cost $C(E)$ of connecting a circuit from an exchange at location (x, y) to an exchange E at (X_E , Y_E) can be expressed as in (2.1).

The decision to which exchange, E, a given exchange should belong can be made by comparison. E should be chosen so that $C(E)$ is minimized. Here E is a higher level transit exchange.

As the influence of the transmission media cost is the most significant in the expression for $C(E)$ we could use the simplified methods for boundary optimization, mentioned above, i.e. to choose the superior exchange so that the circuit cost or the distance is minimum.

A practical approach is to give additional possibility for predefined superior exchanges or superior exchanges obtained in interactive mode.

The distance from any exchange to the superior exchange is calculated as

- the distance $D(x, y, X_E , Y_E)$ along the hypotenuse or the cathetie from exchange in (x, y) to the superior exchange in (X_E, Y_E)
- the shortest path $D(i, j)$ from an exchange in node i to the superior exchange in node j, for network model with nodes and links connecting these nodes.

2.1 DISTANCE CALCULATION

Distance exchange to exchange/node

For networks with nodes and links connecting these nodes, the distances for the links are defined as data, and the distances between any two points are then a result of the shortest path between them.

Different algorithms exist for obtaining the shortest path between two nodes.

The algorithm described here finds the shortest path from any node, i, to all other nodes in the network for given arc lengths; to find the most economic paths, the cost per arc is given instead.

Given :	N	=	number of nodes in the network
	L	=	number of links in the network
	d_{ij}	=	length/cost of arc between nodes i and j, if the link exists.

Find : Shortest path from node, i, to all other nodes.

Method :

Step 1:	Set D_j	=	infinite	for j = 1, 2, ...N
	Set $LAST_j$	=	0	for j = 1, 2, ...N

Step 2: Set D_i = 0

Set D_j = d_{ij} if the link i to j exists; in this case also

set $LAST_j$ = i

Step 3: For any node, K, where $LAST_K$ has been changed in the previous iteration, perform the following :

- find all links having K as an end point;
- for each such link, investigate for the other end point, M;

  if $D_K + d_{K,M} < D_M$
  then the path i to M passes through node K, and therefore set $D_M = D_K + d_{K,M}$
  $LAST_M = K$

- having investigated all such points, K, the iteration is done. If no resetting of D and $LAST$ has occurred during this iteration, proceed to Step 4. Otherwise, start a new iteration according to Step 3.

Step 4: All shortest paths from node i have now been established.

D_j now contains the shortest distance from i to j.

$LAST_j$ contains the last-but-one node on the path i to j.
The path from i to any node can then be reconstructed using the intermediate nodes stored in $LAST$.

The complete path will then be

$$i \rightarrow K(n-1) \rightarrow K(n-2) \rightarrow \Lambda \rightarrow K(2) \rightarrow K(1) \rightarrow j$$

where

$K(1) = LAST_j$

$K(2) = LAST_{K(1)}$

. . .

$K(n-1) = LAST_{K(n-2)}$

$K(n) = LAST_{K(n-1)}$ *which is equal to i.*

For other network models the coordinates of the exchanges/nodes are specified, and the distance between an exchange in (X_E, Y_E) and an exchange/node in (x,y) is defined as

$$D(X_E, Y_E, x, y) = \sqrt{(X_E - x)^2 \cdot L_x^2 + (Y_E - y)^2 \cdot L_y^2} \qquad (2.4)$$

if measured along the hypotenuse

or

$$D(X_E, Y_E, x, y) = |X_E - x| \cdot L_x + |Y_E - y| \cdot L_y \qquad (2.5)$$

if measured along the cathetie,

where L_x and L_y are X- and Y- dimensions of the grid element.

Mean distance from exchange to grid element

For subscribers defined in grid elements a suitable way is to find the *mean distance* from a point, representing an exchange or concentrator, to a subscriber in a certain grid element.

For rectangular grid elements the distance from the exchange in (X_E, Y_E) to an arbitrary point (x,y) is defined as

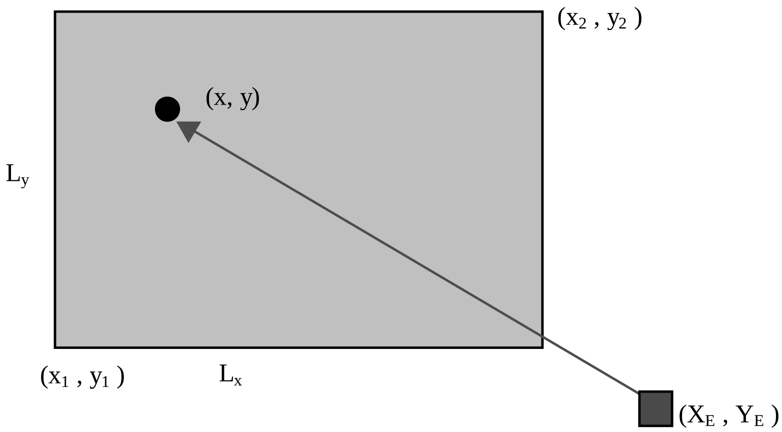
$$D(X_E, Y_E, x, y) = \sqrt{(X_E - x)\ \ \cdot L_x + (Y_E - y)\ \ \cdot L_y}$$

along the hypotenuse

$$D(X_E, Y_E, x, y) = |X_E - x| \cdot L_x + |Y_E - y| \cdot L_y$$

along the cathetie,

where L_x and L_y are the lengths of the side of the rectangle.



The *mean distance* from (X_E, Y_E) to the rectangle can then be found from the double integral

$$D = \frac{1}{area} \int_{x_1}^{x_2} \int_{y_1}^{y_2} d\,(X_E, Y_E, x, y)\, dx\, dy \qquad (2.6)$$

where (x_1, y_1) and (x_2, y_2) are opposite corners of the rectangle,

and $area = (x_2 - x_1) \cdot (y_2 - y_1)$ is the area of the rectangle.

HYPOTENUSE METHOD

The primitive function of D is

$$F(z,u) = 2 \cdot z \cdot u \cdot s + z^3 \cdot log\left(\frac{u+s}{z}\right) + u^3 \cdot log\left(\frac{z+s}{u}\right)$$

where $s = \sqrt{z^2 + u^2}$

We will then get the distance, D, by inserting the integration limits for z and u, thus arriving at

$$D(X_E, Y_E, x_1, y_1, x_2, y_2) = \frac{G(x_2, y_2) - G(x_2, y_1) - G(x_1, y_2) + G(x_1, y_1)}{6 \cdot L_x \cdot L_y} \qquad (2.7)$$

where $G(z,u) = 2 \cdot z \cdot u \cdot s + z^3 \cdot log(u+s) + u^3 \cdot log(z+s)$

The terms containing u^3 and z^3 will cancel out when inserting the integration limits.

CATHETIE METHOD

Formula (2.6) leads to

$$D = \frac{1}{x_2 - x_1} \cdot \int_{x_1}^{x_2} |X_E - x| \cdot L_x \, dx + \frac{1}{y_2 - y_1} \cdot \int_{y_1}^{y_2} |Y_E - y| \cdot L_y \, dy \qquad (2.8)$$

and

$$\frac{1}{x_2 - x_1} \cdot \int_{x_1}^{x_2} |X_E - x| \cdot L_x \, dx = \begin{cases} D_x \cdot L_x & \text{if} \quad D_x \geq 0.5 \\ (D_x^2 + 0.25) \cdot L_x & \text{if} \quad D_x < 0.5 \end{cases}$$

where $D_x = |X_E - x_2 + 0.5|$

$$\frac{1}{y_2 - y_1} \cdot \int_{y_1}^{y_2} |Y_E - y| \cdot L_y \, dy = \begin{cases} D_y \cdot L_y & \text{if} \quad D_y \geq 0.5 \\ (D_y^2 + 0.25) \cdot L_y & \text{if} \quad D_y < 0.5 \end{cases}$$

where $D_y = |Y_E - y_2 + 0.5|$

2.2 JUNCTION NETWORK COST PER SUBSCRIBER

The junction network cost of any subscriber, of a given traffic zone *K* belonging to a given exchange, *E*, is

$$C_j(K,E) = \sum_F N_{EF} \cdot C_c(E,F) + \sum_F N_{FE} \cdot C_c(F,E)$$

where

N is the difference in the circuits, caused by one subscriber

C_C is cost of one circuit; it depends on the cost of the available transmission media and the distance from exchange to exchange.

N could be expressed as

$$N \cong A \cdot \partial N / \partial A$$

$\partial N / \partial A$ is the efficiency of the route; it is calculated when dimensioning or optimising the number of the circuits in a route; a suitable initial value is 1.2 .

Thus the following equation is obtained

$$C_j(K,E) = \sum_F A_{EF} \cdot \partial N / \partial A_{EF} \cdot C_c(E,F) + \sum_F A_{FE} \cdot \partial N / \partial A_{FE} \cdot C_c(F,E)$$

A_{EF} = $NSUB(F,T) \cdot a(K,T)$

A_{FE} = $NSUB(F,T) \cdot a(T,K)$

$NSUB(F,T)$ = number of subscribers for exchange *F*, of traffic zone *T*

$a(K,T)$ = specific traffic interest between one subscriber in traffic zone *K* and one subscriber in traffic zone *T* .

3 EXCHANGE LOCATIONS

Here we shall deal with methods for definition of optimum exchange locations, i.e. finding exchange locations in such a way as to minimize the total network cost.

The exchange area boundaries are considered to be *temporarily fixed*.

Initial exchange locations are also available (from previous iterations). Some of the methods, described here, will improve these locations.

Subscriber distribution on grid

For any given exchange, E, the theoretically optimal location (X_E, Y_E) has the property that the partial derivatives of the total network cost function, C, with regard to X_E and Y_E are equal to zero. As C is dependent on <u>all</u> exchange coordinates, and we want to find the overall minimum of C, we must find a set of exchange coordinates (X_E, Y_E) for E = 1,2,..., so that

$$\left.\begin{aligned} \partial C/\partial X_E &= 0 \\ \partial C/\partial Y_E &= 0 \end{aligned}\right\} \quad \text{for } E = 1,2,\ldots NEX$$

So, for X_E we get

$$\begin{aligned} \partial C/\partial X_E = & \sum_{(i,j)\in E} \left[sub(i,j)\cdot C_c(D_E)\cdot \partial D_E/\partial X_E \right] + \\ & + \sum_{F\neq E} \left[N_{EF}\cdot C_c(E,F) + N_{FE}\cdot C_c(F,E) \right]\cdot \partial D_{EF}/\partial X_E \end{aligned} \tag{3.1}$$

where D_E denotes the mean distance of exchange E from the grid element *(i,j)*, and D_{EF} denotes the distance from exchange E to exchange F. Cc denotes cost per circuit.

A similar expression for Y_E gives $\partial C/\partial Y_E$.

Different methods for solving this 2*E equation system will be employed to cope with the two methods of measuring the distances described in Chapter 2, Exchange boundaries.

HYPOTHENUSE METHOD

The method of calculating the partial derivatives $\partial D_E/\partial X_E$, $\partial D_E/\partial Y_E$, $\partial D_{EF}/\partial X_E$ and $\partial D_{EF}/\partial Y_E$ is described in Appendix 3.1.

Inserting the derivatives in (3.1) we get a system of 2**NEX* non-linear equations, which is not so pleasant to solve. We can, however, expand $\partial C/\partial X_E$ and $\partial C/\partial Y_E$ into a Taylor-series, and truncate after the first-order derivatives. We then get

$$\left.\begin{aligned} \partial C/\partial X_E + \sum_{F=1}^{NEX} \left[\frac{\partial^2 C}{\partial X_E\,\partial X_F}\cdot \Delta X_F + \frac{\partial^2 C}{\partial X_E\,\partial Y_F}\cdot \Delta Y_F \right] = 0 \\ \partial C/\partial Y_E + \sum_{F=1}^{NEX} \left[\frac{\partial^2 C}{\partial Y_E\,\partial X_F}\cdot \Delta X_F + \frac{\partial^2 C}{\partial Y_E\,\partial Y_F}\cdot \Delta Y_F \right] = 0 \end{aligned}\right\} \quad E = 1,2,\ldots, NEX$$

This is now a system of 2**NEX* linear equations in ΔX_F and ΔY_F , Δ denoting improvement, which can easily be solved by standard methods.

The fact that the coefficients on the diagonal of the matrix are large compared to the other coefficients facilitates this process.

The following *iterative method* can then be used to simultaneously improve all exchange locations :

Step 1: Start with the present set of (X_E, Y_E)

Step 2: Calculate derivatives and coefficients (see Appendix 3.1)

Step 3: Solve the system of linear equations, i.e. find all (ΔX_E , ΔY_E) (See Appendix 3.2)

Step 4: Reset the location for any exchange, *E*, to ($X_E + \Delta X_E$, $Y_E + \Delta Y_E$)

Step 5: Repeat from Step 2 until *max* (ΔX_E , ΔY_E) is smaller than a predefined value.

Observe that above mentioned method may

- lead to a *local*, not *global* optimum, if the initial locations are not good enough;
- oscillate depending on changes in the transmission media costs; this can, however, be easily detected and treated.

CATHETIE METHOD

The method of calculating the partial derivatives of the distance is described in Appendix 3.1.

Inserting the derivatives in (3.1) we get :

$$\partial C / \partial X_E = \sum_{(i,j)\in E} sub(i,j) \cdot C_c(D_E) \cdot \begin{cases} -L_x & \text{if } j-1 \geq X_E \\ 2\cdot(X_E - j + 0.5)\cdot L_x & \text{if } j-1 < X_E < j \\ L_x & \text{if } \quad X_E \geq j \end{cases} + \qquad (3.2)$$

$$+ \sum_{F \neq E} \left[N_{EF} \cdot C_c(E,F) + N_{FE} \cdot C_c(F,E) \right] \cdot \begin{cases} -L_x & \text{if } X_E < X_F \\ L_x & \text{if } X_E > X_F \end{cases}$$

and similar expression for $\partial C / \partial Y_E$.

The following method can then be used to find the optimum location of every exchange *E* :

Step 1: Find column (row) *K* for the exchange.

Define

$$S_j = \sum_i sub(i,j) \cdot C_s(D_E) \quad for\ (i,j) \in E \quad + \qquad (3.3)$$

$$\sum_F \left[N_{EF} \cdot C_c(E,F) + N_{FE} \cdot C_c(F,E) \right] \quad for\ j-1 \leq X_F < j$$

and $S = \sum_j S_j$

Then column (row) K is found where

$$\sum_{j=}^{K-} S_j < S\!\!\diagup_{2} \text{ and } \sum_{j=1}^{K} S_j \geq S\!\!\diagup_{2} \ .$$

Step 2 : Find exact location X_E (Y_E) in column (row) K.

Define

$$A = \sum_{j<K} S_j \qquad\qquad B = \sum_{j>K} S_j$$

$$G = \sum P \qquad for\ K - 1 \leq X \quad < X \quad , X \quad < K$$

$$H = \sum P \qquad for \quad X \quad \leq X \quad < K \quad , X \quad > K - 1$$

where $P \quad = N \quad \cdot C_c(E, F) + N \quad \cdot C_c(F, E)$

$$T_K = \sum_i sub(i, K) \cdot C_s(D_E) \qquad for\ (i, K) \in E$$

Then the exact location X_E is

$$X \quad = \left[K - 0.5 + \frac{B - A + H - G}{2 \cdot T_K} \right] \cdot L_x \tag{3.4}$$

Thus, the partial derivatives $\partial C / \partial X_E$ and $\partial C / \partial Y_E$ become zero if the exchange is located at the intersection of the X- and Y- medians with respect to the number of subscribers and junction circuits weighted by the distance-dependent transmission media costs.

As the hypothenuse method, the above-mentioned method may

- lead to a local, not global optimimum, if the initial locations are not good enough;
- oscillate depending on the changes in the transmission media costs.

Simplified methods for exchange location optimization

To speed up the calculations (when tentative exchanges are investigated), a suitable decision is to use the cathetie method for exchange locations optimization, described above, but to disregard the influence of the junction network.

Thus, for S_j in (3.3) we get

$$S_j = \sum_i sub(i, j) \cdot C_s(D_E) \qquad for\ (i, j) \in E$$

and for X_E in (3.4) we have

$$X_E = \left[K - 0.5 + \frac{B - A}{2 \cdot T_K} \right] \cdot L_x$$

or

$$X_E = \left[K + \left(\frac{S}{2} - \sum_{j=1}^{K} S_j \right) \Big/ T_K \right] \cdot L_x \qquad (3.5)$$

Then, we shall follow the same steps : Step 1 to find the column (row) *K* for the exchange, and Step 2 to find the exact location X (Y) .

Moreover, we can disreguard the different transmission media costs in the subscriber network and find the center of gravity, i.e. the point where there is a balance of subscribers to the lef and the right, and above and below.

We shall use the same method, for finding first the column (row) and then the exact location, bit S_j from (3.3) is now :

$$S_j = \sum_i sub(i, j) \qquad for\ (i, j) \in E$$

and X_E from (3.5) is :

$$X_E = \left[K + \left(\frac{S}{2} - \sum_{j=1}^{K} S_j \right) \Big/ S_K \right] \cdot L_x$$

As next simplification, we could approximate to one grid element location, i.e. to find the column ant the row of the exchange location using only Step 1 in the above method.

Besides, we have to treat fixed exchange locations, i.e. predefined exchange locations which cannot be changed and have to be excluded from the optimization process.

Subscribers and exchanges in nodes

If the network model is presented with nodes and links connecting these nodes to the local network cost function, *C*, is a discrete function over all node locations, i.e. it is not possible to use partial derivatives of *C*.

One possibility is to calculate the total network cost, *C*, for all combinations of exchange locations and to find the smallest *C = Cmin* . The exchange locations for *Cmin* are the optimal.

It is obvious that it is not possible to use such a method in practice, except for some very small networks.

Moreover, it is pointless to investigate many of the combinations of exchange locations.

Two ways of solving the problem are possible :

- to eliminate the obvious senseless combinations and to investigate the rest; there will still be too many left;

- to investigate some of the combinations, which could give the optimum exchange locations; an expert decision is recommended for each such combination; using a computer, the interactive mode of operation is the most convenient, i.e. the expert will decide on the exchange locations for each combination and the computer program will dimension the network and will calculate the total network cost *C*.

APPENDIX 3.1

CALCULATION OF DERIVATIVES

Partial derivatives of the distance have to be calculated for optimization of exchange locations when using subscriber distribution on grid.

Distance from exchange to exchange/node

HYPOTENUSE METHOD

The partial derivatives necessary for the expression of the distance measured along the hypotenuse (see (2.4)) are :

$$\partial D / \partial X_E = L_x^2 \cdot (X_E - x) / D$$

$$\partial D / \partial Y_E = L_y^2 \cdot (Y_E - y) / D$$

$$\partial^2 D / \partial X_E \; \partial x = -L_x^2 \cdot L_y^2 \cdot (Y_E - y)^2 / D^3$$

$$\partial^2 D / \partial Y_E \; \partial y = -L_x^2 \cdot L_y^2 \cdot (X_E - x)^2 / D^3$$

$$\partial^2 D / \partial X_E \; \partial y = L_x^2 \cdot L_y^2 \cdot (X_E - x) \cdot (Y_E - y) / D^3$$

$$\partial^2 D / \partial Y_E \; \partial x = L_x^2 \cdot L_y^2 \cdot (X_E - x) \cdot (Y_E - y) / D^3$$

CATHETIE METHOD

The partial derivatives for the expression of the distance measured along the cathetie (see (2.5) are :

$$\partial D / \partial X_E = \begin{cases} L_x & if \;\; X_E > x \\ undefined & if \;\; X_E = x \\ -L_x & if \;\; X_E < x \end{cases}$$

$$\partial D / \partial Y_E = \begin{cases} L_y & if \;\; Y_E > y \\ undefined & if \;\; Y_E = y \\ -L_y & if \;\; Y_E < y \end{cases}$$

Mean distance from exchange to grid element

HYPOTENUSE METHOD

To find the first and second order partial derivatives of the distance with respect to X_E and Y_E, we need the corresponding derivatives of the primitive function G (see (2.7)).

Derivation of G yields :

$$\partial G / \partial X_E = L_x \cdot \left[z^2 + 3 \cdot u \cdot s + 3 \cdot z^2 \cdot log(u + s) \right]$$

$$\partial G / \partial Y_E = L_y \cdot \left[u^2 + 3 \cdot z \cdot s + 3 \cdot u^2 \cdot log(z + s) \right]$$

$$\partial^2 G \big/ \partial X_E \ \partial X_E = L_x^2 \cdot \left[5 \cdot z + 6 \cdot z \cdot log(u+s) \right]$$

$$\partial^2 G \big/ \partial Y_E \ \partial Y_E = L_y^2 \cdot \left[5 \cdot u + 6 \cdot u \cdot log(z+s) \right]$$

$$\partial^2 G \big/ \partial X_E \ \partial Y_E = L_x \cdot L_y \cdot 6 \cdot s$$

The first and second order derivatives of the distance, *D*, are then found by inserting the integration limits for *z* and *u*, i.e.

$$\partial = H(x_2, y_2) - H(x_2, y_1) - H(x_1, y_2) + H(x_1, y_1)$$

where ∂ denotes the value of any partial derivative, and the expressions for *H(z,u)* are as listed below :

$\partial D \big/ \partial X_E$: $$H(z,u) = \frac{u \cdot s + z^2 \cdot log(u+s)}{2 \cdot L_y}$$

$\partial D \big/ \partial Y_E$: $$H(z,u) = \frac{z \cdot s + u^2 \cdot log(z+s)}{2 \cdot L_x}$$

$\partial^2 D \big/ \partial X_E \ \partial X_E$: $$H(z,u) = z \cdot log(u+s) \cdot \frac{L_x}{L_y}$$

$\partial^2 D \big/ \partial Y_E \ \partial Y_E$: $$H(z,u) = u \cdot log(z+s) \cdot \frac{L_y}{L_x}$$

$\partial^2 D \big/ \partial X_E \ \partial Y_E$: $$H(z,u) = s$$

with $s = \sqrt{z^2 + u^2}$

The terms 5*z and 5*u will cancel out when inserting the integration limits.

CATHETIE METHOD

The partial derivatives of the distance (see (2.8)) are :

$$\partial D \big/ \partial X_E = \begin{cases} L_x & if \ X_E \geq x_2 \\ -L_x & if \ X_E \leq x_1 \\ 2 \cdot D_x \cdot L_x & if \ D_x < 0.5 \quad (x_1 < X_E < x_2) \end{cases}$$

where $D_x = X_E - x_2 + 0.5$

$$\partial D \big/ \partial Y_E = \begin{cases} L_y & if \ Y_E \geq y_2 \\ -L_y & if \ Y_E \leq y_1 \\ 2 \cdot D_y \cdot L_y & if \ D_y < 0.5 \quad (y_1 < Y_E < y_2) \end{cases}$$

where $D_y = Y_E - y_2 + 0.5$

APPENDIX 3.2

SOLVING A LINEAR EQUATION SYSTEM

The optimisation of exchange locations, see Chapter 2.3, leads to a linear equation system to be solved. This system has the form

$$
\begin{aligned}
a_{11} \cdot z_1 + a_{12} \cdot z_2 + &\ldots + a_{1n} \cdot z_n = b_1 \\
a_{21} \cdot z_1 + a_{22} \cdot z_2 + &\ldots + a_{2n} \cdot z_n = b_2 \\
a_{31} \cdot z_1 + a_{32} \cdot z_2 + &\ldots + a_{3n} \cdot z_n = b_3 \\
&\vdots \\
a_{n1} \cdot z_1 + a_{n2} \cdot z_2 + &\ldots + a_{nn} \cdot z_n = b_n
\end{aligned}
$$

where the coefficients $a_{..}$ correspond to the $\partial^2 C / \partial X \quad \partial X_F$ described in Chapter 2.3, $b.$ correspond to $\partial C / \partial X$, and $z.$ correspond to ΔX_E or ΔY_E

The coefficients $a_{..}$ have a peculiar, and for our purpose pleasant, property, that is,the elements on the diagonal and the subdiagonals are considerably larger than other elements in the same row or column. This stems, naturally, from the fact that other elements contain derivatives, multiplied by costs, for the routes between exchanges, while the elements around the diagonal contain the corresponding terms for the connection of exchange to subscribers.

As the relation between these terms is usually of magnitude 2 or 3, the *Gauss-Seidel* method with *over-relaxation* converges rapidly.

This iterative method goes as follows :

Step 1 : Set all $z_i = 0$

Step 2 : For every equation, *i*, find an improved value of z_i from

$$z_i(new) = z_i(old) + D_i \cdot w$$

where

$$D_i = (b_i - S_i)/a_{ii}$$

$$S_i = \sum_{j=1}^{n} a_{ij} \cdot z_j$$

and ω is the relaxation factor, a suitable value being ω = 1.2

The *z*-values contained in S_i are to be taken as

$z_j(new)$ for j = 1, 2, ... i-1

and

$z_j(old)$ for $j = i$, i+1, ... n.

Step 3 : After calculating all $z_i(new)$ in this way, the values for D_i are investigated. If all such values are less than a predefined value, the iterations are terminated. Otherwise, the procedure is repeated from Step 2; results should be obtained after 4-5 such iterations, if not, there is something wrong with the calculation of the coefficients.

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