

**Overflow from
Full Availability Group**

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Basic Teletraffic Theory (T)

OVERFLOW FROM FULL AVAILABILITY GROUPS (TOF)

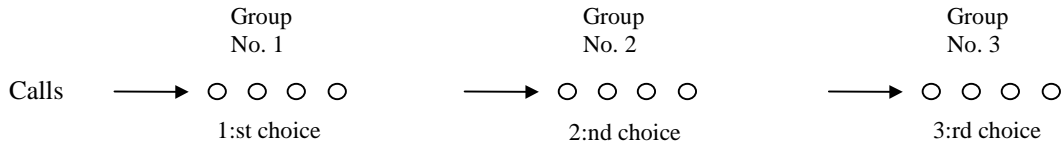
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1. Concept of overflow traffic

This section concerns the theoretical description of overflow traffic, i.e., traffic that overflows from a full availability group.

It is possible in many switching arrangements to let the calls try to find a free circuit in any of a number of groups of circuits. If the calls are tried in a definite order, it is evident that the first group will be used more than the following ones in the hunting chain. The second group will be tried only when the first choice group is fully occupied, and, of course, the third group only when the two previous ones are both fully occupied.



We can understand the situation as follows:

Group No. 1 will accept calls as long as there is a free circuit.

Group No. 2 will receive calls when Group No. 1 is fully occupied, provided it has any free circuits.

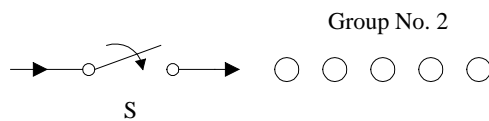
Group No. 3 will receive calls when Groups No. 1 and 2 both are fully occupied, provided it has any free circuit.

Etcetera, until all possible choices have been tried.

If there is no free circuit in any of the groups to be hunted over, the call is lost. (Loss system)

From the theoretical point of view, we can say that Group No. 1 is always offered random traffic. Group No. 2 is offered random traffic only when Group No. 1 is occupied. That is at sporadic occasions, where the frequency and duration depend on how frequently Group No. 1 is congested and how long this congestion state will last. Group No. 3 is also open for random traffic when Group No. 1 and Group No. 2, both, are congested.

If we consider Group No. 2, we can understand it as having a switch, S, which is only operated when Group No. 1 is congested.



For the occupations in Group No. 2, the following holds good:

Switch S operated:

- New calls can occupy free circuits;
- Existing occupations can terminate.

Switch S disconnected (non-operated):

- No new calls are received;
- Existing occupations can terminate.

We understand that the group only behaves as a full availability group offered fresh traffic when the switch is operated. These intervals are succeeded by intervals only when terminations occur. The resulting traffic distribution will

therefore have other properties than the full availability group. The traffic in an overflow group is therefore said to be degenerated.

The degree of degeneration is often described by the ratio between the variance, V, and the mean, M. For a Poissonian input traffic (infinite number of traffic sources), this can be described by:

$$\Theta = \frac{V}{M}$$

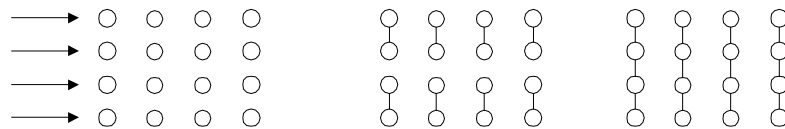
where:

$\Theta < 1$ for traffic carried in a full availability group (Erlang distribution)

$\Theta = 1$ for input traffic, and for the traffic carried if no congestion

$\Theta > 1$ for overflow traffic

The problem of calculating the overflow distribution is further complicated by the fact that the overflow groups are used by more than one first choice group.



This means that our description of an overflow group as having a switch in reality should have one switch for each primary group offering traffic to the overflow group. So, if two first choice groups provide overflow traffic to one second choice group, we should consequently have two switches providing $2 \times 2 = 4$ different combinations of connected and/or disconnected switches. Each combination has its specific properties as concerns new calls and terminations.

The problem of providing a theoretical description of overflow traffic has been attacked for many years by various theoreticians with various success. So far, no calculation method has been developed that is both accurate and simple. The calculation methods are of the following types:

- equations of state approaches;
- weighting between known limits;
- equivalence methods, i.e. determination of an equivalent full availability group which has some characteristics equal to those existing for actual grouping arrangement;
- simulations.

The equations of state methods are generally impractical for grouping arrangements of normal size, because the number of possible states becomes very large. Some weighting methods are simple to use in practice, but may not always be so accurate.

The equivalence methods have hitherto proved to be more accurate and Wilkinson's method, especially, seems to be the most preferred one today. Wilkinson's method is based on characterizing the overflow traffic with its mean and variance. The means and variances for concerned first choice groups are added and the resulting overflow traffic is characterized by an equivalent full availability group which provides an overflow traffic with the same mean and variance as the calculated sums.

The idea of describing an overflow group as having a switch, as above, has also been used. The method is called the Interrupted Poisson Process (Kuczura, 1973). While the Wilkinson's method uses two parameters for the description of overflow traffic, the method of Kuczura uses three parameters which should increase the accuracy. The method has been further developed to use five parameters by Wallström (1979). The IPP-method has not yet been more commonly used for practical calculations.

Before going further into the description of overflow traffic, we will first derive the moments for the traffic carried in a full availability group. This will help the understanding of the much more complicated moments for the overflow traffic.

2. Definitions

The mathematical statistics use frequently the moments to describe the properties and shape of a distribution. These moments have frequently a simple relation to the parameters of the theoretical distribution. The moments are also useful for the description of traffic distributions.

First moment (Mean):

The first moment is defined as:

$$\mu_1 = \sum_p p \cdot [p] \quad (\text{TOF 2.1})$$

The first moment gives the mean value of the distribution. This mean is frequently denoted $\Sigma(p)$ (expectation of p), M or m . For a traffic distribution μ_1 is the traffic carried if p is summed over all possible occupation states.

Second central moment (Variance):

The second central moment is defined as:

$$\mu_2 = \sum_p (p - \mu_1)^2 \cdot [p] \quad (\text{TOF 2.2})$$

Here, μ_2 is also called the variance of the distribution since it describes how much the distribution deviates from its mean, or in other words, the concentration around the mean. The variance is frequently denoted:

$$\varepsilon \{(x - \mu_1)^2\}, \text{ } V \text{ or } \sigma^2$$

Further, the standard deviation of a distribution is:

$$\sigma = \sqrt{\mu_2} \quad (\text{TOF 2.3})$$

For a traffic distribution, the variance also signifies the concentration around the mean.

The ratio μ_2/μ_1 , i.e. V/M , is frequently used to describe the character of a traffic distribution. As already mentioned, for a Poisson input:

$$\Theta = \frac{V}{M} = 1$$

is frequently called "pure chance traffic".

A traffic, for which

$$\Theta = \frac{V}{M} < 1$$

is often called "smooth traffic", and finally, if

$$\Theta = \frac{V}{M} > 1$$

the traffic is said to be rough, or degenerated.

We will, in the following, learn that $\theta < 1$ for carried traffic and that $\theta > 1$ applies for overflow traffic. $\theta = 1$ applies only for a Poisson traffic (infinite number of sources, no congestion). We will also learn that the particular value for θ depends on the type of distribution, so that different values are obtained for Bernoulli, Engset and Erlang distributions.

Higher moments

The third and fourth central moments, μ_3 and μ_4 , are sometimes used to define the skewedness (asymmetry) and the excess of a distribution. These moments have not been used much in the Teletraffic Theory. In fact, the use of higher moments of traffic has mainly been used in the application of the Interrupted Poisson Process method.

The disadvantage of using higher moments in the estimation of the parameters of a statistical distribution from observed data is, of course, that the precision decreases with increased order of the moment.

3. Mean and variance of carried traffic

As defined by (TOF 2.1) and (TOF 2.2), the mean and variance for a traffic distribution are:

$$M = \sum_p p \cdot [p] \quad \text{(TOF 3.1)}$$

$$V = \sum_p (p - M)^2 \cdot [p] \quad \text{(TOF 3.2)}$$

The summations are carried out for all possible values of p.

For the traffic carried by a full availability group (primary group), we will, in the sequel, use the notations m and v for the mean and the variance.

As defined by (TGD 3.5), we know that:

$$m = A^1 = \sum_{p=1}^n p \cdot [p] \quad \text{(TOF 3.1a)}$$

where n is the maximum number of possible occupations. For the calculation of the variance, we can rewrite $(p-m)^2$ as follows:

$$(p - m)^2 = p(p - 1) + p - 2mp + m^2$$

We can then calculate v from:

$$v = \sum_{p=2}^n p(p - 1) \cdot [p] + (1 - 2m) \sum_{p=1}^n p \cdot [p] + m^2 \sum_{p=0}^n p$$

According to (TOF 3.1a), we then have:

$$v = \sum_{p=2}^n p(p - 1) \cdot [p] + m - m^2 \quad \text{(TOF 3.2a)}$$

This expression is useful for the derivation of v for ordinary full availability group distributions. Considering the equilibrium equation (TGD 1.11) and assuming an exponential holding time distribution as defined by (TGD 2.8), the expression (TOF 3.2a) can be transferred to:

$$v = \sum_{p=2}^n s^2 \cdot y(p-1) \cdot y(p-2) \cdot [p-2] + m - m^2 \quad (\text{TOF 3.2b})$$

which sometimes simplifies the derivation of v.

Bernoulli Distribution

From (TFL 4.1B), it follows that:

$$m = \sum_{p=0}^N p \cdot [p] = Na \quad (\text{TOF 3.3B})$$

$$(N \leq n)$$

Using (TFL 2.1B), we obtain

$$v = \sum_{p=0}^n (p-m)^2 \cdot [p] = Na \cdot (1-a) \quad (\text{TOF 3.4B})$$

and consequently

$$\Theta = \frac{v}{m} = 1 - a \quad (\text{TOF 3.5B})$$

which is less than unity.

Engset Distribution ($N > n$)

From (TFL 4.1EB), follows:

$$m = \frac{N\alpha(1-B)}{1+\alpha(1-B)} = \frac{N\alpha}{1+\alpha} \left(1 - \frac{N-n}{N} E \right) \quad (\text{TOF 3.3EB})$$

while the derivation of v is a little more complicated. However, after some transformations, the expression can be written

$$v = \frac{(N-1)\alpha}{1+\alpha} (m-nE) + \frac{n \cdot (n-1) \cdot \alpha E}{n+\alpha} + m - m^2$$

or

$$v = m \frac{\alpha}{1+\alpha} (N + \alpha + nB) - \frac{N\alpha}{1+\alpha} nB - m^2 \quad (\text{TOF 3.4EB})$$

where E is the time congestion and B the call congestion.

The ratio $\Theta = \frac{v}{m}$ is less than unity.

Remark:

The relation between E and B as defined by (TFL 3.1EB) can be written

$$\frac{B}{E} = \frac{N-n}{N-m} \quad (\text{TOF 3.6EB})$$

where m is defined by (TOF 3.3EB).

Erlang Distribution

From (TFL 4.1E), follows that:

$$m = A \cdot (1 - E_n(A)) \quad (\text{TOF 3.3E})$$

and for v, the following expression is arrived at

$$v = A(1 - E_n(A)) - AE_n(A) \cdot (n - A + AE_n(A)) \quad (\text{TOF 3.4E})$$

or

$$v = m - M(n - m)$$

where $M = Ae_n(A)$ and $E_n(A)$ is Erlang's second formula.

The ratio:

$$\Theta = \frac{v}{m} = 1 - \frac{M}{m}(n - m) \quad (\text{TOF 3.5E})$$

is less than unity.

Poisson Distribution

If we let $n \rightarrow \infty$,

$$E_n(A) = 0$$

and we obtain:

$$m = A \quad (\text{TOF 3.3P})$$

$$v = A \quad (\text{TOF 3.4P})$$

$$\Theta = 1 \quad (\text{TOF 3.5P})$$

which describes the properties of a traffic generated by an infinite number of sources ($N = \infty$), not disturbed by congestion, since $n = \infty$.

Negative Binomial Distribution

For this distribution, defined by (TFL 2.1NB), we obtain

$$m = \frac{b\gamma}{1-b} \quad (\text{TOF 3.3NB})$$

$$v = \frac{b\gamma}{(1-b)^2} \quad (\text{TOF 3.4NB})$$

$$\Theta = \frac{v}{m} = \frac{1}{1-b} \quad (\text{TOF 3.5NB})$$

Truncated Negative Binomial Distribution

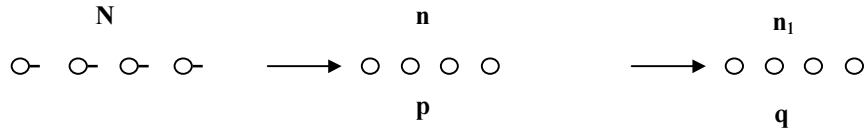
From (TFL 2.1 TNB), we obtain

$$m = \frac{b \cdot \gamma \cdot (1 - E)}{1 - b} - \frac{bnE}{1 - b} \quad (\text{TOF 3.3TNB})$$

$$v = \frac{b \cdot (\gamma + 1)(m - nE)}{1 - b} - \frac{bm \cdot (n - 1) \cdot E}{1 - b} + m - m^2 \quad (\text{TOF 3.4TNB})$$

These two distributions are of interest since $\theta > 1$, for at least (TOF 3.5NB). This provides a similarity with the properties of overflow traffic.

4. Mean and variance of overflow traffic



Consider a full availability group for sequential hunting with N sources and $n + n_1$ circuits, where:

$$n + n_1 > N$$

(This means that if $N = \infty$, also $n_1 = \infty$)

The divided full availability group is used to describe the character of the traffic rejected from the first part of the group (n). This rejected traffic, or overflowing traffic, will be carried by the second part (n_1).

If we denote the state of the group by (p, q) , we have the following limits for p and q

$$0 \leq p \leq n$$

$$0 \leq q \leq n_1$$

$$0 \leq p + q \leq n + n_1 \leq N$$

The state probabilities $[p \ q]$ can be determined by equations of state. As a rule, no simple expressions are obtained. The mean and variance of the overflow traffic are defined as:

$$M = \sum_{p=0}^{n_1} \sum_{q=0}^{n_2} q [p \ q] \quad (\text{TOF 4.1})$$

$$V = \sum_{p=0}^{n_1} \sum_{q=0}^{n_2} (q - M)^2 [p \ q] \quad (\text{TOF 4.2})$$

The most common case is overflow from an Erlang distributed group. Then, we have

$$M = AE_n(A) \quad (\text{TOF 4.1E})$$

$$V = M \left(1 - m + \frac{A}{n + 1 - A + M} \right) \quad (\text{TOF 4.2E})$$

$$\Theta = \frac{V}{M} = 1 - m + \frac{A}{n + 1 - A + M} \quad (\text{TOF 4.3E})$$

This expression has been deduced by Riordan. It is used in the Wilkinson's method for alternative routing calculations.

The ratio Θ is > 1 , which is typical for overflow traffic.

The functional behaviour of M , V and Θ can be summarised as follows:

$$\begin{aligned} \underline{n = 0, A > 0} \\ M = V = A \\ \Theta = 1 \end{aligned}$$

$$\underline{n > 0, A > 0, \Theta > 1}$$

A is increased:

- M and V increase
- Θ increases to a maximum and decreases thereafter
- when $A \rightarrow \infty, \Theta \rightarrow 1$

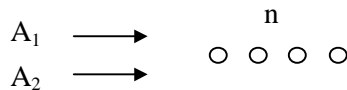
n is increased:

- M and V decrease towards zero
- Θ increases to a maximum and decreases thereafter
- when $n \rightarrow \infty, \Theta \rightarrow 1$
- the maximum value of Θ increases with increased n ; the maximum occurs when n is slightly larger than A .

The existence of a maximum for Θ has been used by Wilkinson in a later simplification of his method. This latter method is called the "Maximum Peakedness Factor Method".

For a limited number of sources, i.e. overflow from an Engset distributed group, a solution has been presented by Schehrer (1972). The solution does, however, not give as simple expressions as for the Erlang case (TOF 4.1E-3E).

5. Addition of overflow traffics

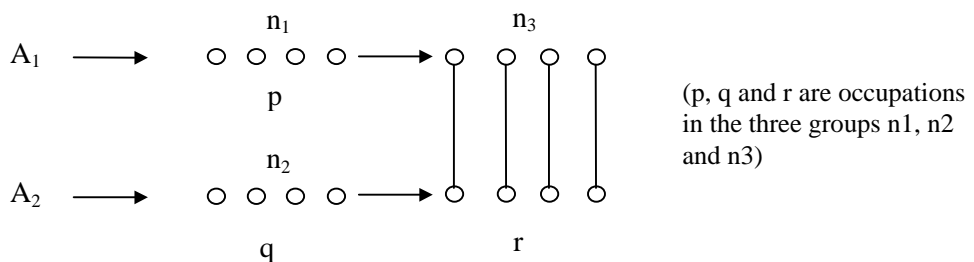


It is a well-known fact that if two Poisson traffics, A_1 and A_2 , are offered to the same full availability group, the case will be equivalent to what happens if the group is offered one traffic of the size $A_1 + A_2$.

Provided that no priority is given to any of the traffics, the state probabilities, the congestion and the traffic carried are all dependent on the parameter $A = A_1 + A_2$.

We can also see that the traffic overflowing from such a group is, according to (TOF 4.1E) and (TOF 4.2E), exclusively dependent on A and not on the individual parts, A_1 and A_2 . This holds also good for the traffic carried by the group as defined by (TOF 3.3E) and (TOF 3.4E).

The next problem is: What happens if two overflow traffics are offered to the same secondary group?



The case is described in the above figure. The traffic A_1 is served by n_1 primary circuits, A_2 by n_2 . Overflowing traffic from both primary groups is offered the secondary group, n_3 .

The state of this system, which is a simple grading, is defined by (p, q, r) where p, q and r are the number of occupations in each part. Analogously to (TOF 4.1) and (TOF 4.2), the mean and the variance of the overflow traffic can be expressed as

$$M_{12} = \sum_{p=0}^{n_1} \sum_{q=0}^{n_2} \sum_{r=0}^{n_3} r [p \ q \ r] \quad (\text{TOF 5.1})$$

$$V_{12} = \sum_{p=0}^{n_1} \sum_{q=0}^{n_2} \sum_{r=0}^{n_3} (r - M_{12})^2 [p \ q \ r] \quad (\text{TOF 5.2})$$

where we can assume $n_3 = \infty$ (no congestion).

We can calculate the mean and variance for the traffic overflowing from each group as given by (TOF 4.1), (TOF 4.1E) and (TOF 4.2E). Let us assume that the values for the first group are M_1 and V_1 and for the second group M_2 and V_2 .

The question then arises, is

$$M_1 + M_2 = M_{12} \quad (?)$$

and

$$V_1 + V_2 = V_{12} \quad (?)$$

We can assume that n_3 is so large that no calls will be rejected. This means that there is no competition between traffic overflowing from n_1 and overflowing from n_2 . The number of occupations in n_3 , r , must therefore be a true measure of the total number of overflowing calls. Furthermore, the two groups n_1 and n_2 act independently of each others. It is, therefore, very likely that

$$M_1 + M_2 = M_{12} \quad (\text{TOF 5.3})$$

should be true as long as $n_3 = \infty$. This agrees with the theories of mathematical statistics saying that the expected value (mean) for the sum of two independent statistical variables should equal the sum of their individual means, i.e.

$$\mathcal{E}\{(x_1 + x_2)\} = \mathcal{E}\{x_1\} + \mathcal{E}\{x_2\}$$

Theories of the mathematical statistics also say that the variance for a sum of independent statistical variables is equal to the sum of the individual variances. Consequently

$$V_1 + V_2 = V_{12} \quad (\text{TOF 5.4})$$

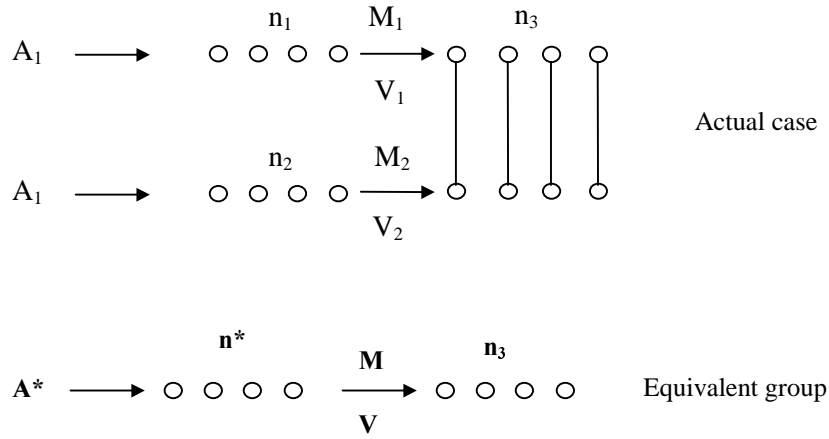
as long as no congestion occurs ($n_3 = \infty$).

We can consequently describe the total traffic overflowing to the same secondary group by adding the means and variances for the rejected traffics from each primary group. This gives a two-parameter description of the overflow traffic offered to a secondary group.

6. Wilkinson's method

For calculating the required number of circuits in alternating routing schemes, almost all administrations and manufacturers use today Wilkinson's method, or further developed applications based on this method.

The method is an equivalence method, where an equivalent full availability group is defined by the mean and variance of its overflow traffic. This mean and variance should equal the sums of the means and variances of the overflow traffics offered to the secondary group.



The values M_1 , M_2 , V_1 and V_2 are calculated from (TOF 4.1E) and (TOF 4.2E).

An equivalent full availability group is sought, which satisfies the condition.

$$\left. \begin{aligned} M &= M_1 + M_2 \\ V &= V_1 + V_2 \end{aligned} \right\} \quad (TOF 6.1)$$

The equivalent group is then defined by its two parameters A^* and n^* , where

$$\left. \begin{aligned} A^* \cdot E_{n^*}(A^*) &= M \\ M \cdot \left(1 - M + \frac{A^*}{n^* + 1 - A^* + M} \right) &= V \end{aligned} \right\} \quad (TOF 6.2)$$

Since the numerical values for M and V are given, (TOF 6.2) implies that A^* and n^* have to be found by trial and error. The solution of (TOF 6.2), therefore, gives some numerical problems which are overcome with suitable graphs and algorithms for the calculations. The precision is further improved by using non-integer values for n^* . Since the Erlang formula, $E_n(A)$, is only defined for whole numbers of circuits n , this implies that a suitable convention has to be introduced for the calculation of the formula for non-integer values.

The congestion for the actual case is then estimated as

$$E = \frac{A^* \cdot E_{n^*+n_3}(A^*)}{A_1 + A_2} \quad (TOF 6.3)$$

This formula provides an estimate of the total congestion. It does not specify how much of the individual traffics, A_1 and A_2 , are rejected. Different methods exist, however, for estimating the congestion for these traffics. It is however heuristically found that the losses in the secondary group are proportional to the value of $V:M$. A more degenerated traffic will namely experience more congestion than a less degenerated one, where the degree of degeneration is expressed by the ratio $V : M$.

Therefore, if we define the losses in the secondary group as e_1 and e_2 , we can write

$$e_1 = k \cdot \frac{V_1}{M_1} \quad e_2 = k \cdot \frac{V_2}{M_2} \quad (TOF 6.4)$$

where k is an unknown constant.

The total loss in group n_3 is now

$$M_1 \cdot e_1 + M_2 \cdot e_2 = A^* \cdot E_{n^*+n_3}(A^*) \quad (\text{TOF 6.5})$$

Introducing (TOF 6.4) in (TOF 6.5), we find that

$$k = \frac{A^* \cdot E_{n^*+n_3}(A^*)}{V_1 + V_2}$$

and that the individual congestions are

$$\left. \begin{aligned} E_1 &= \frac{M_1 \cdot e_1}{A_1} = \frac{V_1}{V_1 + V_2} \cdot \frac{A^* \cdot E_{n^*+n_3}(A^*)}{A_1} \\ E_2 &= \frac{M_2 \cdot e_2}{A_2} = \frac{V_2}{V_1 + V_2} \cdot \frac{A^* \cdot E_{n^*+n_3}(A^*)}{A_2} \end{aligned} \right\} \quad (\text{TOF 6.6})$$

The expression (TOF 6.6) is approximative, but provides rather accurate estimates. The expression was introduced by Elldin & Lind.

The Wilkinson's method defined by (TOF 6.1) - (TOF 6.3) can be repeated. This means that if the switching arrangement contains also a tertiary group, another equivalent group can be determined. This group may then represent the overflow from a number of fictitious traffics, like A^* after $n^* + n_3$ circuits.

7. Other methods

As already mentioned, the calculation methods for overflow traffic can be classified as equations of state, weighting and equivalence methods. A fourth possibility is simulations.

The equations of state methods imply the solution of linear equation systems with a great number of unknowns. It is, therefore, not practical for any overflow arrangement comprising more than, say, 10-15 circuits. Use of certain known relations for the total number of occupations, symmetry assumptions and reasonable approximations may extend the usefulness of the method further, but it must still be considered as unsuitable for practical calculations. The method is however valuable for principal studies and as a check of certain approximations.

The best known weighting method is the O'Dell method. Besides difficulties of arriving at accurate results, it is also less suitable for defining the particular properties of the overflow traffic.

Among the equivalence methods should be mentioned the early method by Berkerly (1934), where the overflow traffic is defined by one parameter only, namely the mean, M , from which A^* is calculated. The calculation procedure is in principle the same as for Wilkinson's method since it permits successive substitution of real traffic with fictitious traffics. The methods based on the Interrupted Poisson Process may also be considered as equivalence methods since the operate/release intervals for the fictitious switch are estimated from the moments of the overflow traffic.

Among the equivalence method should also be mentioned Wilkinson's "Maximum Peakedness Method". This method simplifies the calculation of the equivalent full availability group. By this method, the means, M , are calculated in the ordinary way but the variances are estimated as

$$V = \Theta_{max} \cdot M \quad (\text{TOF 7.1})$$

where Θ_{max} is the maximum value of the ratio $V : M$ for the given number of circuits in the primary group. The calculation will, therefore, always be on the safe side. The method is of course less accurate than the original method given by Wilkinson.

An interesting approximation based on the Maximum Peakedness Method is given by Fredericks (1980). Here, if M , V and Θ are given, an approximate estimation of the congestion in a secondary group can be calculated from Erlang's formula, $E_n(A)$ if n and A are substituted by

$$\left. \begin{aligned} n &= \frac{n_1}{\Theta} \\ A &= \frac{M}{\Theta} \end{aligned} \right\} \quad (\text{TOF 7.2})$$

where n_1 is the actual number of circuits in the secondary group. Further, since

$$\frac{M}{\Theta} = V$$

the variance is introduced in the Erlang formula as a measure of the traffic offered to the secondary group.

Since this method is said to be as accurate as Wilkinson's original method, and since it is numerically simpler to handle, this approximation may be used more in the future.

Further developments and applications of Wilkinson's method have been presented by Bretschneider, Rapp, Wallström, Schehrer, Fried and others.

8. Conclusions

It follows that traffic overflowing from a primary group has other properties than the traffic offered to it.

No simple methods exist for the exact calculation of the distribution and congestion in a group carrying overflow traffic. The method of determining two parameters from the mean and the variance does not give a complete description of the overflow traffic since it does not guarantee that higher moments will agree ¹. This method gives, however, values accurate enough for practical purposes which have been verified by simulations. The only problem is the numerical calculations required for determination of the circuit quantities. Certain diagrams, graphs and calculation algorithms exist, however, which simplifies this work.

The fact that the variance to mean ratio, V/M , is larger than unity indicates that more circuits will be required to carry the same traffic (mean) at the same permitted congestion, if the traffic is degenerated as compared with fresh traffic. It has also been indicated that if traffic with different degree of degeneration is mixed in a secondary group, the traffic with the highest value for V/M will also experience the highest congestion.

¹ Since each primary group is defined by two parameters, A and n , in the Erlang case, a complete description of the aggregated overflow traffic require 2 x moments, if x primary groups overflow to one secondary group.