Shortest Path Problem

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1. Shortest path problem

1.1 *General*

One method which is very effective to investigate the optimum value of a function was presetned by R. Bellman. This method was initially located to economic problems, but its validity is general and can be applied to problems of physics and mathematics. The base of the method is the so-called "principle of optimality". This principle is very simple. It is employed to problems with sequential character.

A full mathematical treatment of the method calls for an important knowledge of "the theory of graphs" which, we believe, is far beyond of the scope of this course. The whole method will be developed by using various examples from telecommunications where this problem is often encountered.

1.2 *Description of algorithm*

The layout of a telecommunication network is shown in Figure 20. The nodes (vertices of the graph) denote either exchange buildings or branching points of the network. Cable runs (branches) are links between nodes.

Given the lengths of cable runs, the problem is to determine the "shortest path" between any two nodes. This problem is often encountered in different other ways. Instead of dealing with length of cable runs, we can assign cost of link to every cable run; then the problem is to determine the minimum cost of a junction circuit between two nodes.

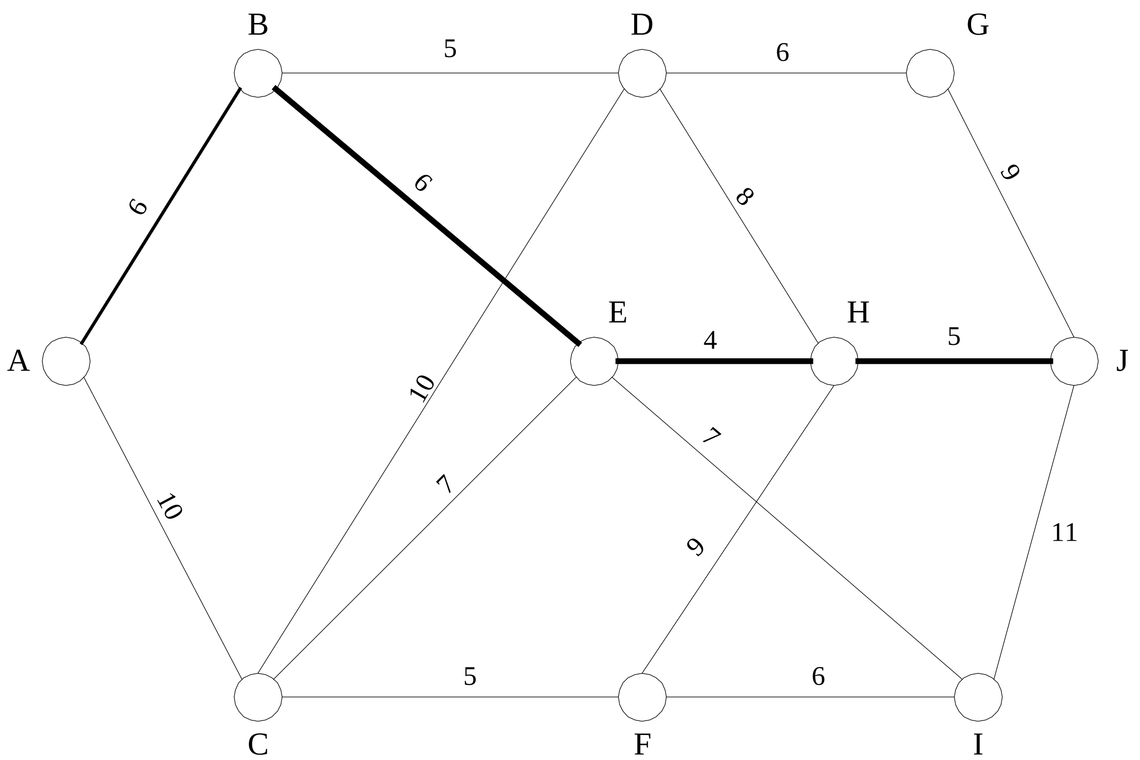


Figure 1

This problem can be tackled as a linear mathematical program, but it is more efficient to use other algorithms. The simplest method is due to Dantzig and the procedure is as follows:

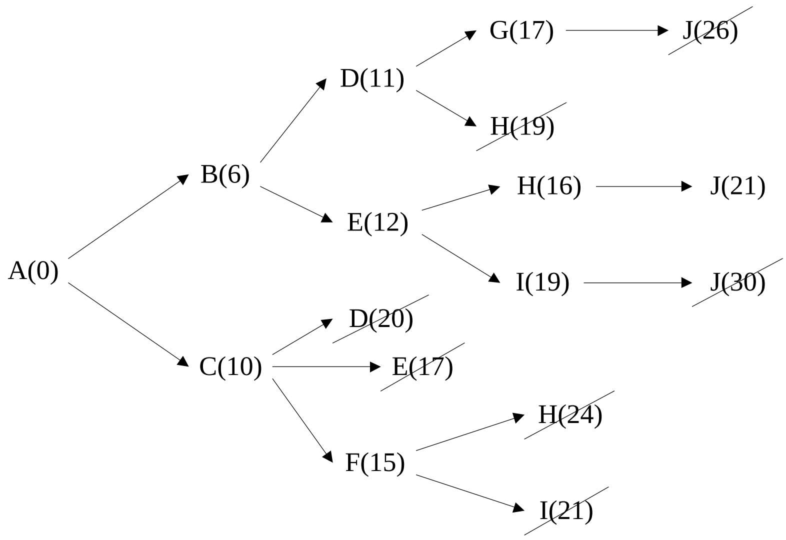
a) Label the source node as "0".

b) Examine the adjacent nodes and label, each one with its distance from the source node.

c) Examine nodes adjacent to those already labelled. When a node has links to two or more labelled nodes, its distance from each node is added to the label of that node. The smallest sum is chosen and used as the label for the new node.

d) Repeat *(C)* until either the destination node is reached (if the shortest route to only one node is required) or until all nodes have been labelled (if the shortest routes to all nodes are required).

Let us try to find the shortest path from node *"A"* to node *"J"* for the network of Figure 1. In Figure 2, all steps to determine the shortest path are illustrated. We label the source node *"A"* with 0.



Shortest path procedure
Figure 2

The adjacent nodes to *A* are *B* and *C*. For those nodes we find the distances by adding the label of *A* with the distance of nodes from *A*. Thus, we get for *B* *0+ 6 = 6*, and for *C* *0+10 = 10*.

These figures are used as labels for *B* and *C* respectively. The next step is to find the adjacent nodes to *B* and *C* and then the labels to these nodes. For *B*, we have the node *D* with the label *11 = (6+5)* and the node *E* with the label *12 = (6+6)*. For *C*, we have the node *D* with the label *20 = (10+10)*. But the node *D* is also reached through *B*. Now we keep the smallest distance, which is *11*, via *B* and eliminate *D(20).* We continue this procedure until the remaining nodes *(E, F)*, adjacent to *C*, are examined. We keep *F(15)* and eliminate *F(17)*. Continuing this way, we stop the procedure when the examination of node(s) we are concerned with is reached. In Figure 1, the shortest path from *A* to *J* is drawn with coarse line.

Consider now all partially paths contained in the path from *A* to *J-(ABEHJ)*. These are: *(AB), (ABE), (ABEH), (BE), (BEH), (BEHJ), (EH), (EHJ), (HJ)*. If we examine these partial paths, we can verify that they are optimal paths. For example, from *B* to *J*, the optimal path is *(BEHJ)*. We can ascertain the fact that every optimal path consists of partially optimal paths.

2. <u>References</u>

1. A. Kaufmann: "Méthodes et modèles de la recherche opérationnelle", Tomes 1 & 2, Dunod 1968.