Traffic Measurements

(Solutions to Exercises)

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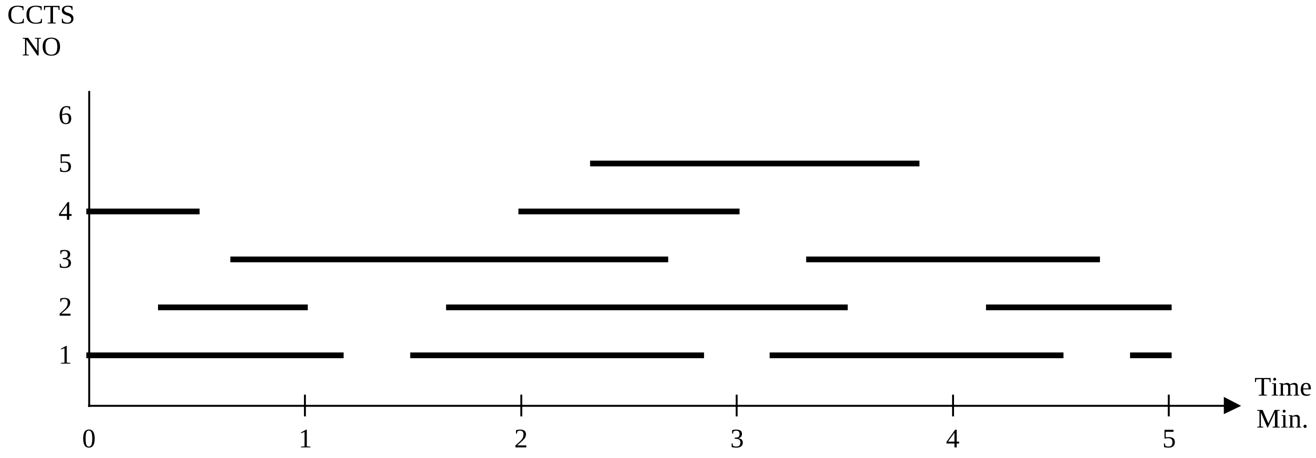
EXERCISES AND SOLUTIONS - TRAFFIC MEASUREMENTS

1. A part of a traffic process is shown in the diagram paper. It concerns the occupations during five minutes of a full availability group of six circuits. The horizontal lines mark occupation of the circuits. The time scale is 10 seconds on the time axis.

   - Mark on the line marked "events" with ↑ every time a new occupation starts and with ↓ every time an occupation terminates.
   - How many events were there?
   - Fill in the diagram for the number of occupied devices for the five minute period.
   - Calculate the traffic carried in the period! (Try different ways !).
   - Assume that the group is scanned every 30 seconds, starting at t = 5 seconds. What would the traffic be according to the scanning result?
   - What is the average occupation time of those occupations that are fully completed within the five minute interval.



Events

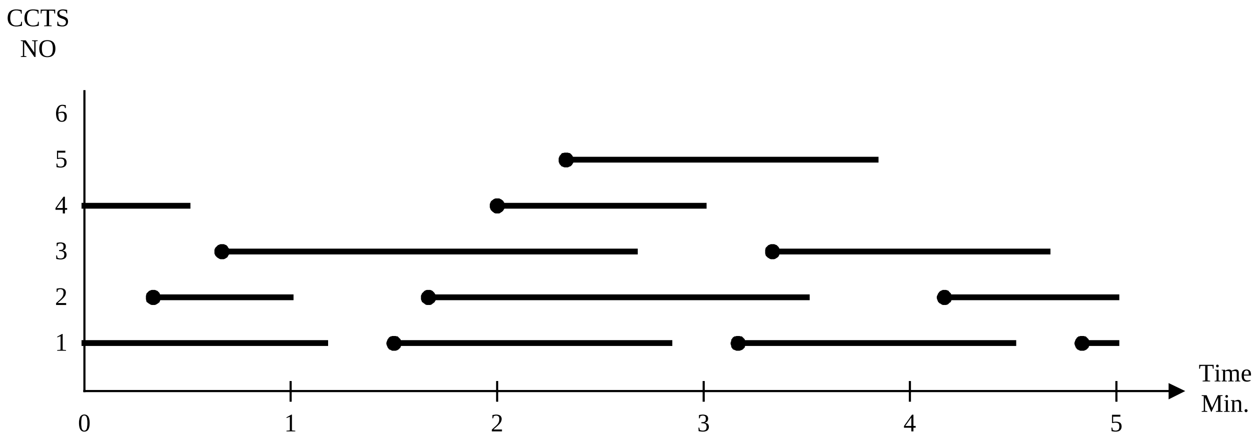
NO. OF OCCUPIED CIRCUITS

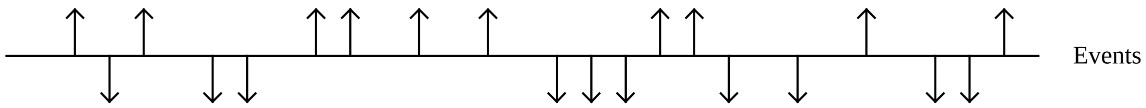
1. SOLUTION

<u>Events</u> The events are marked in the diagram. In all, 10 calls occur in the observation interval and 10 occupations terminate. There are then 20 events in the five minute period.

<u>Number of occupied circuits</u> See attached diagram. Every time a new call occurs the curve makes a step upwards and at each termination a step downwards.

<u>Traffic carried</u> If we add up the total occupation times and divide it with the length of the observation time we get the traffic carried. Taking into account that every square is 10 seconds we have:





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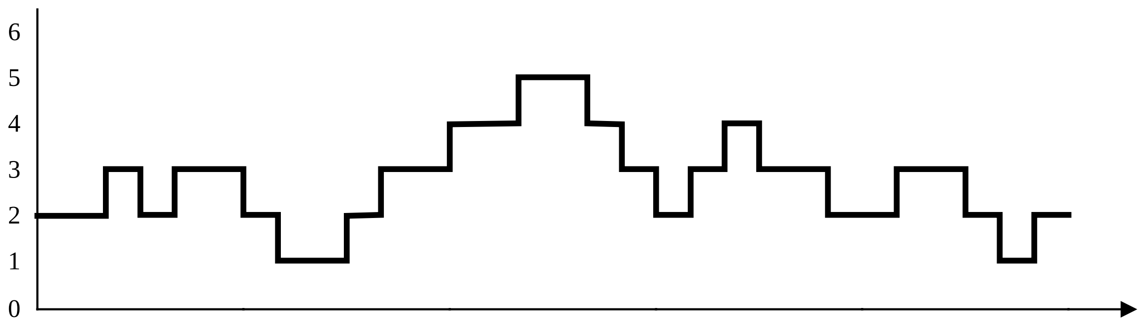

Device	Occupation times		
1	7 + 8 + 8 + 1	=	24 x 10 seconds
2	4 + 11 + 5	=	20 x 10 seconds
3	12 + 8	=	20 x 10 seconds
4	3 + 6	=	9 x 10 seconds
5	9	=	9 x 10 seconds
			82 x 10 seconds

$$A' = \frac{82 \cdot 10}{30 \cdot 10} = 2.73$$

AREA = 82 squares - over 30 steps

$$A' = \frac{82}{30} = 2.73$$

Scans: $2 + 2 + 2 + 2 + 4 + 5 + 2 + 3 + 2 + 2 = 26$

10 scans $A' = \frac{26}{10} = 2.6$

Circuit No.	Occupations in 10 seconds	Total seconds
1	7 + 8 + 8 + 1	240
2	4 + 11 + 5	200
3	12 + 8	200
4	3 + 6	90
5	0	90
6	0	0
		820

Observation time: *5 x 60 = 300 seconds*

Traffic Carried: *820/300 = 2.73 erlang*

Another method is to integrate the area in the diagram for the number of occupied devices.

There are nearly always at least two occupied circuits. The area is then more than *30 x 2 = 60*

Area = 60 + 1 + 2 - 2 + 15 + 5 + 2 - 1 = 82

Traffic Carried = 82/30 = 2.73 erlang

Scanning The points of time when scanning is made is marked in the diagram for the number of occupied circuits.

Scanning result: *2 + 2 + 2 + 2 + 4 + 5 + 2 + 3 + 2 + 2 = 26*

We made 10 scans.

Average number of occupied circuits per scan = *estimated traffic carried = 26/10 = 2.6 erlang*.

<u>Average holding time</u> The total occupation time was 820 seconds. We had in all 12 occupations. There were, however, two outgoing occupations when we started the observations. We do not know their total duration. There were also two not finished occupations at the end of the period. To get a fair estimate of the average holding time we have to exclude these four occupations and take the average on the remaining 8.

The total time for these 8 occupations was:

$$820 - 70 - 30 - 10 - 50 = 660 \text{ seconds}$$

then, $\bar{h} = 660/8 = \underline{82.5} \text{ seconds}$

If we had included the four incomplete holding times we would have got

$$\bar{h} = 820/12 = \underline{68.3} \text{ seconds}$$

If the observation period had been longer than five minutes the difference would be less.

2. A special measurement during one hour gave an estimate of the traffic distribution of a group of five devices. The result was as follows:

No. of circuits occupied	0	1	2	3	4	5
Part of the time	0.086	0.214	0.268	0.222	0.140	0.070

- What was the traffic carried?
- What was the time congestion if the group only had five circuits.
- What was the traffic offered if we can assume that $B = E$?
- How many calls would be rejected during the hour, if we assume the average holding time to be $\overline{h} = 100\ seconds$?

2. SOLUTION

We calculate the traffic carried from the formula

$$A' = \sum_{p=1}^{5} p[p]$$

which gives

$$A = 0.214 \cdot 1 + 0.268 \cdot 2 + 0.222 \cdot 3 + 0.140 \cdot 4 + 0.070 \cdot 5 = 2.326$$

The traffic carried is: $A = \underline{2.326}\ erlang$

Time Congestion According to the measurement all circuits were occupied 0.070 of the time, therefore: $E = 0.070$

Traffic Offered We have to find the traffic offered from the equation;

$$A(1 - E_5(A)) = 2.326 \quad or \quad A(1 - [5]) = 2.326$$

After a few trials we find $A = 2.500$

Number of rejected calls With the average holding time = 100 seconds, we find that there should be;

$$y = \frac{A}{h} = \frac{2.5}{100} \cdot 3600 = 90$$

calls offered during the hours. The estimated number of rejected calls is then;

$$y \cdot E = 90 \cdot 0.070 = \underline{6.3}\ \ calls$$

3. The traffic in a group of circuits was measured by scanning every 30 seconds over a period of two hours. The counter was read off every 30 minutes, without resetting it. The counter was zeroed at the start of the measurement.

- Determine the busiest hour and the Traffic Carried during this hour! What is the standard error of the observed traffic offered? (The average holding time was two minutes.)

Time	Reading	Value
9:30	1	172
10:00	2	434
10:30	3	622
11:00	4	848

3. SOLUTION

The scan results for each half hour were;

Time	Counter
9:00 - 9:30	172
9:30 - 10:00	262
10:00 - 10:30	188
10:30 - 11:00	226

The highest hour is 9:30 - 10:30. During that time the scans gave *262 + 188 = 450*. There is a scan made every 30 seconds, so there will be 120 scans/hour. The traffic carried is then *A' = 450/120 =* 3.75 *erlang*.

Standard error We use the formula (5.5) $\sigma^2 = \frac{A}{T} \cdot h \cdot \frac{e^{\lambda h} + 1}{e^{\lambda h} - 1}$

for $A = 3.75$ $T = 1$ $h = 0.5/60$ $\lambda = 30$

and find $\sigma^2 = 0.2513$ and $\sigma = 0.50$

The standard error is $\sigma = 0.50$ so we can write the result as $A' = 3.75 \pm 0.50$.

4. A measurement on a full availability group is taken by three minute scans for the period 8 a.m. - 12 a.m. on ten working days. For this group of circuits the following 15 - minute totals (added for all 10 days) are found:

340	400	430	440	500	480	470	450	450	435	400	380	365	350	310	340

We assume it is an Erlang loss system and that the holding times are exponentially distributed with the mean = two minutes.

a. Find the time consistent busy hour.

b. Find the expected value (mean) of the carried traffic during the busy hour.

c. Find the variance of the measured traffic (intensity) when this value is obtained by:
   i) scanning
   ii) continuous observations

d. Find the 95 % confidence interval of the traffic intensity.

4. SOLUTION

a. The busy hour is determined by summing four consecutive 15 minute results and choosing those four that give the largest value. In this case $500 + 480 + 470 + 450 = 1900$. This is the 5, 6, 7, and 8th 15 minute period and the busy hour is 9:00 - 10:00 a.m.

b. The average sum for one day (one busy hour) is $1900/10 = 190$. This is the total of $60/3 = 20$ scans so the average result per scan is $190/20 = 9.50$. The average traffic for the 10 days is 9.50 Erlang.

c. The variance of the traffic measured <u>by scans</u> can be calculated from the formula:

$$\sigma^2 = \frac{A}{T} \cdot h \cdot \frac{e^{\lambda h} + 1}{e^{\lambda h} - 1}$$

Where $A = 9.5$

$T = 10 \text{ hours}$

$h = 3/60 \text{ hours}$ (scanning interval)

$s = 1/\lambda = 2/60 \text{ hours}$ (average holding time)

so $\lambda h = \frac{h}{s} = \frac{3}{2} = 1.5$

We then obtain $\sigma^2 = 0.0748$ $\quad\quad$ $\sigma = \underline{0.273}$

If we had measured $A' = 9.5$ *Erlang* by continuous observation the variance is calculated from the formula

$$\sigma^2 = A \cdot E \cdot s/T$$

where
$A = 9.5$
$E = 2$ (exp. distribution)
$s = 2/60 \text{ hours}$
$T = 10 \text{ hours}$
$\sigma^2 = 0.063$
$\sigma = \underline{0.251}$

d. The 95 % confidence intervals are found by multiplying the standard error with 1.96. Consequently:

$8.96 < A < 10.04$ for scanning

$9.00 < A < 10.00$ for continuous observations

Comment The calculation of the standard errors is based on the assumption that every one of the 10 daily observations are samples from the same population with the same mean. This is generally not true since the traffic varies. For example, if the 10 values were taken on two successive weeks, Monday to Friday, we know that certain weekdays have systematically higher traffic than others. On the other hand, if the 10 values were collected on the same week day in 10 consecutive weeks, we know that this is such a long period that the seasonal variations will be appreciable during the period.

It follows that there is no point in using too much mathematics for the determination of measuring accuracy.

5. The number of calls arriving on a group of devices in a telephone system was recorded on a counter. The counter was read off every three minutes. The following values we obtained during the busy hour:

16	13	21	17	23	22	13	18	23	21	19	18	19	28	23	22	20	29	17

a. Calculate the mean value (expected value) of the number of arriving calls per two minutes.

b. Calculate the variance of the number of arriving calls per two minutes.

c. Estimate the 95 % confidence interval of the call intensity.

5. SOLUTION

a. The total number of calls during the hour is summed up to; 398 calls/hour.
b.
During a two minute period it would consequently occur an average 398/30 = 13.27 calls.

c. If we consider a single two minute period we can use the given statistics per three minute interval for estimation of the variance per two minute interval. Another method would be to apply a suitable statistical distribution.

The 20 values for the 3 minutes periods gave:

the mean = 19.9 The variance = 17.88

Assuming that the number of calls in a two minute period is Poisson distributed we have:

mean = 13.27 variance = 13.27

This is an upper limit for the variance. Further statistic analysis would reduce it somewhat.

d. The 95 % confidence limits for the two minute periods are arrived at by multiplying the standard error with 1.96.

The standard error is; $\sigma = \sqrt{13.27} = 3.64$

Therefore the 95 % confidence limits for the number of calls per two minute period is; $6.1 < y_2 < 20.4$.

This means that in one case in 20, the number of calls per two minute period can fall outside this interval without contradicting our hypothesis.

6. The number of calls carried by a group of circuits are counted at intervals of 10 minutes during one hour and the average holding time is 3 minutes. The number of calls in progress simultaneously were;

13, 10, 15, 10, 12

a. Find the traffic carried!

b. Find the average number of calls during the hour!

c. Find the number of calls during a three minute period!

d. What accuracy has the measurement?

6. SOLUTION

a. The average number of occupations are $72/6 = 12$, so the traffic carried is $A' = 12$.

b. The expected number of occupations during one hour are:

$$y = A'/h = 12 \cdot 60/3 = 240$$

c. The number of occupations during a three minute period is $240/20 = 12$ *calls*. Actually, the number of calls per mean holding time is equal to the value of the offered traffic in erlangs. Since we do not know the offered traffic we cannot estimate the number of calls offered to the group per mean holding time. It may be more than 12.

d. We calculate the accuracy from the formula;

$$\sigma^2 = \frac{A}{T} \cdot h \cdot \frac{e^{\lambda} \quad +1}{e^{\lambda} \quad -1}$$

with $A = 12$ $T = 1$ $h = 10$ *minutes* $= 1/6$ *hour*

$s = 3$ *minutes* $= 1/\lambda$ $\lambda h = h/s = 10/3$

$\sigma^2 = 2.15$ $\sigma = 1.47$

The 95 % confidence interval is then $9.1 < A' < 14.9$

7. In a full availability group of 10 circuits the load on the last circuit was observed to be 0.05 erlang during one hour.

   What is your estimate of the traffic offered to the group?

7. SOLUTION

   If we assume Erlang distribution the load on the 10'th circuit is;

   $$a_{10} = A \cdot \left(E_9(A) - E_{10}(A)\right)$$

   We can then try different values of A until it agrees with $a_{10} = 0.05$

A	a_{10}
4	0.0321
5	0.0954
4.5	0.0588
4.3	0.0469
4.35	0.0498

   The offered traffic is about *4.35 Erlangs*. The measured value $a_{10} = 0.05$ *Erlangs* is, however, a poor traffic estimate so we cannot rely much on the above value.

8. A long distance route between two big cities A and B is also used for transit to six other smaller cities. If we want to find the dispersion of the calls with 90% assurance and 90% confidence interval, how many of the calls from A to B must be analysed? The dispersion is roughly as follows;

Town	% of calls
B	60
C	10
D	10
E	4
F	6
G	5
H	5

8. SOLUTION

The precision of such call statistics is least satisfactory for the smallest proportion. The critical part is consequently the calls to town E. We denote:

n = total number of observations on route A B
x = number of calls to E
P = unknown proportion of calls to E then
P = x/n

The variance $v = p(1-p)/n$ standard deviation $\sigma = \sqrt{v}$

We assume a priori that the proportion 4% is correct. Then the 90% confidence interval is:

$$P\left\{-1.645 \cdot \sigma < x/n = p < +1.645 \cdot \sigma\right\} = 0.90$$

The length of the confidence interval is $2 \cdot 1.645 \cdot \sigma$

According to the requirements stated this confidence interval is not permitted to exceed *10% of 4% = 0.004*.

We can then calculate *n* if we set $p = 0.04$

$$2 \cdot 1.645 \cdot \sqrt{\frac{p(1-p)}{n}} \leq 0.004$$

$$\sqrt{\frac{0.04 \cdot 0.96}{n}} \leq 0.001216$$

$$n \leq \frac{0.04 \cdot 0.96}{0.001216^2} = 25978$$

It is consequently necessary to analyse 26,000 calls on the *AB* route to arrive at desired precision for the smallest route.

<u>Comment</u> If the *AB* route carries *100 Erlang* in the busy hour and the average holding time is three minutes, there will be 2000 calls per busy hour. If the study is limited to the busy hours, the observations must cover 13 busy hours. If the observations are made over the whole day we can expect about 16,000 calls per 24 hours. It will then be enough with two days measurements.

It can, however, be questioned if the call dispersion is really the same over the whole day and for the busy hour. Different categories of subscribers may use the route. As concerns observations extended over 13 busy hours one can ask oneself if the results may be influenced, or not, by seasonal variations. Only extensive studies can answer these questions.

What may happen in reality is that the administration reduces the number of calls to be analysed and accepts a lesser precision on the smaller routes. They may try to get complementary information for the route *ABE* in the *E* exchange.

9. A meter was connected so as to receive a pulse every six seconds when a group is fully occupied. During a certain hour the meter counter increased from 2430 to 2439. How much was the time congestion?

9. SOLUTION

The meter indicates that all circuits have been busy during $9 \cdot 6 = 54 \text{ seconds}$. The time congestion is, therefore,

$$E = 54/3600 = 0.015$$

For estimating the precision of this value, we must know the traffic, the number of circuits and the average holding time of the group.

10. A group of 40 circuits is connected to an Ah - meter. The resistances used are 100k-ohm and the voltage is 50V. How many circuits are occupied if the current to the Ah - meter is 10mA? What error is introduced if the voltage goes up to 52V?

10. SOLUTION

The current per circuit is $I = \dfrac{50}{100000} = 0.0005\ A = 0.5\ mA$

If the resulting current is 10mA there are evidently *10/0.5 = 20 circuits* busy.

If the voltage is 52V then the current per device is 0.52mA. The value 10mA then corresponds to $10/0.52 = 19.23$ which means that we will underestimate the traffic if the meter is calibrated for 50V.

11. During one hour three types of measurements were made on a group of circuits.

a. Every 36 seconds (start at $t = 0$) the number of occupations is scanned and added to counter A.

b. Every 2 seconds (start at $t = 0$) the group is scanned. If all devices are occupied counter B is moved one step.

c. The number of calls is registered on counter C. The readings were:

Counter A:	1500
Counter B:	54
Counter C:	500

Estimate the traffic carried, the average holding time and the time congestion!

11. SOLUTION

Scanning is executed every 36 seconds. There are consequently 100 scans/hour. The traffic carried is;

$$A' = 15.00$$

The group had 500 occupations so the average holding time is

$$h = \frac{A}{y} = \frac{15}{500} \cdot 3600 = 108 \text{ seconds}$$

Counter B checks every two seconds if the group is fully occupied. The congestion time is $54 \cdot 2 = 108 \text{ seconds}$ and the time congestion $E = 108/3600 = 0.03$

12. In the country Ut-O-Pia the telecommunication administration decided to apply the CCITT recommendation E 500 for measurements on international automatic relation. They applied it on a national long distance route (STD). The busy hour traffic was recorded on every normal working day during a year. Find how many circuits would be required if the CCITT grade of service standards are applied.

$$E\left(\overline{A}_{30}\right) \leq 0.01 \qquad\qquad E\left(\overline{A}_{5}\right) \leq 0.07$$

The records for the year were the following after doubtful and erroneous records had been sorted out:

January:	33	37	43	48	46	33	38	30	40	45	
February:	43	49	43	45	38	39	53	49	50		
March:	51	42	56	46	59	45	55	52	45	40	
April:	48	49	60	64	47	60	63	57	53	53	51
May:	60	51	66	56	66	65	57	63			
June:	55	58	54	48	55	59	44	45	40	55	
July:	39	26	38	30	25	31	43				
August:	21	28	35	28	27	35	32	26			
September:	42	36	41	45	44	48	46	36	52	51	
October:	48	54	59	50	45	53	64	61	53		
November:	57	60	64	52	56	56	56	66	60	69	
December:	68	58	73	62	61	66	63	61	69	70	

Find the 30 and 5 highest values during the year and estimate how many circuits would be required.

12. SOLUTION

There are 112 values given in the table of which only the 30 highest are of interest.

- Underline the highest value for each month.
- Underline all values ≥ 60: There are 24
- Underline all values ≥ 57: There are 33
- Count how many values = 57: There are 2
- Exclude one value = 58 and there are 30 left.
- Sum up the rest: The sum is 1887 and $A_{30} = 62.9$
- List the values ≥ 68: They are 68, 73, 69, 70
- Look for a value = 67 or 66: They are 66, 66, 66
- The sum of the highest is = 346 and $\overline{A}_5 = 69.2$

January:	33	37	43	48	46	33	38	30	40	45	
February:	43	49	43	45	38	39	53	49	50		
March:	51	42	56	46	59	45	55	52	45	40	
April:	48	49	60	64	47	60	63	57	53	53	51
May:	60	51	66	56	66	65	57	63			
June:	55	58	54	48	55	59	44	45	40	55	
July:	39	26	38	30	25	31	43				
August:	21	28	35	28	27	35	32	26			
September:	42	36	41	45	44	48	46	36	52	51	
October:	48	54	59	50	45	53	64	61	53		
November:	57	60	64	52	56	56	56	66	60	59	
December:	68	58	73	62	61	66	63	61	69	70	

$A_{30} = 62.9$ $\qquad A_5 = 69.2$

$A = 62.59$ $\qquad E_{77}(A) = 0.01$ $\qquad A = 68.53$ $\qquad E_{71}(A) = 0.07$

$A = 63.51$ $\qquad E_{78}(A) = 0.01$ $\qquad A = 69.58$ $\qquad E_{72}(A) = 0.07$

The E_{30} condition requires $n = 78$ *circuits* while the E_5 condition requires only $n = 72$ *circuits*, which may lead to the suspicion that the congestion is high for the very highest value, since only the traffic carried is measured.

However, if we take into account that we deal with carried traffic, our estimate for the traffic offered should be;

$A_{30} = \overline{A}_{30} / (1 - 0.01) = 63.54$ $\qquad$ we get $n = 79$

$A_5 = \overline{A}_5 / (1 - 0.07) = 74.41$ $\qquad$ we get $n = 77$

Therefore, the route should have had 79 circuits.

Remark We observe from the statistics that certain months have very low traffic. The highest value for each month is:

January:	48	July:	39
February:	53	August:	35
March:	59	September:	52
April:	64	October:	64
May	66	November:	66
June:	59	December:	73

The lowest months are July and August and the highest is December. This reflects a certain seasonal variation pattern. It is also possible that the general trend to increase with time is included in the values.

13. Consider the traffic data given in the table below, which provides 3 x 12 monthly traffic values. We assume that the individual values in the table are the results of monthly measurements executed after a defined routine.

- Discuss what monthly measuring routine should be applied to obtain representative figures for forecasting.
- What data would suit the forecaster best if he is going to forecast the traffic 5 years ahead?

<u>TOTAL ORIGINATING TRAFFIC</u>

MONTH	1979	1980	1981
January	38.6	39.4	45.6
February	37.9	43.7	46.2
March	42.1	48.7	47.2
April	40.6	43.8	46.2
May	40.1	40.2	45.6
June	38.1	42.6	48.5
July	37.7	41.1	44.4
August	39.9	44.2	47.4
September	40.4	41.0	49.1
October	40.7	43.8	48.7
November	40.8	41.8	45.0
December	42.2	49.5	49.5

13. SOLUTION

The traffic values given in the table are arrived at according to a given measuring routine. The records have then been treated after a defined routine. We can, therefore, understand that monthly figures are the result of more than one busy hour measurement per month.

A very common practice is to measure during the busy hour one week per month and take the average of these records as the representative value for the month. This is acceptable if the highest week is chosen and the individual traffic values do no vary much. This is a practice which gives risks that the highest values during the month cannot be seen in the monthly traffic value presented.

The forecasting is made so as to avoid too high congestion too frequently. The forecaster is, therefor, interested in the highest traffic values that occur every year. Therefore, hiding these values in averages is not very helpful to him. It would have been better to give him a list of the 10 - 20 highest traffic values observed during the year. This list should be supplemented with observed congestion, etc. on these occasions. This defines a reliable measuring practice!

We can analyse the figures given in order to find out what may be most useful for the forecaster:

If the rows and columns in the table are summed up, we get the following result.

MONTH	1979	1980	1981	TOTAL
January	38.6	39.4	45.6	123.6
February	37.9	43.7	46.2	127.8
March	42.1	48.7	47.2	138.0
April	40.6	43.8	46.2	130.6
May	40.1	40.2	45.6	125.9
June	38.1	42.6	48.5	129.2
July	37.7	41.1	44.4	123.2
August	39.9	44.2	47.4	131.5
September	40.4	41.0	49.1	130.5
October	40.7	43.8	48.7	133.2
November	42.8	41.8	45.0	127.6
December	42.2	49.5	49.5	141.2
<u>TOTALS</u>	479.1	519.8	563.4	1562.3
		1.085	1.084	

The sum of all monthly values for each year shows that this sum or its average, increases rather steadily for each year; 8.5 and 8.4% per annum. We see also that December values are highest every year and that certain other months seem to give almost always low values.

This is further confirmed if we calculate the ratio between each value and its annual mean:

MONTH	1979	1980	1981	AVERAGE
January	0.967	0.910	0.971	0.949
February	0.949	1.009	0.984	0.981
March	1.055	1.124	1.005	1.061
April	1.017	1.011	0.984	1.004
May	1.004	0.928	0.971	0.968
June	0.954	0.984	1.033	0.990
July	0.994	0.749	0.946	0.946
August	0.999	1.020	1.010	1.010
September	1.012	0.947	1.046	1.001
October	1.019	1.011	1.037	1.023
November	1.022	0.965	0.959	0.982
December	1.057	1.143	1.054	1.085

It is evident that measurements in certain months are almost always lower than others, at least in the monthly values that are presented here! They may be averages for a number of measurements. In an average for a low month, such as July, a few odd high values may be hidden.

We conclude that the December values are the most interesting for finding the highest values during the year. These values should, therefore, be given to the forecaster, if the individual busy hour records are not available.

We assume that we are now in the beginning of 1982. We know that the forecaster is going to forecast the traffic up to 1987. We know further that three years historical material is too poor to base a five year forecast on. (But it is enough for December 1983.) We should, therefore, try to find records from earlier years, 1978, 1977 etc.

As concerns the annual increase we see that the mean traffic each year increases 8.4 - 8.5 %. As a coincidence we find that December values from 1979 to 1981 also has increased 8.3%, if we disregard the value for 1980. It is, therefore, likely that the forecaster will estimate such an increase in his forecast.

<u>REMARK:</u> The data presented here is a typical example of the type of data given to the forecasters. Before establishing the measurement practice for records to be taken for forecasting purposes, it is advisable that the problem is discussed with the forecaster.

14. In the neighbour country Teleria measurements were made on eight routes, as shown in the table below. Check the records and point out if any of the data are erroneous.

Observations on some Telerian trunk groups

18 August 1980; 9:30 - 10:30

Trunk group	No. of circuits	Observed traffic Erlangs	Observed congestion %	No. of occupations	Complaints	Other observations
1	18	10.51	1.8	300	None	
2	24	12.03	12	827	Yes	Installation work ongoing
3	36	24.5	6.8	503	None	
4	10	11.52	10.5	27	None	
5	20	18.6	31.5	1865		
6	16	5.0	0	148	None	
7	75	68.0	4	2101	Yes	
8	75	60.0	2.1	1487	None	Completion rate low

14. SOLUTION

Before accepting the records as correct, we can make some checks. Simple checks are:

- Is $A' < n$?
- Are the average holding times credible?
- Does the measured congestion agree with the expected theoretical value?

a. Recorded carried traffic as compared with the number of circuits. Route Number 4 has $A' = 11.52$ *erlang* on $n = 10$ *circuits*, which is impossible. This record is not correct. In all other cases $A' < n$

b. Calculation of the average holding times gives the following results.

Route	h	Credibility
1	126 sec.	OK
2	52	?
3	175	OK
4	1536	NO
5	36	?
6	122	OK
7	117	OK
8	145	OK

Except Route number 4 the holding times for routes number 2 and 5 are exceptionally low, which may mean that there is some error in the records or technical faults in the route.

c. We calculate the theoretical value of the congestion and compare with the recorded losses. We assume that all routes are full availability groups and apply Erlang's first formula. To do so we have to find the offered traffic from the observed carried traffic and the specified number of circuits. We calculate also the expected number of lost calls and its 95% confidence interval and compare with the observed number. We assume that the number of rejected calls, x, are Poisson distributed with the standard error = $\sqrt{x}$ **.** The confidence interval is then:

$$\left(x - 1.96 \cdot \sqrt{x},\ \ x + 1.96 \cdot \sqrt{x}\right)$$

x is calculated for the specified number of occupations and y the number of lost calls.

$$x = (y + x) \cdot E_n(A) \qquad\qquad x = \frac{y \cdot E_n(A)}{1 - E_n(A)}$$

Route	n	A'	A	$E_n(A)$	x	x_{min}-x_{max}	x_{OBS}	Comment
1	18	10.51	10.63	0.0115	3.49	0 - 7.2	5.5	OK
2	24	12.03	12.04	0.0008	0.68	0 - 2.3	113	? - NO
3	36	25.50	24.76	0.0069	3.49	0 - 7.2	37	? - NO
5	20	18.60	30.03	0.3806	1146	1080 - 1212	858	? - ?
6	16	5.00	5.00	0.00005	0.007	0 - 0.2	0	OK
7	75	68.00	73.65	0.0767	175	149 - 200	88	? - NO
8	75	60.00	60.00	0.0096	14.5	7.0 - 22.0	32	? - ?

We can now summarise the outcome of the three checks made:

Route	A' < n	h ?	E	Conclusion
1	OK	OK	OK	OK
2	OK	?	NO	NO
3	OK	OK	NO	NO
4	NO	NO	--	NO
5	OK	?	?	NO
6	OK	OK	OK	OK
7	OK	OK	NO	NO
8	OK	OK	?	NO

Consequently the records of routes number 1 and 6 seem to be alright, all the other records are doubtful. If we go back to the given table it follows that there were really complaints on routes number 2 and 7. It is also noted in the table that the completion rate was low on route number 8. So for these three routes it was expected that there could be something wrong. We further obtained by our checks that the records of routes number 3, 4 and 5 were not correct.

Epilogue The check of the traffic records caused the administration to start fault hunting. After cleaning the routes from disturbances the traffic was served without complaints for some time.

15. On a Tuesday, during the busy hour, six “errors” occurred in an exchange. This was regarded as being too much, so on Wednesday certain adjustments were made. The following Thursday during the busy hour two “errors” occurred. Assume that the traffic offered to the exchange in both busy hours is the same and that the occurrence of errors can be described by a Poisson-process.

    a. Is this reduction in the number of “errors” evidence of improved reliability of the system?

    b. The same question, if the number of errors were 22 before and 9 after the adjustments.

15. SOLUTION

    a. $x_1 = 6$ $\qquad$ $x_2 = 2$

    We apply the hypothesis that both outcomes come from the same Poisson distribution with the same mean.

    We can calculate the probability that a still larger deviation can occur for the same both days, from the formula:

$$P(>) = \left(\frac{1}{2}\right)^x \cdot 2 \cdot \sum_{v1=0}^{x_2} \binom{x}{v_1}$$

    where $x = x_1 + x_2$ $\qquad$ $x_1 = 6$ $\qquad$ $x_2 = 2$

    We then get: $P(>) = (1/2)^8 \cdot 2 \cdot (1 + 8 + 28) = 0.289$

    We have, therefore, no statistical motivation that an improvement has occurred. Further statistics must be taken!

    b. $x_1 = 22$ $\qquad$ $x_2 = 9$

The numbers are so big that we dare to consider the variable:

$$u = \frac{x_1 - x_2 - 1}{\sqrt{x_1 + x_2}} \qquad (x_1 > x_2)$$

as normal distributed (0,1). We find u = 2.16, which means that it is a rather large deviation from the mean = 0. For $u = 1.96$ we have 5% probability of a larger deviation. In this case we can conclude that an improvement has occurred.