Basic Concepts of Teletraffic Theory

(Exercises included)

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Basic Concepts of Teletraffic Theory

Traffic in Erlang = Average no. of simultaneous occupations in a trunk group during a defined period of time.

1 $$\boxed{A = y \cdot s}$$

A = Traffic in Erlang.
y = Call intensity (calls/time unit)
s = Mean holding time

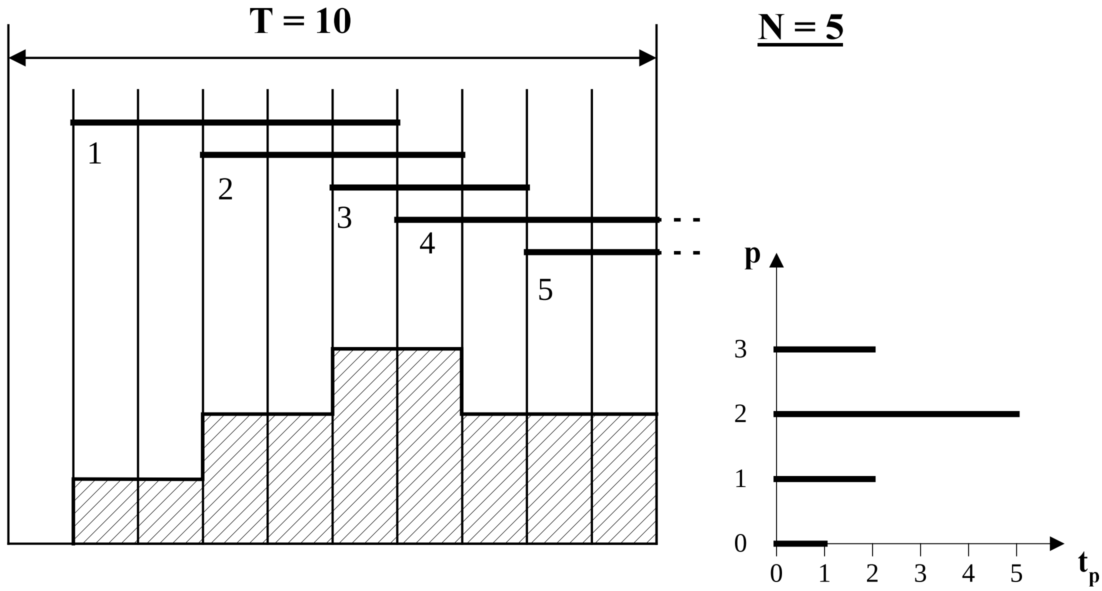
2 $$\boxed{A = \frac{1}{T} \cdot \sum_{v=1}^{N} t_v}$$

T = Length of the time period
t_v = Length of occupation no. v
N = Total no. of occupations

3 $$\boxed{A = \frac{1}{T} \cdot \sum_{p=0}^{n} p \cdot t_p}$$

p = No. of simultaneous occupations in the group.
t_p = Total time with exactly p occupations.
n = Max. no of occupations = no. of trunks.

Example



1 $\boxed{A = y \cdot s}$

$$y = \frac{N}{T} = \frac{5}{10} = \underline{0.5 \text{ calls / time unit}}$$

$$s = \frac{1}{N} \cdot \sum t_v = \frac{1}{5} \cdot (5 + 4 + 3 + 4 + 2) =$$

$$= \frac{1}{5} \cdot 18 = \underline{3.6 \text{ time units}}$$

$$A = y \cdot s = 0.5 \cdot 3.6 = \underline{1.8 \text{ Erlang}}$$

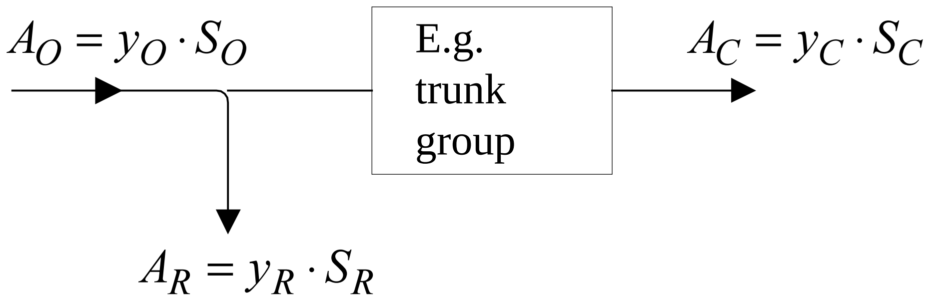
2 $\boxed{A = 1/T \cdot \sum t_v}$

$$A = \frac{1}{10} \cdot 18 = \underline{1.8 \text{ Erlang}}$$

3 $\boxed{A = 1/T \cdot \sum p \cdot t_p}$

$$A = \frac{1}{10} \cdot (0 \cdot 1 + 1 \cdot 2 + 2 \cdot 5 + 3 \cdot 2) =$$

$$= \frac{1}{10} \cdot 18 = \underline{1.8 \text{ Erlang}}$$



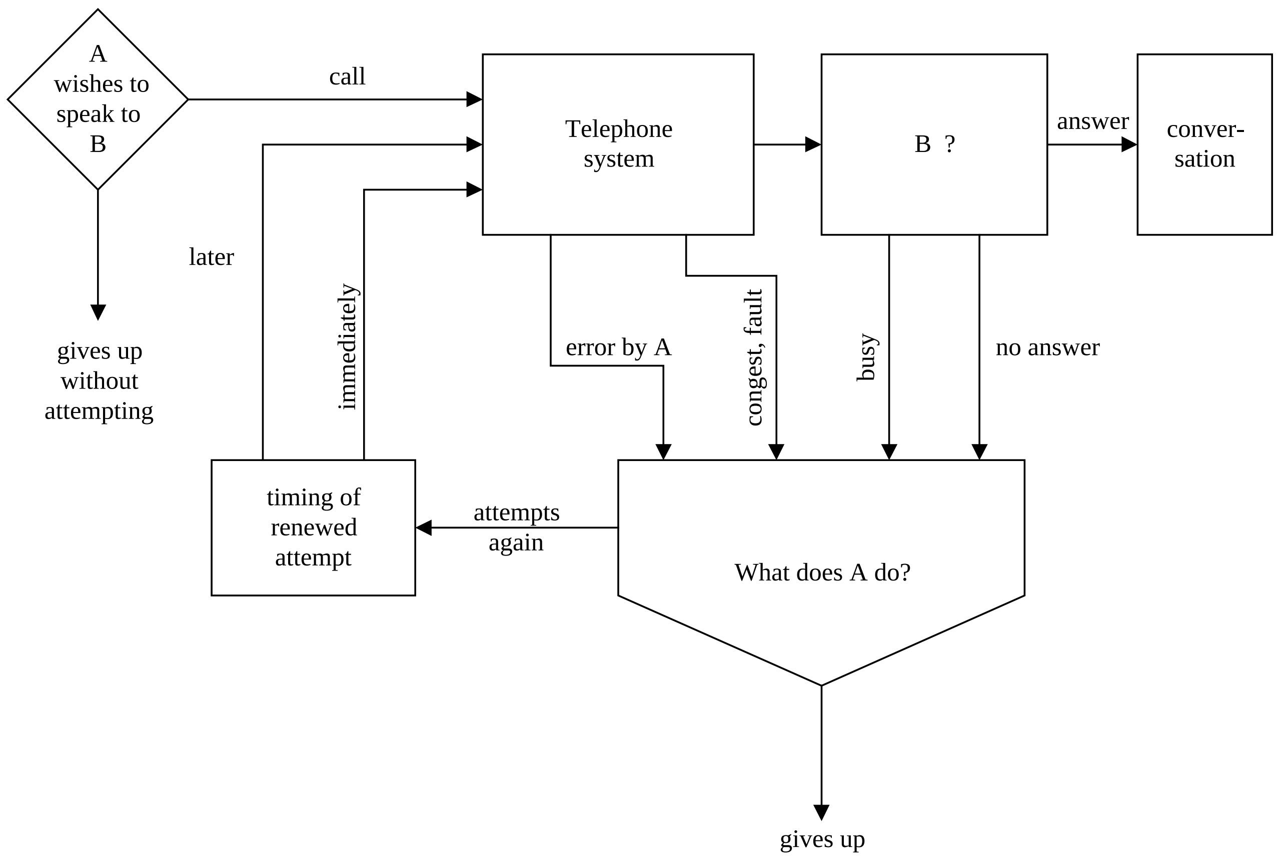
A_O = Offered Traffic
A_C = Carried Traffic
A_R = Rejected Traffic

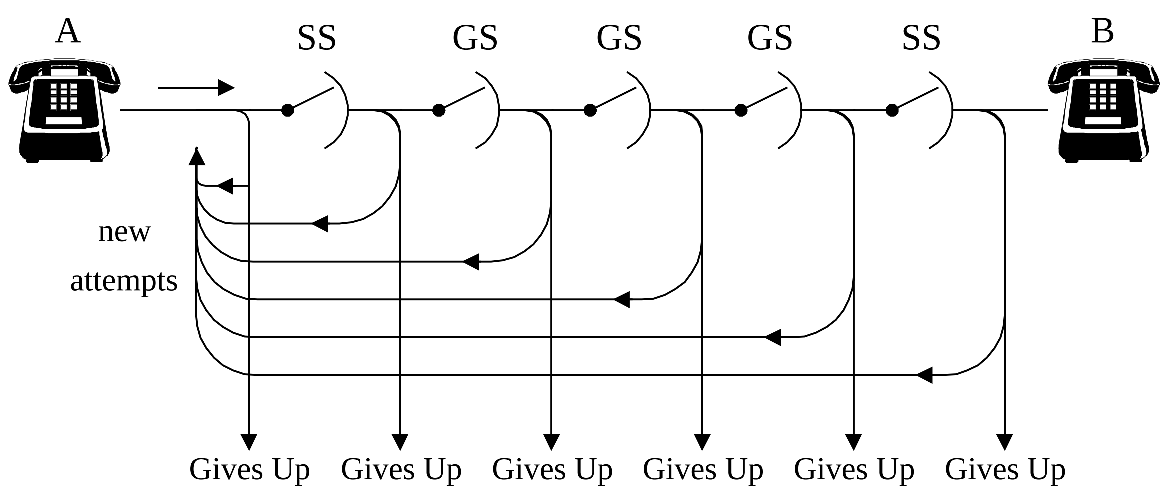
① $y_O = y_C + y_R$ is true!

② $A_O = A_C + A_R$ is convenient for traffic calculations!

③ $S_O = S_C = S_R = S$ is not true, but the consequence of ① + ②!

Therefore, be careful when traffic congestion is high!


A wishes to speak to B
call
Telephone system
B ?
answer
conver-sation
later
immediately
gives up without attempting
error by A
congest, fault
busy
no answer
timing of renewed attempt
attempts again
What does A do?
gives up


A
SS
GS
GS
GS
SS
B
new attempts
Gives Up
Gives Up
Gives Up
Gives Up
Gives Up
Gives Up

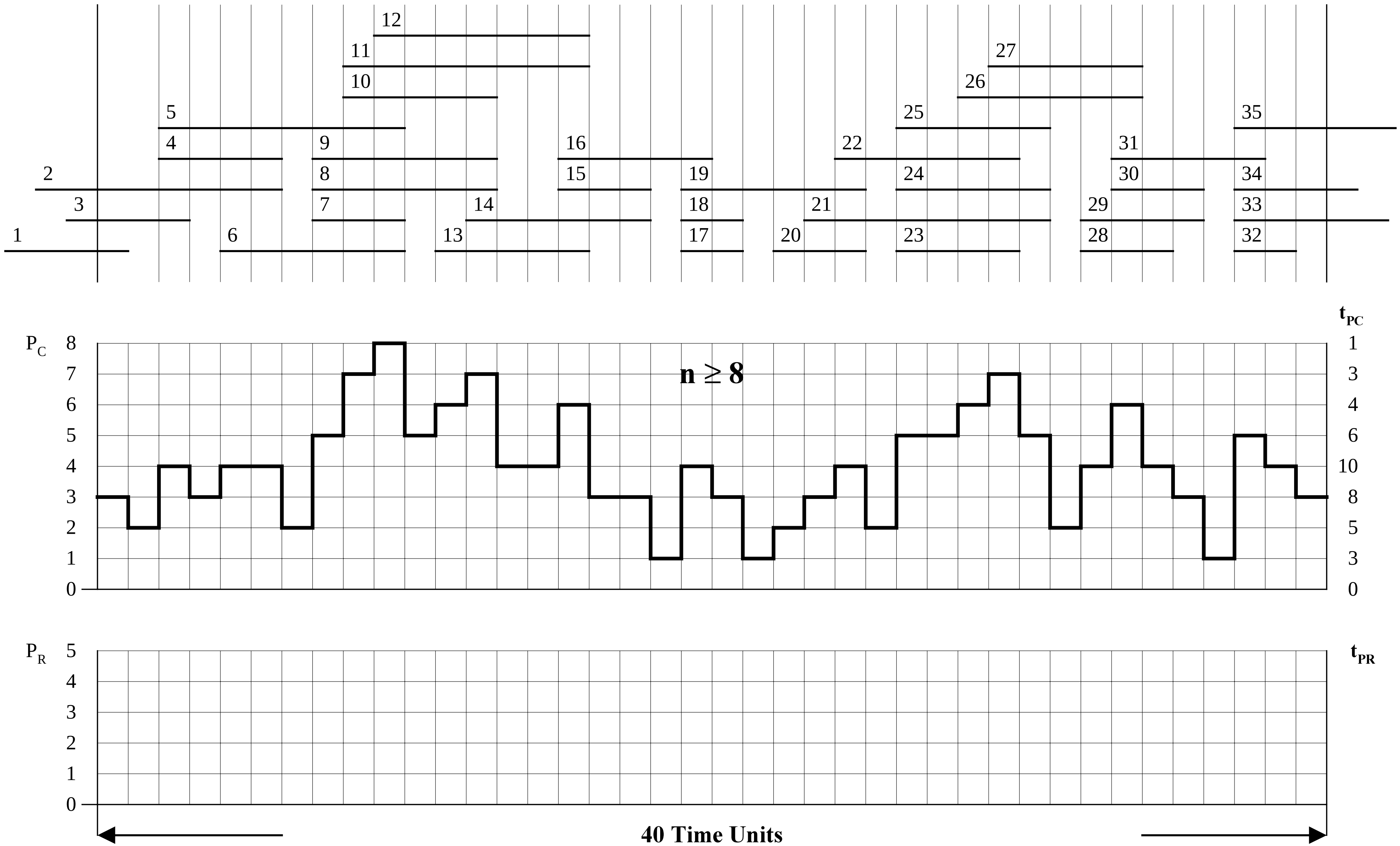
No. of successful calls at 1.st trial

No. of attempts	Original Calls			TOT. No. of Attempts	
i	Totally B	Conversations C	No Conver. A	Totally $T = i \cdot B$	Failures $N = T - C$
1	140	57	83	140	83
2	63	37	26	126	89
3	41	22	19	123	101
4	22	7	15	88	81
5	6	3	3	30	27
6	15	3	12	90	87
7	2	-	2	14	14
8	3	1	2	24	23
9	3	1	2	27	26
11	1	-	1	11	11
19	1	1	-	19	18
Totally	297	132	165	692	560

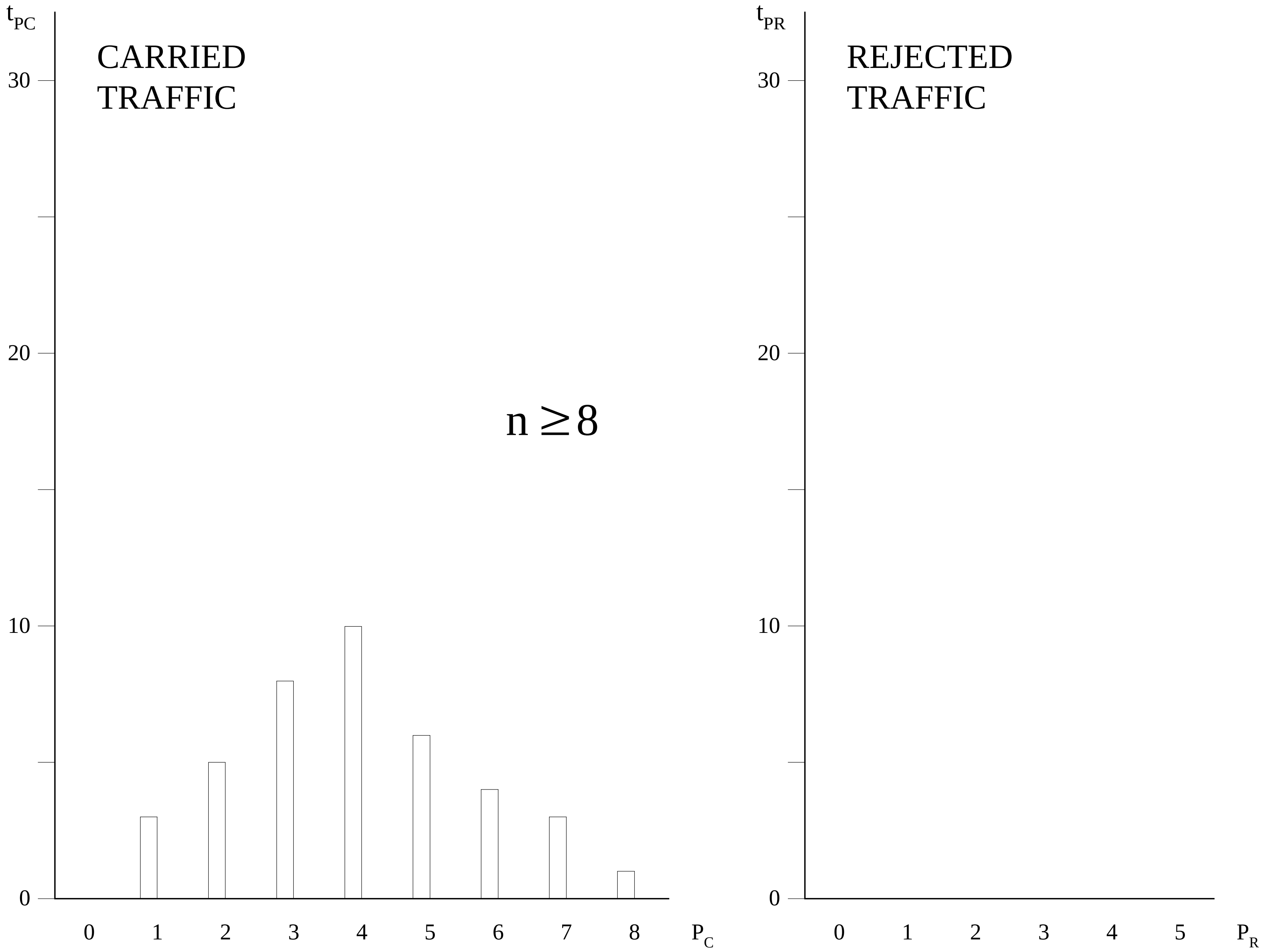
Total no. of desired calls

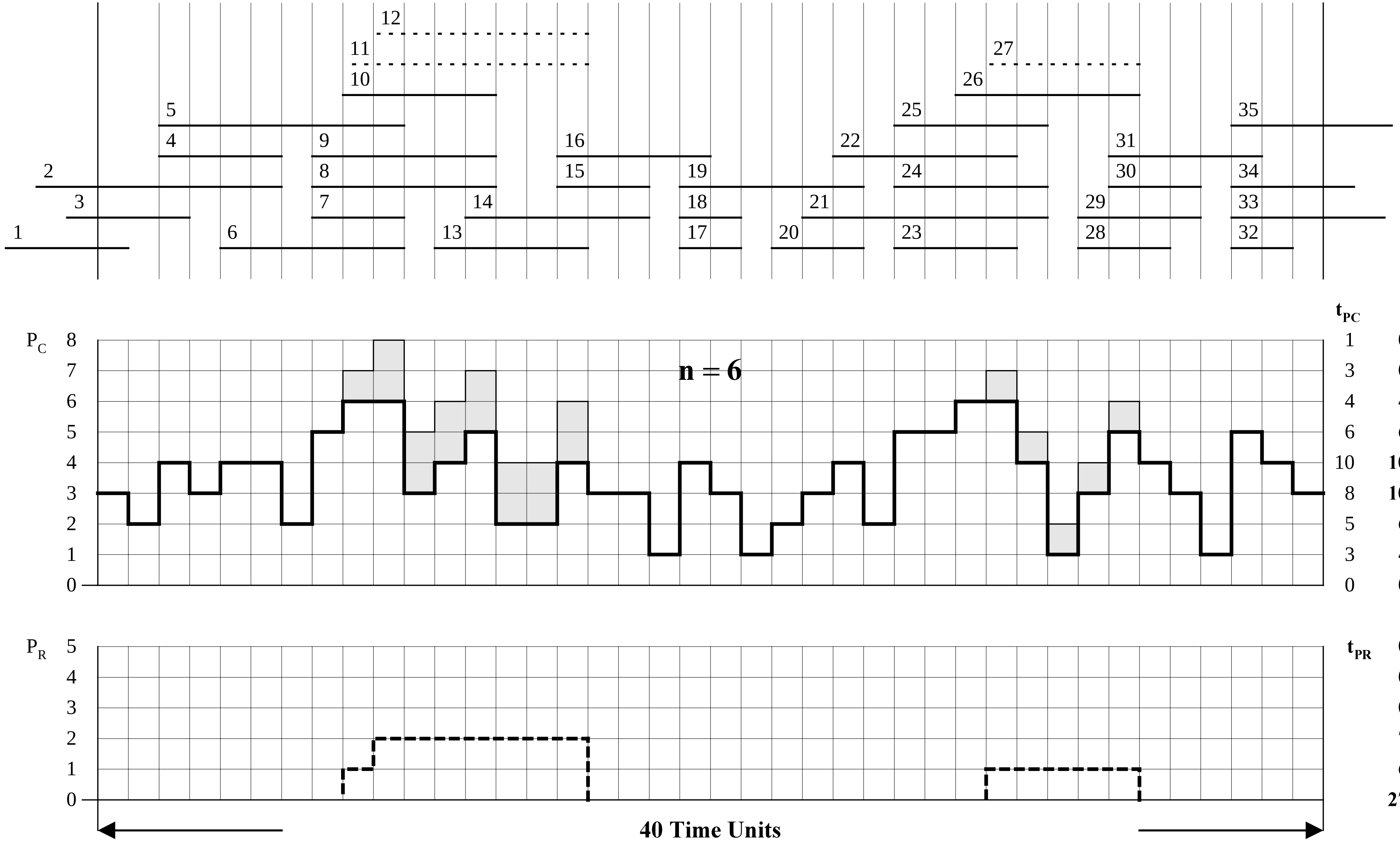
Total no. of successful calls

Total no. of offered calls

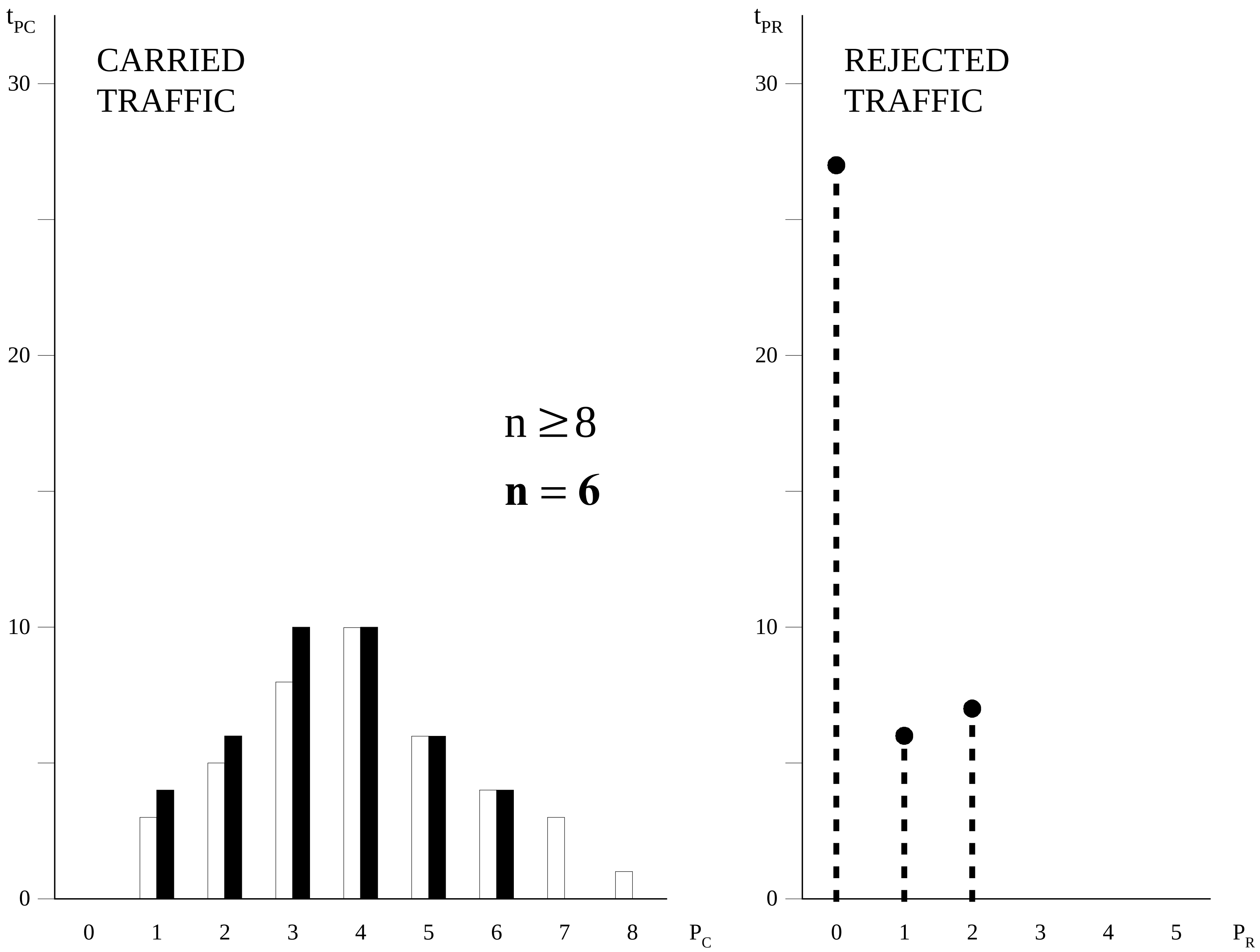


12
11
27
10
26
5
25
35
4
9
16
22
31
2
8
15
19
24
30
34
3
7
14
18
21
29
33
1
6
13
17
20
23
28
32
t_PC
P_C
8 7 6 5 4 3 2 1 0
1 3 4 6 10 8 5 3 0
n ≥ 8
P_R
5 4 3 2 1 0
t_PR
40 Time Units

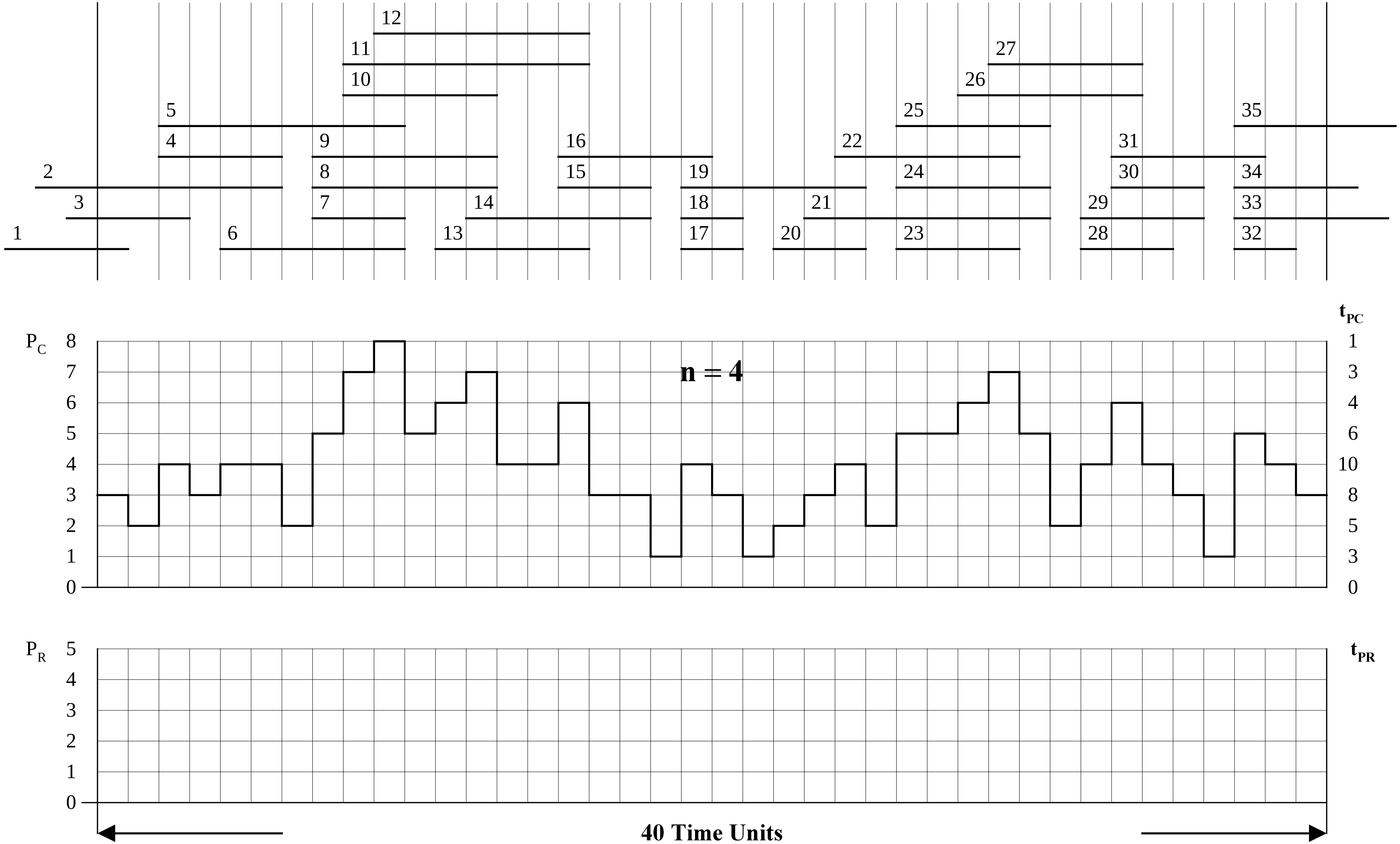

t_{PC}
CARRIED
TRAFFIC
30
20
10
0
$n \geq 8$
0 1 2 3 4 5 6 7 8 P_C
t_{PR}
REJECTED
TRAFFIC
30
20
10
0
0 1 2 3 4 5 P_R

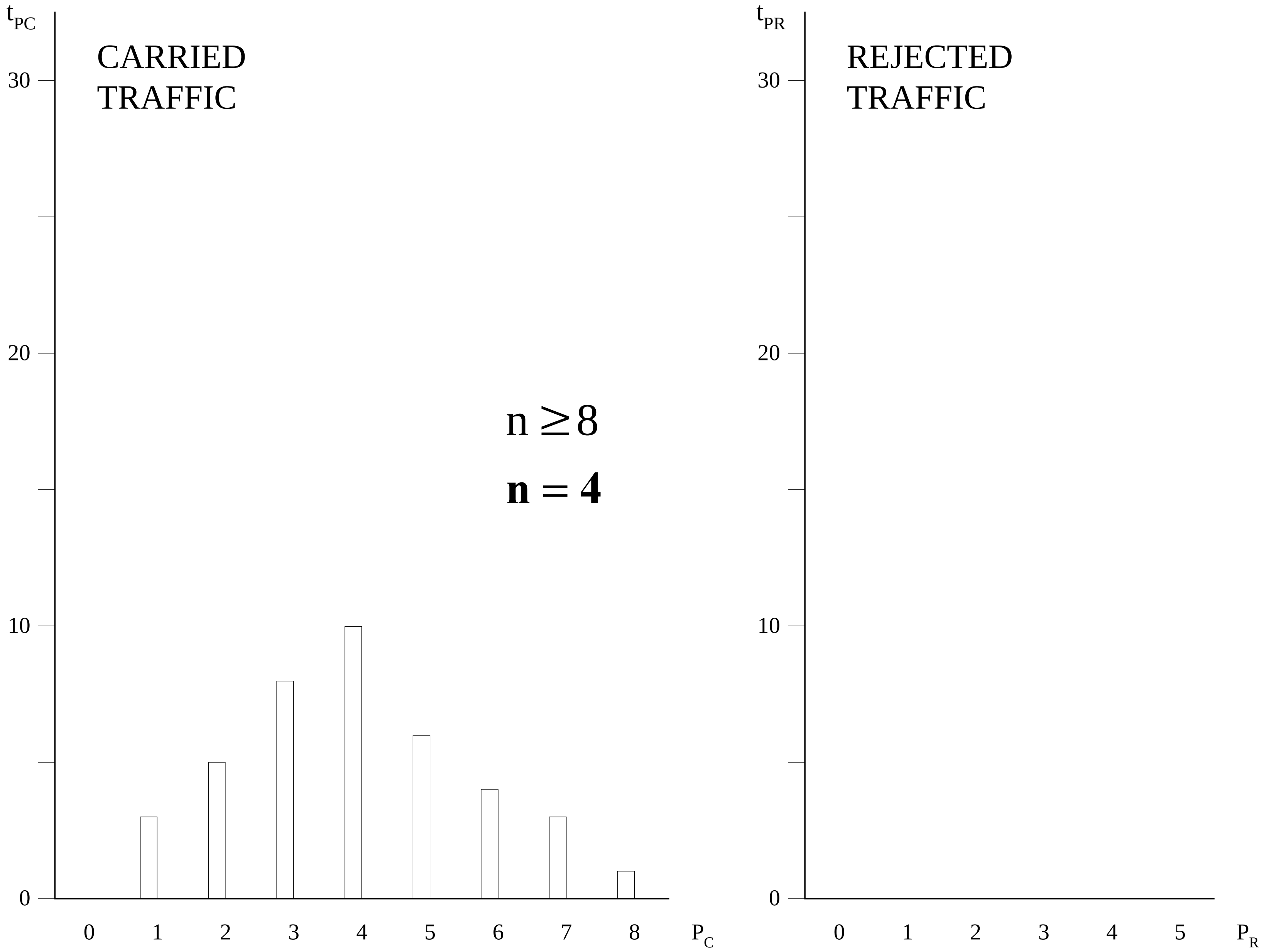


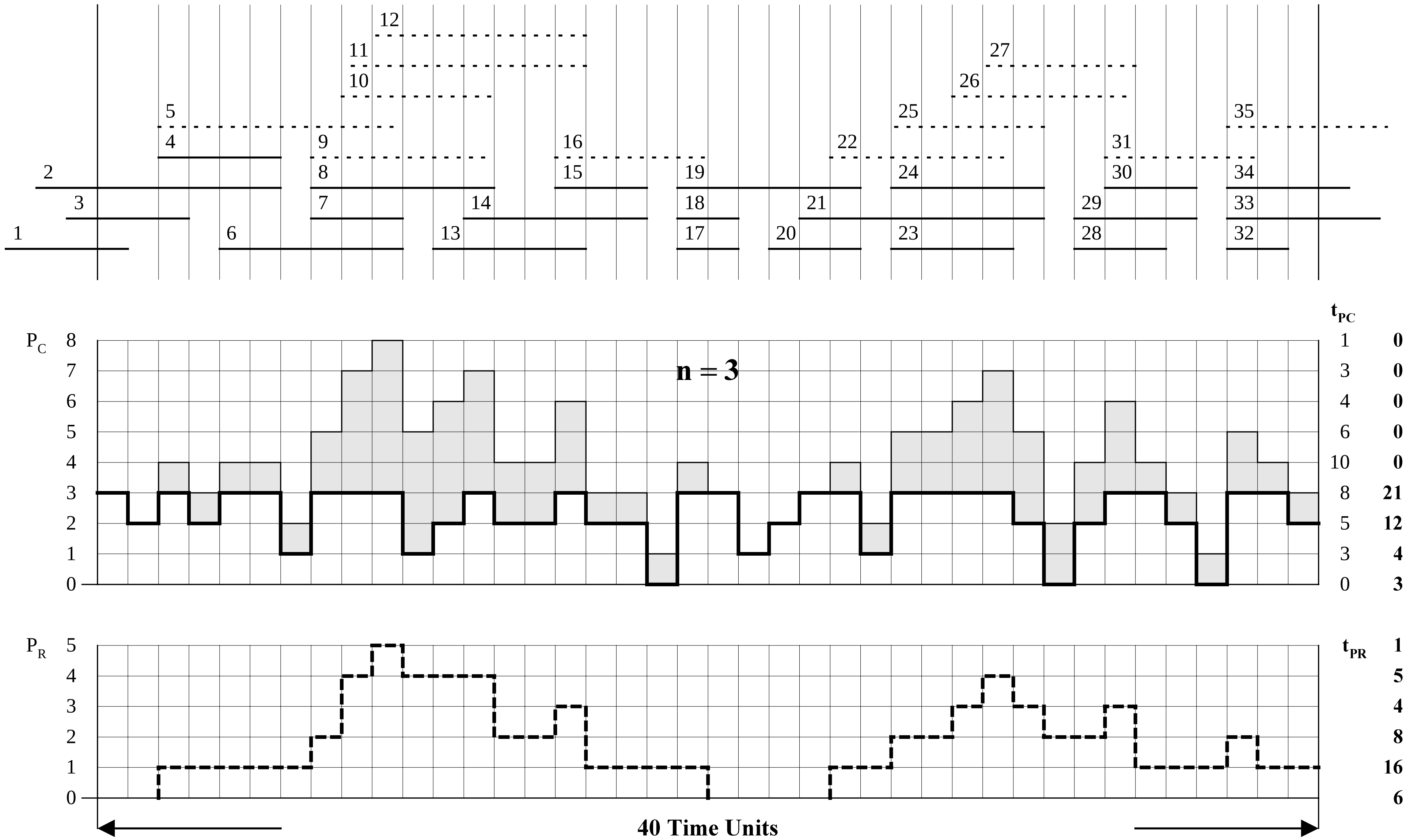
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
P_C
t_{PC}
n = 6
P_R
t_{PR}
40 Time Units

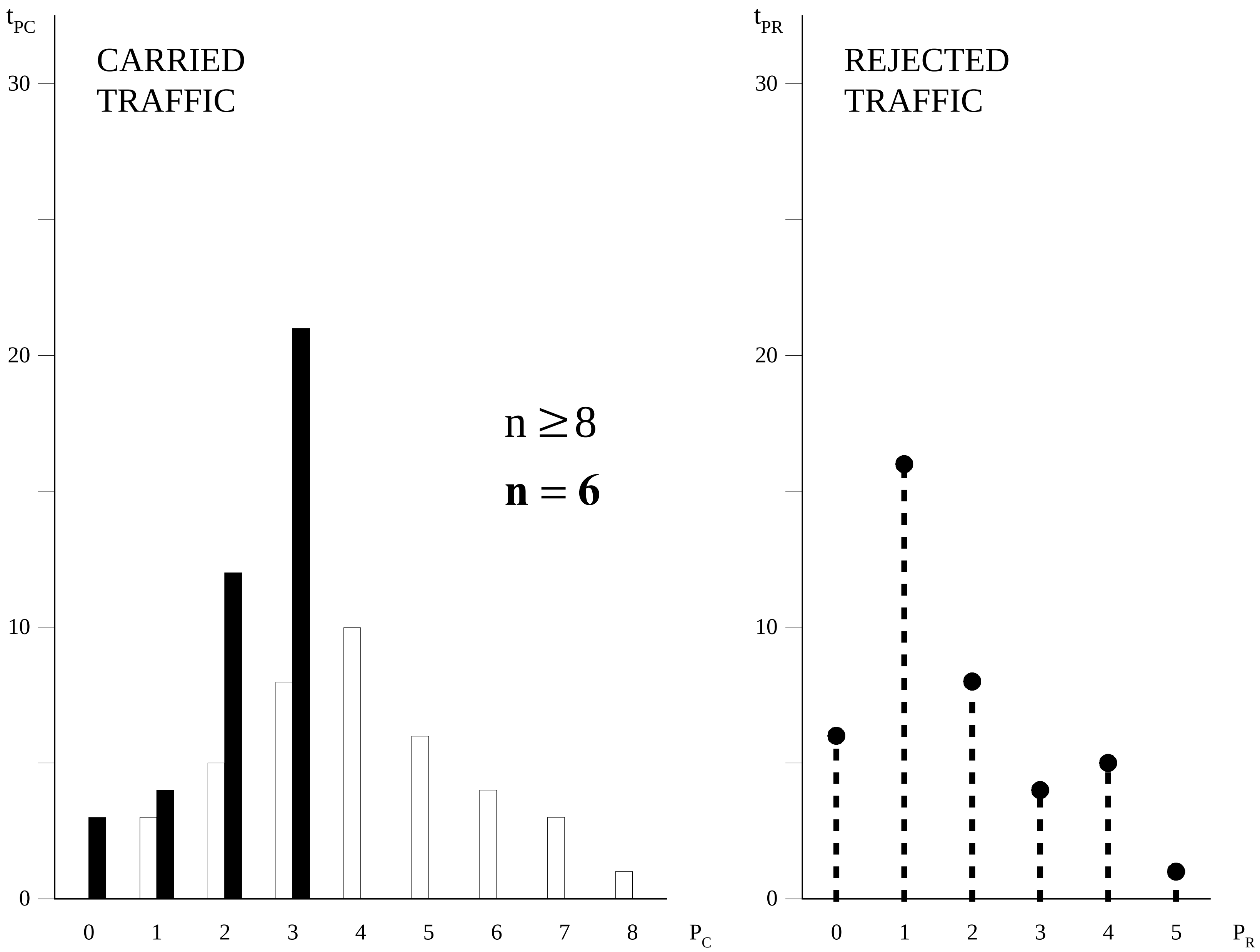

t_{PC}
CARRIED
TRAFFIC
30
20
10
0
$n \geq 8$
n = 6
0 1 2 3 4 5 6 7 8
P_C
t_{PR}
REJECTED
TRAFFIC
30
20
10
0
0 1 2 3 4 5
P_R

Exercise : complete the diagrams for n = 4




t_PC
CARRIED
TRAFFIC
30
20
10
0
n ≥ 8
n = 4
0
1
2
3
4
5
6
7
8
P_C
t_PR
REJECTED
TRAFFIC
30
20
10
0
0
1
2
3
4
5
P_R


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
P_C
t_PC
n = 3
P_R
t_PR
40 Time Units



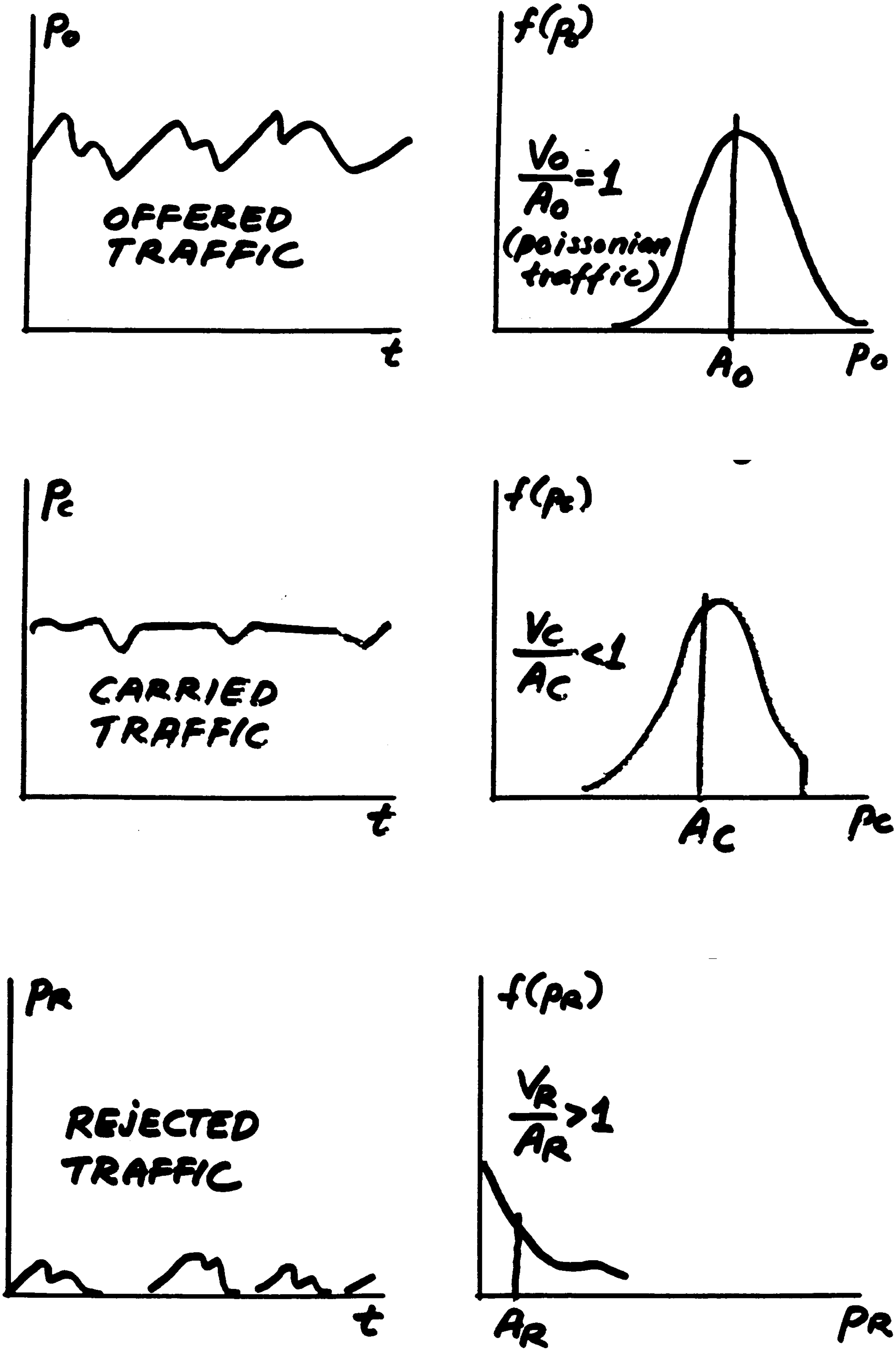
t_PC
CARRIED
TRAFFIC
30
20
10
0
n ≥ 8
n = 6
0 1 2 3 4 5 6 7 8
P_C
t_PR
REJECTED
TRAFFIC
30
20
10
0
0 1 2 3 4 5
P_R

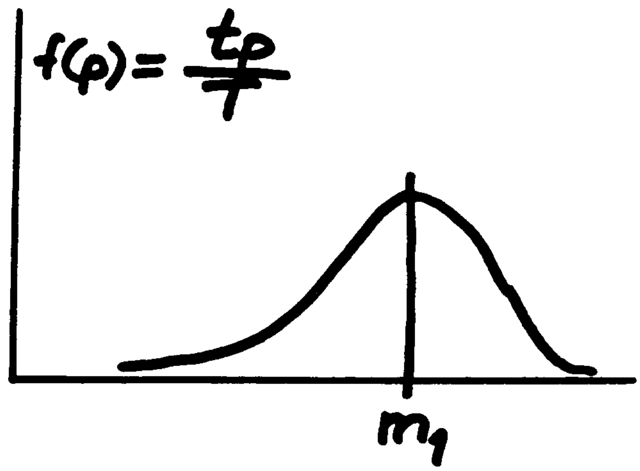
Exercise (cont.)

P_x (x = O, C or R)	Offered Traffic ($n \geq 8$)		n = 6				n = 4				n = 3			
			Carried Traffic		Rejected Traffic		Carried Traffic		Rejected Traffic		Carried Traffic		Rejected Traffic	
	t_{P_o}	$P_o \cdot t_{P_o}$	t_{P_C}	$P_C \cdot t_{P_C}$	t_{P_R}	$P_R \cdot t_{P_R}$	t_{P_C}	$P_C \cdot t_{P_C}$	t_{P_R}	$P_R \cdot t_{P_R}$	t_{P_C}	$P_C \cdot t_{P_C}$	t_{P_R}	$P_R \cdot t_{P_R}$
0					27	0					3	0	6	0
1	3	3	4	4	6	6					4	4	16	16
2	5	10	6	12	7	14					12	24	8	16
3	8	24	10	30							21	63	4	12
4	10	40	10	40									5	20
5	6	30	6	30									1	5
6	4	24	4	24										
7	3	21												
8	1	8												
$\sum$	40	160	40	140	40	20					40	91	40	69
$A_x = \frac{\sum p_x \cdot t_{p_x}}{\sum t_{p_x}}$	160/40 = 4.0 A_O		140/40 = 3.5 A_C		20/40 = 0.5 A_R		A_C		A_R		91/40 =2.3 A_C		69/4 = 1.7 A_R	

Exercise (cont.)

n	"MEASURED"					The Erlang Table:
	Rejected Calls, Nos.	No. of rejected calls	B	E	A_R/A_O	E(=B)
8	-	0	0	1/40 = 0.03	0	0.03
6	11, 12, 27	3	3/32 = 0.09	4/40 = 0.10	05/40 = 0.13	0.12
4						
3	5, 9, 10, 11, 12, 16, 22, 25, 26, 27, 31, 35	12	12/32 = 0.38	21/40 = 0.53	1.7/4.0 = 0.43	0.45


Po
OFFERED TRAFFIC
t
f(Po)
Vo/Ao = 1
(poissonian traffic)
Ao
Po
Pc
CARRIED TRAFFIC
t
f(Pc)
Vc/Ac < 1
Ac
Pc
PR
REJECTED TRAFFIC
t
f(PR)
VR/AR > 1
AR
PR



$$m_1 = m = \sum_{p=0}^{n} p \cdot f(p)$$

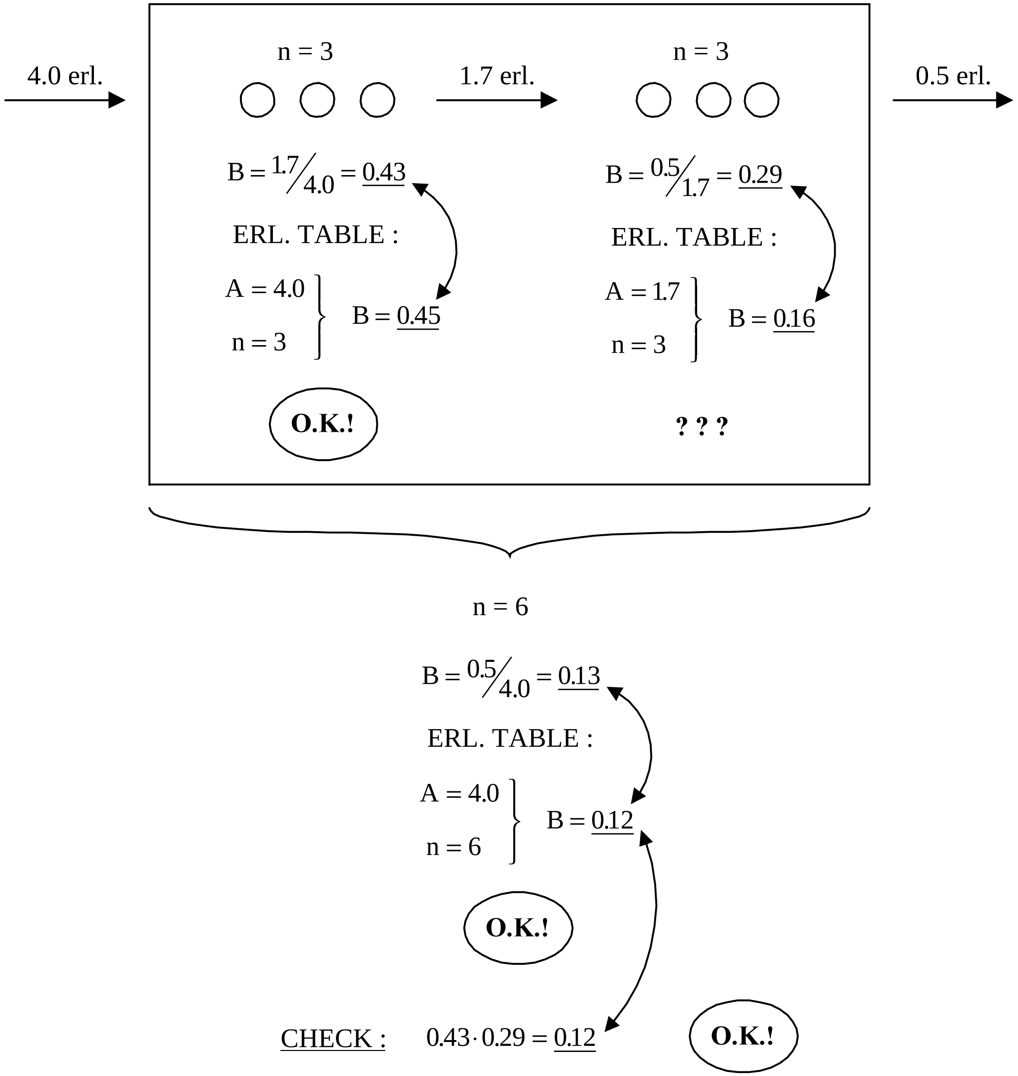
$$m_2 = v = \sum_{p=0}^{n} (p-m)^2 \cdot f(p)$$

for

POISSON TRAFFIC : $\quad$ **m = v = A**

**(= fresh traffic
from many sources)**

	$n \geq 8$ $m = A_0 = 4.0$				$n = 6$ $m=A_C=3.5$				$m=A_R=0.5$			
p	$\|p-m\|$	$\|p-m\|^2$	t_p	$t_p \cdot \|p-m\|^2$	$\|p-m\|$	$\|p-m\|^2$	t_p	$t_p \cdot \|p-m\|^2$	$\|p-m\|$	$\|p-m\|^2$	t_p	$t_p \cdot \|p-m\|^2$
0									0.5	0.25	27	6.75
1	3	9	3	27	2.5	6.25	4	25	0.5	0.25	6	1.5
2	2	4	5	20	1.5	2.25	6	13.5	1.5	2.25	7	15.75
3	1	1	8	8	0.5	0.25	10	2.5				
4	0	0	10	0	0.5	0.25	10	2.5				
5	1	1	6	6	1.5	2.25	6	13.5				
6	2	4	4	16	2.5	6.25	4	25				
7	3	9	3	27								
8	4	16	1	16								
$\sum$				120				82.0				24
v	v=120/40 = 3.0				v=82/40 = 2.1				v=24/40 = 0.6			
v/m	v/m = 3.0/4.0 = 0.75				v/m = 2.1/3.5 = 0.6				v/m = 0.6/0.5 = 1.2			



★ ERL. Table NOT VALID for the last group!
Explanation : the offered traffic (1.7 erl.) NOT FRESH!

<u>Exercise:</u>

Assume T = 10 min

In all 32 new calls arrived.

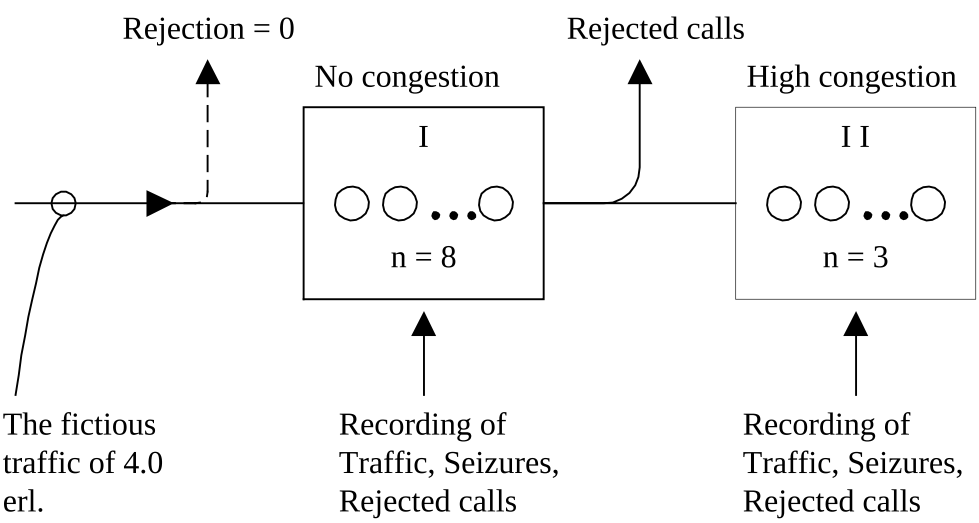
$$y_O = \frac{32}{10} = \underline{3.2\,\text{calls / min.}}$$

$$\underline{A_O = 4.0\,\text{erl}}$$

Then we calculate S_0:

$$S_O = \frac{A_O}{y_O} = \frac{4.0}{3.2} = \underline{1.25\,\text{min}}$$

What values would have been recorded and calculated for the following (fictitious) arrangements?
Each call will occupy a circuit in group I. Immediately after the seizure, group II will be called. If a free circuit is found, the connection will be set up and so both the circuits are occupied the whole holding time. Should however group II reject the call, the circuit in group I would be released immediately.



(cont)

		Group I	Group II
RECORDED VALUES	A_C		
	y_C		
	y_R		
CALC. VALUES	S_C		
	B		
	A_O		