INTERNATIONAL TELECOMMUNICATION UNION

CCITT

E.506 (rev.1)

THE INTERNATIONAL
TELEGRAPH AND TELEPHONE
CONSULTATIVE COMMITTEE

TELEPHONE NETWORK AND ISDN

**QUALITY OF SERVICE,
NETWORK MANAGEMENT AND TRAFFIC
ENGINEERING**

FORECASTING INTERNATIONAL TRAFFIC

Recommendation E.506 (rev.1)

Geneva, 1992

FOREWORD

The CCITT (the International Telegraph and Telephone Consultative Committee) is a permanent organ of the International Telecommunication Union (ITU). CCITT is responsible for studying technical, operating and tariff questions and issuing Recommendations on them with a view to standardizing telecommunications on a worldwide basis.

The Plenary Assembly of CCITT which meets every four years, establishes the topics for study and approves Recommendations prepared by its Study Groups. The approval of Recommendations by the members of CCITT between Plenary Assemblies is covered by the procedure laid down in CCITT Resolution No. 2 (Melbourne, 1988).

Recommendation E.506 was prepared by Study Group II and was approved under the Resolution No. 2 procedure on the 16th of June 1992.

CCITT NOTE

In this Recommendation, the expression "Administration" is used for conciseness to indicate both a telecommunication Administration and a recognized private operating agency.

Recommendation E.506

FORECASTING INTERNATIONAL TRAFFIC[1)]

(revised 1992)

1 Introduction

This Recommendation is the first in a series of three Recommendations that cover international telecommunications forecasting.

In the operation and administration of the international telephone network, proper and successful development depends to a large degree upon estimates for the future. Accordingly, for the planning of equipment and circuit provision and of telephone plant investments, it is necessary that Administrations forecast the traffic which the network will carry. In view of the heavy capital investments in the international network, the economic importance of the most reliable forecast is evident.

The purpose of this Recommendation is to give guidance on some of the prerequisites for forecasting international telecommunications traffic. Base data, not only traffic and call data but also economic, social and demographic data, are of vital importance for forecasting. These data series may be incomplete; strategies are recommended for dealing with missing data. Different forecasting approaches are presented including direct and composite methods, matrix forecasting, and top down and bottom up procedures.

Recommendation E.507 provides guidelines for building forecasting models and contains an overview of various forecasting techniques. Recommendation E.508 covers the forecasting of new international telecommunica-tions services.

2 Base data for forecasting

An output of the international traffic forecasting process is the estimated number of circuits required for each period in the forecast horizon. To obtain these values, traffic engineering techniques are applied to forecast erlangs, a measure of traffic. Figure 1/E.506 outlines two different approaches for determining forecasted erlangs.

The two different strategies for forecasting are the direct strategy and the composite strategy. The first step in either process is to collect raw data. These raw data, perhaps adjusted, will be the base data used to generate the traffic forecasts. Base data may be hourly, daily, monthly, quarterly, or annual. Most Administrations use monthly accounting data for forecasting purposes.

With the direct strategy, the traffic carried in erlangs, or measured usage, for each relation would be regarded as the base data in forecasting traffic growth. These data may be adjusted to account for such occurrences as regeneration (see Recommendation E.500).

In both strategies (direct and composite) it is necessary to convert the carried traffic into offered traffic erlangs. The conversion formula can be found in Recommendation E.501 for the direct strategy and in this Recommendation for the composite strategy.

[1)] The old Recommendation E.506 which appeared in the *Red Book* was split into two Recommendations, revised E.506 and new E.507 and considerable new material was added to both.

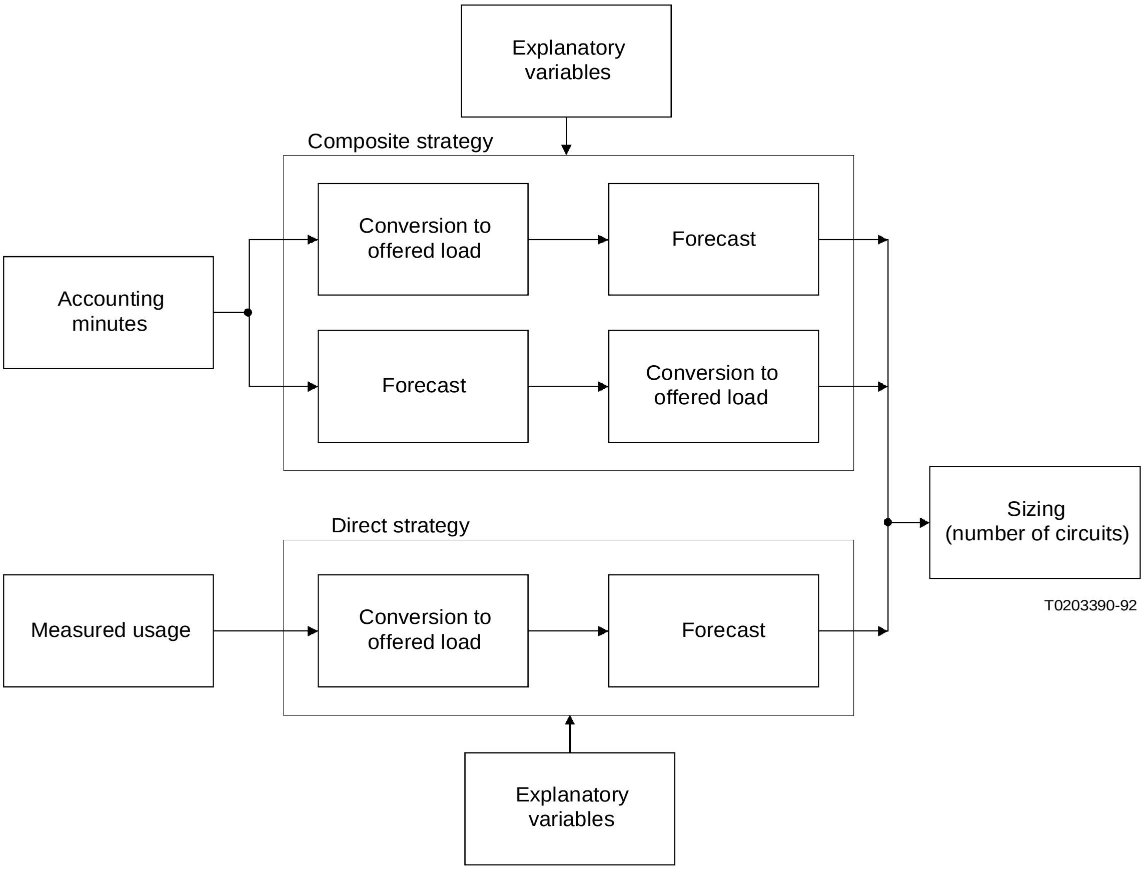

FIGURE 1/E.506

Direct and composite strategy

Composite forecasting uses historical international accounting data of monthly paid minute traffic as the base data. The data may be adjusted by a number of factors, either before or after the forecasting process, that are used for converting paid minutes on the basis of the accounting data into busy-hour erlang forecasts.

As seen in Figure 1/E.506, the forecasting process is common to both the direct and composite strategy. However, the actual methods or models used in the process vary. Forecasts can be generated, for example, using traffic matrix methods (see § 4), econometric models or autoregressive models (see § 3, Recommendation E.507). There are various other data that are input to the forecasting process. Examples of these are explanatory variables, market segmentation information and price elasticities.

Wherever possible, both the direct and composite forecasting strategies should be used and compared. This comparison may reveal irregularities not evident from the use of only one method. Where these are significant, in particular in the case of the busy hour, the causes for the differences should be identified before the resulting forecast is adopted.

In econometric modelling especially, explanatory variables are used to forecast international traffic. Some of the most important of these variables are the following:

– exports,

– imports,

– degree of automation,

– quality of service,

– time differences between countries,

– tariffs,

– consumer price index, and

– gross national product.

Other explanatory variables, such as foreign business travellers and nationals living in other countries, may also be important to consider. It is recommended that data bases for explanatory variables should be as comprehensive as possible to provide more information to the forecasting process.

Forecasts may be based on market segmentation. Base data may be segmented, for example, along regional lines, by business, non-business, or by type of service. Price elasticities should also be examined, if possible, to quantify the impact of tariffs on the forecasting data.

3 Composite strategy – Conversion method

Where both composite and direct strategies are being used and, in any other case, where past data is available, an overall paid-minute/erlang ratio should be derived based on the present value for the particular relation, observed trends and future objectives.

If data is not available, the conversion should be carried out in accordance with the formula.

$$A = Mdh/60e \tag{3-1}$$

where

A is the estimated mean traffic in the busy hour,

M is the monthly paid-minutes,

d is day-to-month ratio,

h is the busy hour-to-day ratio, and

e is the efficiency factor.

The formula is described in detail in Annex A.

4 Procedures for traffic matrix forecasting

4.1 *Introduction*

To use traffic matrix or point-to-point forecasts the following procedures may be used:

– direct point-to-point forecasts,

– Kruithof’s method,

– extension of Kruithof’s method,

– weighted least squares method.

It is also possible to develop a Kalman filter model for point-to-point traffic taking into account the aggregated forecasts. Tu and Pack describe such a model in [16].

The forecasting procedures can be used to produce forecasts of internal traffic within groups of countries, for example, the Nordic countries. Another application is to produce forecasts for national traffic on various levels.

4.2 *Direct point-to-point forecasts*

It is possible to produce better forecasts for accumulated traffic than forecast of traffic on a lower level.

Hence, forecasts of outgoing traffic (row sum) or incoming traffic (column sum) between one country and a group of countries, will give a relatively higher precision than the separate forecasts between countries.

In this situation it is possible to adjust the individual forecasts by taking into account the aggregated forecasts.

On the other hand, if the forecasts of the different elements in the traffic matrix turn out to be as good as the accumulated forecasts, then it is not necessary to adjust the forecasts.

Evaluation of the relative precision of forecasts may be carried out by comparing the ratios $\hat{\sigma}(X)/X$ where X is the forecast and $\hat{\sigma}(X)$ the estimated forecast error.

4.3 *Kruithof 's method*

Kruithof's method [11] is well known. The method uses the last known traffic matrix and forecasts of the row and column sum to make forecasts of the traffic matrix. This is carried out by an efficient iteration procedure.

Kruithof's method does not take into account the change over time in the point-to-point traffic. Because Kruithof's method only uses the last known traffic matrix, information on the previous traffic matrices does not contribute to the forecasts. This would be disadvantageous, especially when the growth of the distinct point-to-point traffic varies. Also when the traffic matrices reflect seasonal data, Kruithof's method may give poor forecasts.

4.4 *Extension of Kruithof 's method*

The traditional Kruithof's method is a projection of the traffic based on the last known traffic matrix and forecasts of the row and column sums.

It is possible to extend Kruithof's method by taking into account not only forecasts of the row and column but also forecasts of point-to-point traffic. Kruithof's method is then used to adjust the point-to-point traffic forecasts to obtain consistency with the forecasts of row and column sums.

The extended Kruithof's method is superior to the traditional Kruithof's method and is therefore recommended.

4.5 *Weighted least squares method*

Weighted least squares method is again an extension of the last method. Let $\{C_{ij}\}$, $\{C_{i\cdot}\}$ and $\{C_{\cdot j}\}$ be forecasts of point-to-point traffic, row sums and column sums respectively.

The extended Kruithof's method assumes that the row and column sums are "true" and adjust $\{C_{ij}\}$ to obtain consistency.

The weighted least squares method [2] is based on the assumption that both the point-to-point forecasts and the row and column sum forecasts are uncertain. A reasonable way to solve the problem is to give the various forecasts different weights.

Let the weighted least squares forecasts be $\{D_{ij}\}$. The square sum Q is defined by:

$$Q = \sum_{ij} a_{ij} (C_{ij} - D_{ij})^2 + \sum_{i} b_i (C_{i\cdot} - D_{i\cdot})^2 + \sum_{j} c_j (C_{\cdot j} - D_{\cdot j})^2 \qquad (4\text{-}1)$$

where $\{a_{ij}\}$, $\{b_i\}$, $\{c_j\}$ are chosen constants or weights.

The weighted least squares forecast is found by:

$$\min_{D_{ij}} Q(D_{ij})$$

subject to

$$D_{i\cdot} = \sum_j D_{ij} \qquad i = 1, 2, \ldots \tag{4-2}$$

and

$$D_{\cdot j} = \sum_i D_{ij} \qquad j = 1, 2, \ldots$$

A natural choice of weights is the inverse of the variance of the forecasts. One way to find an estimate of the standard deviation of the forecasts is to perform ex-post forecasting and then calculate the root mean square error.

The properties of this method are analyzed in [14].

5 Top down and bottom up methods

5.1 *Choice of model*

The object is to produce forecasts for the traffic between countries. For this to be a sensible procedure, it is necessary that the traffic between the countries should not be too small, so that the forecasts may be accurate. A method of this type is usually denoted as “bottom up”.

Alternatively, when there is a small amount of traffic between the countries in question, it is better to start out with forecasting the traffic for a larger group of countries. These forecasts are often used as a basis for forecasts for the traffic to each country. This is done by a correction procedure to be described in more detail below. Methods of this type are called “top down”. The following comments concern the preference of one method over another.

Let $\sigma^{2,T}$ be the variance of the aggregated forecast, and $\sigma^{2,i}$ be the variance of the local forecast No. i and γ_{ij} be the covariance of the local forecast No. i and j. If the following inequality is true:

$$\sigma_T^2 < \sum_i \sigma_i^2 + \sum_{i \neq j} \sum \gamma_{ij} \tag{5-1}$$

then, in general, it is not recommended to use the bottom up method, but to use the top down method.

In many situations it is possible to use a more advanced forecasting model on the aggregated level. Also, the data on an aggregated level may be more consistent and less influenced by stochastic changes compared to data on a lower level. Hence, in most cases the inequality stated above will be satisfied for small countries.

5.2 *Bottom up method*

As outlined in § 5.1 the bottom up method is defined as a procedure for making separate forecasts of the traffic between different countries directly. If the inequality given in § 5.1 is not satisfied, which may be the case for large countries, it is sufficient to use the bottom up method. Hence, one of the forecasting models mentioned in Recommendation E.507 can be used to produce traffic forecasts for different countries.

5.3 *Top down procedure*

In most cases the top down procedure is recommended for producing forecasts of international traffic for a small country. In Annex D a detailed example of such a forecasting procedure is given.

The first step in the procedure is to find a forecasting model on the aggregated level, which may be a rather sophisticated model. Let X_T be the traffic forecasts on the aggregated level and σ_T the estimated standard deviation of the forecasts.

The next step is to develop separate forecasting models of traffic to different countries. Let X_i be the traffic forecast to the i-th country and $\hat{\sigma}_i$ the standard deviation. Now, the separate forecasts $[X_i]$ have to be corrected by taking into account the aggregated forecasts X_T. We know that in general,

$$X_T \neq \sum_i X_i \tag{5-2}$$

Let the corrections of $[X_i]$ be $[X'_i]$, and the corrected aggregated forecast then be $X'_T = \Sigma X'_i$.

The procedure for finding $[X'_i]$ is described in Annex C.

6 Forecasting methods when observations are missing

6.1 *Introduction*

Most forecasting models are based on equally spaced time series. If one observation or a set of observations are missing, it is necessary either to use an estimate of missing observations and then use the forecasting model, or to modify the forecasting model.

All smoothing models are applied on equally spaced observations. Also autoregressive integrated moving average (ARIMA)-models operate on equally spaced time series, while regression models work on irregularly spaced observations without modifications.

In the literature it is shown that most forecasting methods can be formulated as dynamic linear models (DLM). The Kalman filter is a linear method to estimate states in a time series which is modelled as a dynamic linear model. The Kalman filter introduces a recursive procedure to calculate the forecasts in a DLM which is optimal in the sense of minimizing the mean squared one step ahead forecast error. The Kalman filter also gives an optimal solution in the case of missing data.

6.2 *Adjustment procedure based on comparable observations*

In situations when some observations are missing, it may be possible to use related data for estimating the missing observations. For instance, if measurements are carried out on a set of trunk groups in the same area, then the traffic measurements on various trunk groups are correlated, which means that traffic measurements on a given trunk group, to a certain degree, explain traffic measurements on other trunk groups.

When there is high correlation between two time series of traffic measurements, the relative change in level and trend will be of the same size.

Suppose that a time series x_t of equidistant observations from 1 to n has an inside gap · x_t is, for instance, the yearly increase. The gap consists of k missing observations between r and $r + k + 1$.

A procedure for estimating the missing observations is given by the following steps:

i) Examine similar time series to the series with missing observations and calculate the cross correlation.

ii) Identify time series with high cross correlation at lag zero.

iii) Calculate the growth factor Δ_{r+i} between r and $r + k$ of the similar time series y_t:

$$\Delta_{r+i} = \frac{y_{r+i} - y_r}{y_{r+k+1} - y_r} \qquad i = 1, 2, \ldots k \tag{6-1}$$

iv) Estimates of the missing observations are then given by:

$$\hat{x}_{r+i} = x_r + \Delta_{r+i} (x_{r+k+1} - x_r) \qquad i = 1, 2, \ldots k \tag{6-2}$$

Example

Suppose we want to forecast the time series x_t. The series is observed from 1 to 10, but the observations at time 6, 7 and 8 are missing. However a related time series y_t is measured. The measurements are given in Table 1/506.

TABLE 1/E.506

Measurements of two related time series; one with missing observations

t	1	2	3	4	5	6	7	8	9	10
x_t	100	112	125	140	152	–	–	–	206	221
y_t	300	338	380	422	460	496	532	574	622	670

The last known observation of x_t before the gap at time 5 is 152, while the first known observation after the gap at time 9 is 206.

Hence $r = 5$ and $k = 3$. The calculation gives:

$$\Delta_6 = \frac{496 - 460}{622 - 460} = \frac{36}{162}$$

$$\Delta_7 = \frac{532 - 460}{622 - 460} = \frac{72}{162}$$

$$\Delta_8 = \frac{574 - 460}{622 - 460} = \frac{114}{162}$$

$$\hat{x}_6 = 152 + \frac{36}{162} (206 - 152) = \underline{164}$$

$$\hat{x}_7 = 152 + \frac{72}{162} (206 - 152) = \underline{176}$$

$$\hat{x}_8 = 152 + \frac{114}{162} (206 - 152) = \underline{190}$$

6.3 *Modification of forecasting models*

The other possibility for handling missing observations is to extend the forecasting models with specific procedures. When observations are missing, a modified procedure, instead of the ordinary forecasting model, is used to estimate the traffic.

To illustrate such a procedure we look at simple exponential smoothing. The simple exponential smoothing model is expressed by:

$$\hat{\mu}_t = (1 - a)\, y_t + a\hat{\mu}_{t-1} \qquad (6\text{-}3)$$

where

y_t is the measured traffic at time t,

$\hat{\mu}_t$ is the estimated level at time t,

a is the discount factor [and $(1 - a)$ is the smoothing parameter].

Equation (6-3) is a recursive formula. The recursion starts at time 1 and ends at n if no observation is missing. Then a one step ahead forecast is given by:

$$\hat{y}_t(1) = \hat{\mu}_t \tag{6-4}$$

If some observations lying in between 1 and n are missing, then it is necessary to modify the recursion procedure. Suppose now that $y_1, y_2, \ldots, y_r, y_{r+k+1}, y_{r+k+2}, \ldots, y_n$ are known and $y_{r+1}, y_{r+2}, \ldots, y_{r+k}$ are unknown. Then the time series has a gap consisting of k missing observations.

The following modified forecasting model for simple exponential smoothing is proposed in Aldrin [2].

$$\hat{\mu}_t = \begin{cases} (1 - a)\, y_t + a\,\hat{\mu}_{t-1} & t = 1, 2, \ldots r \\ (1 - a_k)\, y_t + a_k\,\hat{\mu}_t & t = r + k + 1 \\ (1 - a)\, y_t + a\,\hat{\mu}_{t-1} & t = r + k + 2, \ldots n \end{cases} \tag{6-5}$$

where

$$a_k = \frac{a}{1 + k(1 - a)^2} \tag{6-6}$$

By using the (6-5) and (6-6) it is possible to skip the recursive procedure in the gap between r and $r + k + 1$.

In Aldrin [2] similar procedures are proposed for the following forecasting models:

– Holt's method,

– Double exponential smoothing,

– Discounted least squares method with level and trend,

– Holt-Winters seasonal methods.

Wright [17] and [18] also suggests specific procedures to modify the smoothing models when observations are missing.

As mentioned in the first paragraph, regression models are invariant of missing observations. When using the least squares method, all observations are given the same weight. Hence, missing observations do not affect the estimation procedure and forecast are made in the usual way.

On the other hand it is necessary to modify ARIMA models when observations are missing. In the literature several procedures are suggested in the presence of missing data. The basic idea is to formulate the ARIMA model as a dynamic linear model. Then the likelihood function is easy to obtain and the parameters in the model can be estimated recursively. References to work on this field are Jones [9] and [10], Harvey and Pierse [8], Ansley and Kohn [3] and Aldrin [2].

State space models or dynamic linear models and the Kalman filter are a large class of models. Smoothing models, ARIMA models and regression models may be formulated as dynamic linear models. This is shown, for instance, in Abraham and Ledolter [1]. Using dynamic linear models and the Kalman filter the parameters in the model are estimated in a recursive way. The description is given, for instance, in Harrison and Stevens [7], Pack and Whitaker [13], Moreland [12], Szelag [15] and Chemouil and Garnier [6].

In Jones [9] and [10], Barham and Dunstan [4], Harvey and Pierse [8], Aldrin [2] and Bølviken [5], it is shown how the dynamic linear models and the Kalman filter handle missing observations.

ANNEX A

(to Recommendation E.506)

Composite strategy – Conversion method

A.1 *Introduction*

This annex describes a method for estimating international traffic based on monthly paid-minutes and a number of conversion factors. It demonstrates the method by examining the factors and showing their utility.

The method is seen to have two main features:

1) Monthly paid-minutes exchanged continuously between Administrations for accounting purposes, provide a large and continuous volume of data.

2) The conversion factors are directly impacted by operating changes such as tariffs and network improvements allowing forecasts to reflect goals for programmes in these areas.

A.2 *Basic procedure*

A.2.1 *General*

The conversions should be specific to each traffic relation. They may also be done separately for each traffic direction and class, but in such cases care must be taken in the handling of collect, credit card, home direct and freephone calls which may appear in the opposite direction from actual traffic in the accounting records.

The estimated mean carried busy-hour traffic (in erlangs) is derived from the monthly paid-minutes using the formula:

$$A = Mdh/60e \tag{A-1}$$

where

A is the estimated mean traffic in erlangs carried in the busy-hour,

M is the total monthly paid-minutes,

d is the day/month ratio, i.e. the ratio of average weekday paid-time to monthly paid-time,

h is the busy-hour/day ratio, i.e. the ratio of the busy-hour paid-time to the average daily paid-time,

e is the efficiency factor, i.e. the ratio of busy-hour paid-time to busy-hour occupied-time.

A.2.2 *Monthly paid-minutes (M)*

Monthly paid-minute figures are exchanged between Administrations for accounting purposes and consequently, historical records covering many years, are normally readily available. In some applications (e.g. long-term forecasts) it may be desirable to use annual paid-minutes. In this case, the same formula is applied but with an additional factor to relate annual paid-minutes to busy month paid-minutes.

A.2.3 *Day/month ratio (d)*

This ratio is related to the amount of traffic carried on a typical weekday compared with the total amount of traffic carried in a month.

This factor reflects both the number of weekdays in a month and the relative levels of weekday and non-weekday traffic.

It may be based on:

- "typical" global averages (e.g. d in range 0.03-0.04); or
- sample measurements for the relation; or
- number of days and social interest calculations for the relation and month; or
- objectives for tariff and promotion programmes to reduce day to day variations.

It should be noted that where non-weekday traffic exceeds weekday traffic, it may be desirable to change the forecasting and dimensioning base to take this into account.

A.2.4 *Busy-hour/day ratio (h)*

The relative amount of average weekday traffic in the busy-hour primarily depends on the difference between the local time at origin and destination. Moderately successful attempt have been made to predict the diurnal distribution of traffic based on this information together with supposed "degree of convenience" at origin and destination. However, sufficient discrepancies exist to warrant measuring the diurnal distribution, from which the busy-hour/day ratio may be calculated.

Where measurement data is not available, a good starting point is Recommendation E.523. From the theoretical distributions found in Recommendation E.523, one finds variations in the busy-hour/day ratio from 10% for 0 to 2 hours time difference and up to 13.5% for 7 hours time difference.

This ratio is influenced by subscriber perceptions of quality and tariff policies. For some applications it may be desirable to choose a value based on the objectives for service improvement, traffic promotion or tariff programmes. Based on experience in the long-term this could reduce h to 6% or less.

A.2.5 *Efficiency factor (e)*

The efficiency factor (ratio of busy-hour paid time to busy-hour occupied time) e converts the paid time into a measure of total circuit occupancy.

There is a tendency for the efficiency to change with time. In this regard, efficiency is mainly a function of operating method (manual, semi-automatic, international subscriber dialling), in the B subscriber's availability, and the quality of the distant network.

Forecasts of the efficiency can be made on the basis of extrapolation of past trends together with adjustments for planned improvements.

The detailed consideration of efficiency including measurements is also an advantage from an operational viewpoint in that it may be possible to identify improvements that may be made, and quantify the benefits deriving from such improvements.

For automatic working with modern signalling systems, e can attain values in excess of 0.9.

A.2.6 *Mean offered busy hour traffic*

It should be noted that A is the mean offered busy-hour traffic expressed in erlangs.

Offered traffic can be approximated by:

- considering it equal to carried traffic (where blocking is not significant or is unknown); or
- using the methods of Recommendation E.501.

A.3 *Overall ratios*

The detailed conversion described above provides insight into the factors influencing total traffic efficiency and increases the accuracy of short-term forecasts.

For long-term forecasts and other applications where detailed measurements are not available or inappropriate, it is sufficient to consider "typical" or target values for the overall erlang to paid-minute ratio. These might range from 1/10 000 for inefficient existing international/relations in 1990, to 1/25 000 as a long-term objective for more efficient relations.

For some applications it might be practical to do long-term forecasts directly on the basis of circuit to paid-minute ratios.

ANNEX B

(to Recommendation E.506)

Example using weighted least squares method

B.1 *Telex data*

The telex traffic between the following countries has been analyzed:

– Germany (D)

– Denmark (DNK)

– USA (USA)

– Finland (FIN)

– Norway (NOR)

– Sweden (S).

The data consists of yearly observations from 1973 to 1984 [19].

B.2 *Forecasting*

Before using the weighted least squares method, separate forecasts for the traffic matrix have to be made. In this example a simple ARIMA (0,2,1) model with logarithmic transformed observations without explanatory variables is used for forecasting. It may be possible to develop better forecasting models for the telex traffic between the various countries. However, the main point in this example only is to illustrate the use of the weighted least squares technique.

Forecasts for 1984 based on observations from 1973 to 1983 are given in Table B-1/E.506.

It should be noticed that there is no consistency between row and column sum forecasts and forecasts of the elements in the traffic matrix. For instance, the sum of forecasted outgoing telex traffic from Germany is 28 005, while the forecasted row sum is 27 788.

To adjust the forecasts to get consistency and to utilize both row/column forecasts and forecasts of the traffic elements, the weighted least squares method is used.

TABLE B-1/E.506

Forecasts for telex traffic between Germany (D), Denmark (DNK), USA (USA), Finland (FIN), Norway (NOR) and Sweden (S) in 1984

From / To	D	DNK	USA	FIN	NOR	S	Sum	Forecasted sum
D	–	4869	12 630	2879	2397	5 230	28 005	27 788
DNK	5 196	–	1 655	751	1270	1 959	10 831	10 805
USA	11 103	1313	–	719	1657	2 401	17 193	17 009
FIN	2 655	715	741	–	489	1 896	6 496	6 458
NOR	2 415	1255	1 821	541	–	1 548	7 580	7 597
S	4 828	1821	2 283	1798	1333	–	12 063	12 053
Sum	26 197	9973	19 130	6688	7146	13 034		
Forecasted sum	26 097	9967	19 353	6659	7110	12 914		

B.3 *Adjustment of the traffic matrix forecasts*

To be able to use the weighted least squares method, the weights and the separate forecasts are needed as input. The separate forecasts are found in Table B-2/E.506, while the weights are based on the mean squared one step ahead forecasting errors.

Let y_t be the traffic at time t. The ARIMA (0,2,1) model with logarithmic transformed data is given by:

$$z_t = (1 - B)^2 \ln y_t = (1 - \theta B)\, a_t$$

or

$$z_t = a_t - \theta a_{t-1}$$

where

$z_t = \ln y_t - 2 \ln y_{t-1} + \ln y_{t-2}$,

a_t is white noise,

θ is a parameter,

B is the backwards shift operator.

The mean squared one step ahead forecasting error of z_t is:

$$MSQ = \frac{1}{n} \sum (z_t - \hat{z}_{t-1}(1))^2$$

where

$\hat{z}_{t-1}(1)$ is the one step ahead forecast.

The results of using the weighted least squares method is found in Table B-3/E.506 and show that the factors in Table B-1/E.506 have been adjusted. In this example only minor changes have been performed because of the high conformity in the forecasts of row/column sums and traffic elements.

TABLE B-2/E.506

Inverse weights as mean as squared one step ahead forecasting errors of telex traffic (100^{-4}) between Germany (D), Denmark (DNK), USA (USA), Finland (FIN), Norway (NOR) and Sweden (S) in 1984

From / To	D	DNK	USA	FIN	NOR	S	Sum
D	–	28.72	13.18	11.40	8.29	44.61	7.77
DNK	5.91	–	43.14	18.28	39.99	18.40	10.61
USA	23.76	39.19	–	42.07	50.72	51.55	21.27
FIN	23.05	12.15	99.08	–	34.41	19.96	17.46
NOR	21.47	40.16	132.57	24.64	–	17.15	20.56
S	6.38	12.95	28.60	28.08	8.76	–	6.48
Sum	6.15	3.85	14.27	9.55	12.94	8.53	

TABLE B-3/E.506

Adjusted telex forecasts using the weighted least squares method

From / To	D	DNK	USA	FIN	NOR	S	Sum
D	–	4850	12 684	2858	2383	5 090	27 865
DNK	5 185	–	1 674	750	1257	1 959	10 825
USA	11 001	1321	–	717	1644	2 407	17 090
FIN	2 633	715	745	–	487	1 891	6 471
NOR	2 402	1258	1 870	540	–	1 547	7 617
S	4 823	1817	2 307	1788	1331	–	12 066
Sum	26 044	9961	19 280	6653	7102	12 894	

ANNEX C

(to Recommendation E.506)

Description of a top down procedure

Let

X_T be the traffic forecast on an aggregated level,

X_i be the traffic forecast to country i,

$\hat{\sigma}_T$ be the estimated standard deviation of the aggregated forecast,

$\hat{\sigma}_i$ be the estimated standard deviation of the forecast to country i.

Usually

$$X_T \neq \sum_i X_i \qquad \text{(C-1)}$$

so that it is necessary to find a correction

$$[X'_i] \text{ of } [X_i] \text{ and } [X'_T] \text{ of } [X_T]$$

by minimizing the expression

$$Q = \alpha_0 (X_T - X'_T)^2 + \sum_i \alpha_i (X_i - X'_i)^2 \qquad \text{(C-2)}$$

subject to

$$X'_T = \sum_i X'_i \qquad \text{(C-3)}$$

where α and $[\alpha_i]$ are chosen to be

$$\alpha_0 = \frac{1}{\hat{\sigma}_T^2} \text{ and } \alpha_i = \frac{1}{\hat{\sigma}_i^2} \qquad i = 1, 2, \ldots \qquad \text{(C-4)}$$

The solution of the optimization problem gives the values $[X'_i]$:

$$X'_i = X_i - \hat{\sigma}_i^2 \frac{\sum_i X_i - X_T}{\sum_i \hat{\sigma}_i^2 + \hat{\sigma}_T^2} \qquad \text{(C-5)}$$

A closer inspection of the data base may result in other expressions for the coefficients $[\alpha_i]$, $i = 0, 1, \ldots$ On some occasions, it will also be reasonable to use other criteria for finding the corrected forecasting values $[X'_i]$. This is shown in the top down example in Annex D.

If, on the other hand, the variance of the top forecast X_T is fairly small, the following procedure may be chosen:

The corrections $[X_i]$ are found by minimizing the expression

$$Q' = \sum_i \alpha_i (X_i - X'_i)^2 \qquad \text{(C-6)}$$

subject to

$$X_T = \sum X'_i \tag{C-7}$$

If α_i, $i = 1, 2, \dots$ is chosen to be the inverse of the estimated variances, the solution of the optimization problem is given by

$$X'_i = X_i - \hat{\sigma}_i^2 \frac{\sum X_i - X_T}{\sum \hat{\sigma}_i^2} \tag{C-8}$$

ANNEX D

(to Recommendation E.506)

Example of a top down modelling method

The model for forecasting telephone traffic from Norway to the European countries is divided into two separate parts. The first step is an econometric model for the total traffic from Norway to Europe. Thereafter, we apply a model for the breakdown of the total traffic on each country.

D.1 *Econometric model of the total traffic from Norway to Europe*

With an econometric model we try to explain the development in telephone traffic, measured in charged minutes, as a function of the main explanatory variables. Because of the lack of data for some variables, such as tourism, these variables have had to be omitted in the model.

The general model may be written:

$$X_t = e^K \cdot GNP_t^a \cdot P_t^b \cdot A_t^c \cdot e^{u_t} \qquad (t = 1, 2, \dots, N) \tag{D-1}$$

where:

- X_t is the demand for telephone traffic from Norway to Europe at time t (charged minutes);
- GNP_t is the gross national product in Norway at time t (real prices);
- P_t is the index of charges for traffic from Norway to Europe at time t (real prices);
- A_t is the percentage direct-dialled telephone traffic from Norway to Europe (to take account of the effect of automation). For statistical reasons (i.e. impossibility of taking logarithm of zero) A_t goes from 1 to 2 instead of from 0 to 1;
- K is the constant;
- a is the elasticity with respect to GNP;
- b is the price elasticity;
- c is the elasticity with respect to automation;
- u_t is the stochastic variable, summarizing the impact of those variables that are not explicitly introduced in the model and whose effects tend to compensate each other (expectation of $u_t = 0$ and var $u_t = \sigma^2$).

By applying regression analysis (OLSQ) we have arrived at the coefficients (elasticities) in the forecasting model for telephone traffic from Norway to Europe given in Table D-1/E.506 (in our calculations we have used data for the period 1951-1980).

The t statistics should be compared with the Student's Distribution with $N - d$ degrees of freedom, where N is the number of observations and d is the number of estimated parameters. In this example, $N = 30$ and $d = 4$.

The model "explains" 99.7% of the variation in the demand for telephone traffic from Norway to Europe in the period 1951-1980.

From this logarithmic model it can be seen that:

– an increase in GNP of 1% causes an increase in the telephone traffic of 2.80%,

– an increase of 1% in the charges, measured in real prices, causes a decrease in the telephone traffic of 0.26%, and

– an increase of 1% in A_t causes an increase in the traffic of 0.29%.

We now use the expected future development in charges to Europe, in GNP, and in the future automation of traffic to Europe to forecast the development in telephone traffic from Norway to Europe from the equation:

$$X_t = e_t^{-16.095} \cdot GNP_t^{2.80} \cdot P_t u^{-0.26} \cdot A_t^{0.29} \qquad \text{(D-2)}$$

TABLE D-1/E.506

Coefficients	Estimated values	t statistics
K	–16.095	–4.2
a	2.799	8.2
b	–0.264	–1.0
c	0.290	2.1

D.2 *Model for breakdown of the total traffic from Norway to Europe*

The method of breakdown is first to apply the trend to forecast the traffic to each country. However, we let the trend become less important the further into the period of forecast we are, i.e. we let the trend for each country converge to the increase in the total traffic to Europe. Secondly, the traffic to each country is adjusted up or down, by a percentage that is equal to all countries, so that the sum of the traffic to each country equals the forecasted total traffic to Europe from equation (D-2).

Mathematically, the breakdown model can be expressed as follows:

Calculation of the trend for country i:

$$R_{it} = b_i + a_i \cdot t, \qquad i = 1, \ldots, 34 \qquad t = 1, \ldots, N \qquad \text{(D-3)}$$

where

$R_{it} = \dfrac{X_{it}}{X_t}$, i.e country i's share of the total traffic to Europe;

X_{it} is the traffic to country i at time t;

X_t is the traffic to Europe at time t;

t is the trend variable;

a_i and b_i are two coefficients specific to country i; i.e. a_i is country i's trend. The coefficients are estimated by using regression analysis, and we have based calculations on observed traffic for the period 1966-1980.

The *forecasted shares* for country i is then calculated by

$$R_{it} = R_{iN} + a_i \cdot (t - N) \cdot e^{-\frac{t-5}{40}} \qquad \text{(D-4)}$$

where N is the last year of observation, and e is the exponential function.

The factor $e^{-\frac{t-5}{40}}$ is a correcting factor which ensures that the growth in the telephone traffic to each country will converge towards the growth of total traffic to Europe after the adjustment made in equation (D-6).

To have the sum of the countries' shares equal one, it is necessary that

$$\sum_i R_{it} = 1 \qquad \text{(D-5)}$$

This we obtain by setting the adjusted share, $\tilde{R}_{it}$, equal to

$$\tilde{R}_{it} = R_{it} \frac{1}{\sum_i R_{it}} \qquad \text{(D-6)}$$

Each country's forecast traffic is then calculated by multiplying the total traffic to Europe, X_t, by each country's share of the total traffic:

$$X_{it} = \tilde{R}_{it} \times X_t \qquad \text{(D-7)}$$

D.3 *Econometric model for telephone traffic from Norway to Central and South America, Africa, Asia and Oceania*

For telephone traffic from Norway to these continents we have used the same explanatory variables and estimated coefficients. Instead of gross national product, our analysis has shown that for the traffic to these continents the number of telephone stations within each continent are a better and more significant explanatory variable.

After using cross-section/time-series simultaneous estimation, we have arrived at the coefficients in Table D-2/E.506 for the forecasting model for telephone traffic from Norway to these continents (for each continent we have based our calculations on data for the period 1961-1980):

TABLE D-2/E.506

Coefficients	Estimated values	*t* statistics
Charges	–1.930	–5.5
Telephone stations	2.009	4.2
Automation	0.5	–

We then have $R^2 = 0.96$. The model may be written:

$$X_t^k = e^K \cdot (TS_t^k)^{2.009} \cdot (P_t^k)^{1.930} \cdot (A_t^k)^{0.5} \tag{D-8}$$

where

X_t^k is the telephone traffic to continent k (k = Central America, . . ., Oceania) at time t,

e^K is the constant specific to each continent. For telephone traffic from Norway to:

Central America:	$K^1 = -11.025$
South America:	$K^2 = -12.62$
Africa:	$K^3 = -11.395$
Asia:	$K^4 = -15.02$
Oceania:	$K^5 = -13.194$

TS_t^k is the number of telephone stations within continent k at time t,

P_t^k is the index of charges, measured in real prices, to continent k at time t, and

A_t^k is the percentage direct-dialled telephone traffic to continent k.

Equation (D-8) is now used – together with the expected future development in charges to each continent, future development in telephone stations on each continent and future development in automation of telephone traffic from Norway to the continent – to forecast the future development in telephone traffic from Norway to the continent.

References

[1] ABRAHAM (A.) and LEDOLTER (J.): Statistical methods for forecasting. *J. Wiley*, New York, 1983.

[2] ALDRIN (M.): Forecasting time series with missing observations. *Stat 15/86 Norwegian Computing Center*, 1986.

[3] ANSLEY (C. F.) and KOHN (R.): Estimation, filtering and smoothing in state space models with incomplete specified initial conditions. *The Annals of Statistics*, 13, pp. 1286-1316, 1985.

[4] BARHAM (S. Y.) and DUNSTAN (F. D. J.): Missing values in time series. *Time Series Analysis: Theory and Practice 2*: Anderson, O. D., ed., pp. 25-41, North Holland, Amsterdam, 1982.

[5] BØLVIKEN (E.): Forecasting telephone traffic using Kalman Filtering: Theoretical considerations. *Stat 5/86 Norwegian Computing Center*, 1986.

[6] CHEMOUIL (P.) and GARNIER (B.): An adaptive short-term traffic forecasting procedure using Kalman Filtering. *XI International Teletraffic Congress*, Kyoto, 1985.

[7] HARRISON (P. J.) and STEVENS (C. F.): Bayesian forecasting. *Journal of Royal Statistical Society*. Ser B 37, pp. 205-228, 1976.

[8] HARVEY (A. C.) and PIERSE (R. G.): Estimating missing observations in econometric time series. *Journal of American Statistical As.*, 79, pp. 125-131, 1984.

[9] JONES (R. H.): Maximum likelihood fitting of ARMA models to time series with missing observations. *Technometrics*, 22, No. 3, pp. 389-396, 1980.

[10] JONES (R. H.): Time series with unequally spaced data. *Handbook of Statistics 5*. Ed. Hannah, E. J. et al., pp. 157-177, North Holland, Amsterdam, 1985.

[11] KRUITHOF (J.): Telefoonverkeersrekening. *De Ingenieur*, 52, No. 8, 1937.

[12] MORELAND (J. P.): A robust sequential projection algorithm for traffic load forecasting. *The Bell Technical Journal*, 61, pp. 15-38, 1982.

[13] PACK (C. D.) and WHITAKER (B. A.): Kalman Filter models for network forecasting. *The Bell Technical Journal*, 61, pp. 1-14, 1982.

[14] STORDAHL (K.) and HOLDEN (L.): Traffic forecasting models based on top down and bottom up models. *ITC 11*, Kyoto, 1985.

[15] SZELAG (C. R.): A short-term forecasting algorithm for trunk demand servicing. *The Bell Technical Journal*, 61, pp. 67-96, 1982.

[16] TU (M.) and PACK (D.): Improved forecasts for local telecommunications network. 6th International Forecasting Symposium, Paris, 1986.

[17] WRIGHT (D. H.): Forecasting irregularly spaced data: An extension of double exponential smoothing. *Computer and Engineering*, 10, pp. 135-147, 1986.

[18] WRIGHT (D. H.): Forecasting data published at irregular time intervals using an extension of Holt's method. *Management science*, 32, pp. 499-510, 1986.

[19] *Table of international telex relations and traffic*, ITU, Geneva, 1973-1984.