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Jitter and wander accumulation in digital networks

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NOTES

1 Supplement 36 to the G-series Recommendations was approved in Melbourne (1988) and published in Fascicle III.5 of the *Blue Book*. This file is an extract from the *Blue Book.* While the presentation and layout of the text might be slightly different from the *Blue Book* version, the contents of the file are identical to the *Blue Book* version and copyright conditions remain unchanged (see below).

2 In this Supplement, the expression "Administration" is used for conciseness to indicate both a telecommunication administration and a recognized operating agency.

Supplement No. 36

JITTER AND WANDER ACCUMULATION IN DIGITAL NETWORKS

(Referred to in Recommendation G.824)

The present Supplement describes a model which has been used to compute jitter/wander accumulation in digital networks arising from cascaded digital regenerators and asynchronous digital multiplexes. This model does not include other sources of wander generation; e.g., environmental, disruptions in synchronization reference distribution, etc.

1 Jitter and wander accumulation – Digital regenerator component

The most widely used model of regenerator jitter/wander accumulation, attributed to Chapman [1], treats the regenerator as linear, shift-invariant system. In order to compute the accumulated jitter/wander after N cascaded regenerators, intrinsic regenerator jitter/wander is categorized in terms of "random" and "systematic" components. Chamzas model of regenerator jitter/wander accumulation [2] addresses how stochastic variations in regenerator retiming circuits affect jitter/wander accumulation. The results of this study demonstrate that use of the appropriate *mean* jitter/wander transfer characteristic in the identical regenerator accumulation model, summarized above, provides a very good estimate to jitter/wander accumulation computed assuming a stochastic variation of retiming circuits.

Using Chapman's model for a chain of N *identical* regenerators, defining $H_1(j\omega)$ as the jitter/wander transfer characteristic for one regenerator, and redefining the random and systematic components as completely uncorrelated and correlated components, respectively,

– the power spectral density of the random jitter/wander component is:

$$\Phi_N^R(\omega) = \Phi_{i1}^R |H_1(j\omega)|^2 \frac{1 - |H_1(j\omega)|^{2N}}{1 - |H_1(j\omega)|^2} \qquad (1)$$

where Φ_{i1}^R is the constant, internally generated, random (pattern independent plus uncorrelated pattern dependent) jitter/wander power spectral density for one regenerator.

– the power spectral density of the systematic jitter/wander component is:

$$\Phi_N^S(\omega) = \Phi_{i1}^S |H_1(j\omega)|^2 \frac{|1 - H_1(j\omega)^N|^2}{|1 - H_1(j\omega)|^2} \qquad (2)$$

where Φ_{i1}^S is the constant, internally generated, systematic (correlated pattern dependent) jitter/wander power spectral density for one regenerator. Φ_{i1}^R and Φ_{i1}^S can be estimated from practical measurements based upon the regenerator's jitter/wander response to short and long word lengths from a pattern generator, and correlation studies.

When there is no peaking in the regenerator jitter/wander transfer characteristic, the systematic jitter/wander accumulates much more rapidly than the random jitter/wander [1], [4], [5]; as a result, random jitter/wander accumulation is often ignored. However, for a large number of regenerators with peaking in the jitter/wander transfer characteristic, the total jitter/wander accumulation can be dominated by the random component.

2 Jitter and wander accumulation – Asynchronous digital multiplex component

With Gaussian input jitter/wander, having an rms amplitude of σ, and double-sided power spectral density $\theta_{in}(f)$, the unfiltered multiplex intrinsic jitter/wander is given by [6].

$$\Phi_{out}(f) = sinc^2 f\, rep\, \Phi_{in}(f) + \sum_{n=1}^{x} \frac{p^2}{(2\pi n)^2}\left[\delta(f-n) + \delta(f+n)\right]$$
$$+ \sum_{n=1}^{x} \frac{sinc^2 f}{(2\pi n)^2}\left[rep\, Z_n(f-np) + rep\, Z_n(f+np)\right] \tag{3}$$

where $rep\, X(f) = \sum_{k=x}^{x} X(f-k)$

$$Z_n(f) = e^{-2\pi n\sigma}\left[\delta(f) + \sum_{k=1}^{x}\left[\frac{2\pi n}{k!}\right]^{2k} \underbrace{\Phi_{in}(f) * \ldots * \Phi_{in}(f)}_{k\ terms}\right]$$

p Multiplexer stuffing ratio

f jitter/wander frequency normalized by the multiplexer maximum suffising frequency

3 Method of combination

Assuming that the jitter/wander accumulation from each component part can be modeled by filtered Gaussian randam variables, the power spectrum and rms amplitude after each component part[1)] is computed as the accumulation due to the preceding parts according to the following rules [3]:

i) The jitter/wander spectrum at the output of a chain of regenerators is the power sum of the jitter/wander generated by the regenerators (equations [1] and [2]) and any jitter/wander at the input of the chain, appropriately filtered by the equivalent jitter/wander transfer characteristic. Thus, for input jitter/wander, $\theta_{in}(\omega)$, the output jitter/wander, $\theta_{out}(\omega)$, is given by

$$\Phi_{out}(\omega) = \Phi_{i1}^{R}\, |H_1(j\omega)|^2 \frac{1 - |H_1(j\omega)|^{2y}}{1 - |H_1(j\omega)|^2} +$$
$$+ \Phi_{i1}^{S}\, |H_1(j\omega)|^2 \frac{|1 - H_1(j\omega)^N|^2}{|1 - H_1(j\omega)|^2} + \Phi_{in}(\omega)\, |H_1(j\omega)^N|^2 \tag{4}$$

ii) The jitter/wander spectrum at the output of a demultiplexer is the power sum of the unfiltered intrinsic multiplex jitter/wander and the accumulated higher rate input jitter/wander, attenuated by the desynchronizer jitter/wander transfer characteristic. Thus, if $\theta_{in,1}(\omega)$ is the unfiltered intrinsic multiplex jitter/wander and $\theta_{in,2}(\omega)$ is the accumulated higher rate input jitter/wander, the output jitter/wander, $\theta_{out}(\omega)$ is given by

$$\Phi_{out}(\omega) = \left\{\Phi_{in,1}(\omega) + \frac{\Phi_{in,2}(\omega)}{r^2}\right\} |G(j\omega)|^2 \tag{5}$$

where r is the ratio of the multiplexer output frequency to tributary frequency, and $G(j\omega)$ represents the desynchronizer jitter/wander transfer characteristic.

1) The following equations are valid for both single- and double-sided power spectra and corresponding transfer characteristics.

4 Definition of peak-to-peak jitter/wander amplitude

The probability that the jitter/wander exceeds a particular threshold amplitude $|x|$ n times in the time interval $(t, t + \Delta t)$ may be described by a Poisson density function [3].

$$Pr\left\{n(\pm x) \text{ crossings in } (t, t+\Delta t)\right\} = \frac{\{\overline{N(x)}\Delta t\}^n}{n!} e^{-\overline{N(x)}\Delta t} \tag{6}$$

where $\overline{N(x)}$ is the average number of times/second that the threshold $|x|$ is exceeded.

For Gaussian jitter/wander, with double-sided power spectral density $\theta(\omega)$, $\overline{N(x)}$ is given by [7]

$$\overline{N(x)} = N_0 \, e^{-x^2 2\Omega\sigma 2}$$

$$\text{where} \quad \sigma^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega) d\omega \tag{7}$$

$$N_0 = \frac{1}{\pi} \left\{ \frac{\int_{-\infty}^{\infty} \omega^2 \Phi(\omega) d\omega}{2\pi\sigma^2} \right\}^{1/2}$$

The probability that the jitter/wander doesn't exceed the threshold during the time interval $(t, t + \Delta t)$ is

$$1 - P_0 = e^{-\overline{N(x)}\Delta t} \tag{8}$$

Solving for the threshold,

$$|x| = \left\{ 2\sigma^2 \ln \left[N_0 \frac{\Delta t}{\ln\left(\frac{1}{1 - P_\sigma}\right)} \right] \right\}^{1/2} \tag{9}$$

If we assume that each time the threshold is crossed, an undesirable event (impairment) may result, the mean time between impairments, MTBI, may be taken as

$$MTBI = \frac{1}{\overline{N(x)}} \tag{10}$$

Thus, equation (9) may be expressed as

$$|x| = \left\{ 2\sigma^2 \ln(N_0 \, MTBI) \right\}^{1/2} \tag{11}$$

References

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