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SERIES O: SPECIFICATIONS OF MEASURING EQUIPMENT

Equipment for the measurement of digital and analogue/digital parameters

Equipment to assess error performance on Optical Transport Network interfaces

ITU-T Recommendation O.182

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SPECIFICATIONS OF MEASURING EQUIPMENT

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ITU-T Recommendation O.182

Equipment to assess error performance on Optical Transport Network interfaces

Summary

ITU-T Recommendation O.182 specifies measuring equipment for monitoring defects and anomalies and for measuring error performance and forward error correction (FEC) efficiency at the OCh layer of the optical transport network (OTN).

Source

ITU-T Recommendation O.182 was approved on 22 July 2007 by ITU-T Study Group 4 (2005-2008) under the ITU-T Recommendation A.8 procedure.

Keywords

Error performance, Forward Error Correction, In-service, Measuring equipment, Measurement mode, Optical Transport Network, Out-of-service, Test Signal Structure.

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ITU-T Recommendation O.182

Equipment to assess error performance on Optical Transport Network interfaces

1 Scope

This Recommendation specifies the functions of measuring equipment which has the capability of assessing OCh layer (OTUk, ODUk, OPUk) frame-error performance at OTN interfaces.

[ITU-T G.959.1] defines the OTN physical interface characteristics. [ITU-T G.709] defines the OTN signals applicable to these measurements.

Depending on the network entity under test, this Recommendation specifies the use of either in-service or out-of-service measurement modes.

Annex A of [ITU-T G.709] specifies FEC for OTUk frame. This Recommendation defines the functions of Forward Error Correction (FEC) performance verification.

Furthermore, this Recommendation defines the required functions of the generator and receiver part of the error performance measuring equipment.

This Recommendation does not cover OTUkV framing.

2 References

The following ITU-T Recommendations and other references contain provisions which, through reference in this text, constitute provisions of this Recommendation. At the time of publication, the editions indicated were valid. All Recommendations and other references are subject to revision; users of this Recommendation are therefore encouraged to investigate the possibility of applying the most recent edition of the Recommendations and other references listed below. A list of the currently valid ITU-T Recommendations is regularly published. The reference to a document within this Recommendation does not give it, as a stand-alone document, the status of a Recommendation.

[ITU-T G.707] ITU-T Recommendation G.707/Y.1322 (2007), *Network node interface for the synchronous digital hierarchy (SDH)*.

[ITU-T G.709] ITU-T Recommendation G.709/Y.1331 (2003), *Interfaces for the Optical Transport Network (OTN)*.

[ITU-T G.798] ITU-T Recommendation G.798 (2006), *Characteristics of optical transport network hierarchy equipment functional blocks*.

[ITU-T G.806] ITU-T Recommendation G.806 (2006), *Characteristics of transport equipment – Description methodology and generic functionality*.

[ITU-T G.870] ITU-T Recommendation G.870/Y.1352 (2004), *Terms and definitions for optical transport networks (OTN)*.

[ITU-T G.872] ITU-T Recommendation G.872 (2001), *Architecture of optical transport networks*.

[ITU-T G.959.1] ITU-T Recommendation G.959.1 (2006), *Optical transport network physical layer interfaces*.

[ITU-T G.972] ITU-T Recommendation G.972 (2004), *Definition of terms relevant to optical fibre submarine cable systems*.

[ITU-T G.975.1] ITU-T Recommendation G.975.1 (2004), *Forward error correction for high bit-rate DWDM submarine systems*.

[ITU-T G.8201] ITU-T Recommendation G.8201 (2003), *Error performance parameters and objectives for multi-operator international paths within the Optical Transport Network (OTN)*.

[ITU-T G.8251] ITU-T Recommendation G.8251 (2001), *The control of jitter and wander within the optical transport network (OTN)*.

[ITU-T M.2401] ITU-T Recommendation M.2401 (2003), *Error performance limits and procedures for bringing-into-service and maintenance of multi-operator international paths and sections within an optical transport network*.

[ITU-T O.3] ITU-T Recommendation O.3 (1992), *Climatic conditions and relevant tests for measuring equipment*.

[ITU-T O.150] ITU-T Recommendation O.150 (1996), *General requirements for instrumentation for performance measurements on digital transmission equipment*.

3 Definitions

3.1 Terms defined elsewhere

This Recommendation uses the following terms defined elsewhere:

3.1.1 optical transport network (OTN): See [ITU-T G.872]

3.1.2 forward error correction (FEC): See [ITU-T G.972]

3.1.3 CBR2G5, CBR10G, CBR40G: See [ITU-T G.709]

3.2 Terms defined in this Recommendation

This Recommendation defines the following terms:

3.2.1 black box testing: Black box testing does not explicitly use the knowledge of the internal implementation.

3.2.2 in-service measurement modes: The digital signal transported by the entity under test (EUT) is analysed at a given access point of the EUT.

3.2.3 out-of-service measurement modes: After setting up a path through the entity under test (EUT) by appropriate means, a suitable test sequence is applied to the input at one side of the EUT. The received information is analysed at an access point at the other side of the EUT.

3.2.4 test signal structure (TSS): A test signal structure specifies the mapping of a defined test pattern onto an OPUk.

3.2.5 white box testing: White box testing allows one to peek inside the "box" (the actual EUT) and to use the knowledge of the internal implementation of the EUT.

Out-of-service measurement is performed in absence of user traffic. Consequently, the test signal to be used in case of out-of-service mode must be defined. Annex B defines the test signal structure named TSSx (x being a number) to be used.

Table 3-1 presents the four test signal structures. They are detailed in Annex B.

Table 3-1 – Test signal structures

TSS1	Test signal structure TSS1 applied to all bytes of a CBR2G5
TSS2	Test signal structure TSS2 applied to all bytes of a CBR10G
TSS3	Test signal structure TSS3 applied to all bytes of a CBR40G
TSS4	Test signal structure TSS4 applied to NULL/PRBS

4 Abbreviations and acronyms

This Recommendation uses the following abbreviations and acronyms:

AIS Alarm Indication Signal

BBE Background Block Error

BBER Background Block Error Ratio

BDI Backward Defect Indicator

BEI Backward Error Indicator

BIAE Backward Incoming Alignment Error

BIP-8 Bit Interleaved Parity

CBR Constant Bit Rate

CBR2G5 Constant Bit Rate of 2.5 Gigabits

CBR10G Constant Bit Rate of 10 Gigabits

CBR40G Constant Bit Rate of 40 Gigabits

EB Errored Block

EDC Error Detection Code

EUT Entity Under Test

FA Frame Alignment

FEC Forward Error Correction

FUEB FEC Uncorrected Error Blocks

GCC General Communication Channel

IAE Incoming Alignment Error

JC Justification Control

LCK Locked Defect

LOF Loss of Frame

LOM Loss of Multiframe

LOS Loss Of Signal

LSS Loss of Sequence Synchronization

LTC Loss of Tandem Connection

ME Measuring Equipment

NJO Negative Justification Opportunity

OCh	Optical Channel with full functionality
OCI	Open Connection Indication
ODTUjk	Optical channel Data Tributary Unit j into k
ODU	Optical channel Data Unit
ODUk	Optical channel Data Unit-k
OH	OverHead
ONNI	Optical Network Node Interface
OOF	Out Of Frame
OOM	Out of Multiframe
OPU	Optical channel Payload Unit
OPUk	Optical channel Payload Unit-k
OTN	Optical Transport Network
OTUk	Optical channel Transport Unit -k
PJO	Positive Justification Opportunity
PLM	Payload Mismatch
PM	Path Monitoring
PRBS	Pseudo Random Binary Sequence
PSI	Payload Structure Identifier
PT	Payload Type
RES	Reserved for future international standardization
RS	Reed-Solomon
SES	Severely Errored Second
SESR	Severely Errored Second Ratio
SM	Section Monitoring
STM(-N)	Synchronous Transport Module (-N)
TC	Tandem Connection
TCM	Tandem Connection Monitoring
TCMi	Tandem Connection Monitoring i (i = 1,2,...,6)
TIM	Tail trace Identifier Mismatch
TSE	Test Sequence Error
TSS	Test Signal Structure
VC-n	Virtual Container-n

5 Conventions and background

5.1 Conventions

The following conventions are directly taken from [ITU-T G.709]:

Transmission order: The order of transmission of information in all the diagrams in this Recommendation is first from left to right and then from top to bottom. Within each byte the most significant bit is transmitted first. The most significant bit (bit 1) is illustrated at the left in all the diagrams.

Value of reserved bit(s): The value of an overhead bit, which is reserved or reserved for future international standardization, shall be set to "0".

Value of non-sourced bit(s): Unless stated otherwise, any non-sourced bits shall be set to "0".

5.2 Background

The following figures illustrate OTN framing. They are directly taken from [ITU-T G.709]:

OTUk, ODUk and OPUk overhead assignment: The assignment of overhead in the optical channel transport/data/payload unit signal to each part is defined in Figure 5-1.

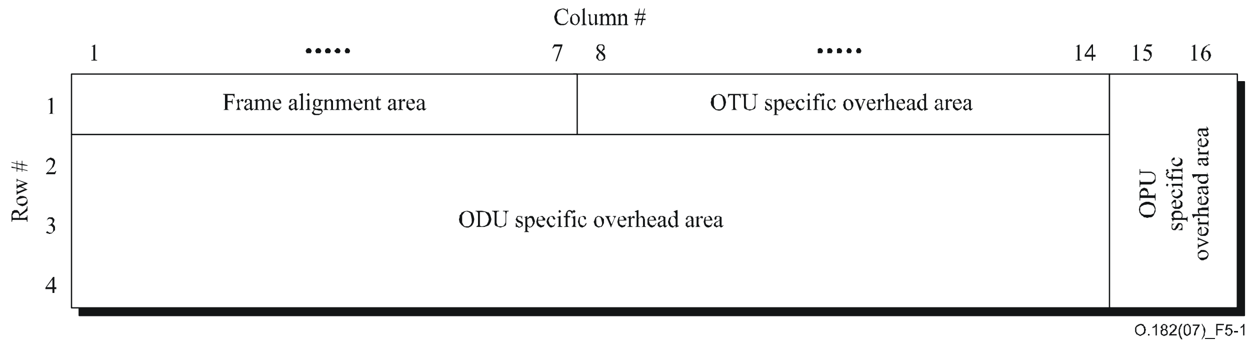

Figure 5-1 – OTUk, ODUk and OPUk overhead (taken from ITU-T G.709)

OTUk, ODUk and OPUk frame structure: The frame structures in the optical channel transport/data/payload unit signal are defined in Figures 5-2, 5-3 and 5-4.

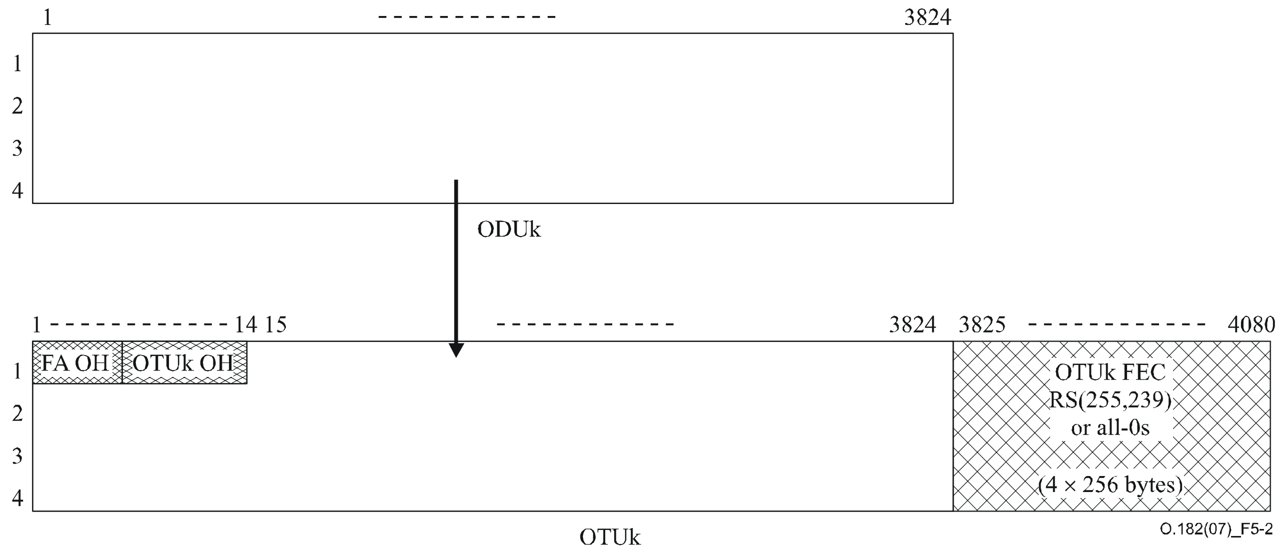

Figure 5-2 – OTUk frame structure (taken from ITU-T G.709)

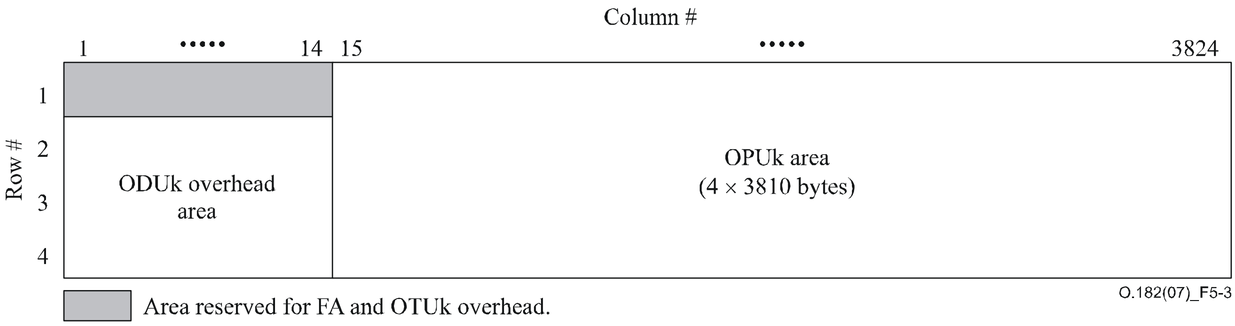

Figure 5-3 – ODUk frame structure (taken from ITU-T G.709)

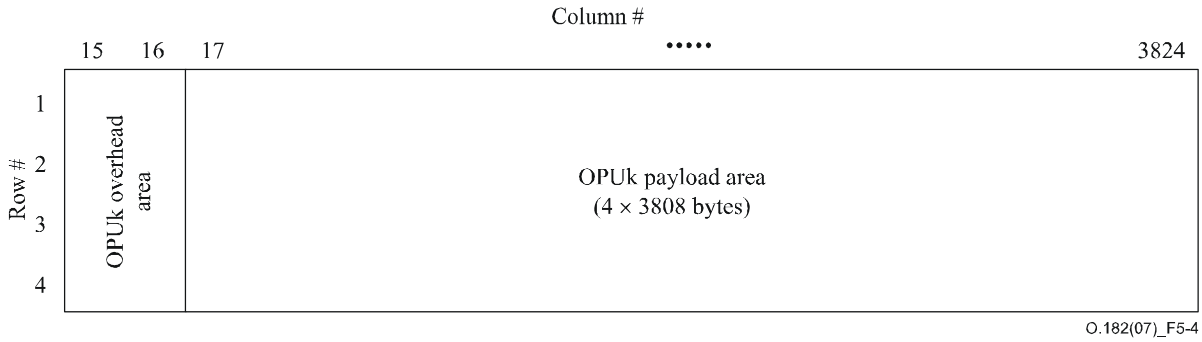

Figure 5-4 – OPUk frame structure (taken from ITU-T G.709)

6 Measurement modes

Two measurement modes shall be supported by OTN measurement equipment:

1) *Out-of-service measurement mode (OSM)*

   After setting up a path through the entity under test (EUT) by appropriate means, a suitable test sequence is applied to the input at one side of the EUT. The received information is analysed at an access point at the other side of the EUT. Annex B defines the test signal structures (TSSx) to be used.

2) *In-service measurement mode (ISM)*

   The digital signal transported by the EUT is analysed at a given access point of the EUT by observing the overhead and, optionally, the FEC.

The advantage of ISM is that customer traffic is not affected by the measurement, i.e., ISM is non service affecting. However, because of the way the overhead, i.e., the BIP or FEC, is computed, there is a non-zero probability that not all bit errors are detected.

With OSM, the possibility to do a bit-by-bit comparison of the signal applied to the EUT and the signal extracted at the other side of the EUT, allows detection of all individual bit errors. However, because the test sequence replaces the customer signal, this measurement mode is service affecting.

Both measurement modes can be used simultaneously: It is possible to apply the in-service measurement mode to a test path which has been set up in order to perform out-of-service measurements.

7 Events to be monitored

This clause defines the network events that shall be monitored in order to specify the error performance measurement procedures.

Measurement equipment defined in this Recommendation shall be able to test OTUk signals without FEC.

Measurement equipment defined in this Recommendation shall be able to test OTUk signals with FEC defined in [ITU-T G.709].

The following subclauses list defects, anomalies and other events related to error performance which shall be detected by the measuring equipment on OCh layer signals. Criteria for detecting anomalies and defects are given in Annex A.

A subset of these events is selected for each measurement mode according to the OTN network entity under test.

7.1 Network events

A measurement equipment shall monitor the anomalies and the defects recommended in the latest agreed version of [ITU-T G.709] and [ITU-T G.798].

Tables 7-1 and 7-2 summarize the anomalies and the defects monitored. They are explained in Annex A.

Table 7-1 – Network anomalies

Name	Meaning
OOF	Out Of Frame
OOM	Out of multiframe
SM-BIP-8	Section monitoring – Bit interleaved parity error
PM-BIP-8	Path monitoring – Bit interleaved parity error
SM-BEI	Section monitoring – Backward error indicator
TCMi-BIP-8	Tandem connection monitoring i – Bit interleaved parity error (i = 1,2,...,6)
TCMi-BEI	Tandem connection monitoring i – Backward error indication (i = 1,2,...,6)

Table 7-2 – Network defects

Name	Meaning
LOS	Loss of signal
LOF	Loss of frame
LOM	Loss of multiframe
OTU-AIS	Optical channel transport unit – Alarm indication signal
SM-BDI	Section monitoring – Backward defect indicator
SM-IAE	Section monitoring – Incoming alignment error
SM-TIM	Section monitoring – Tail trace identifier mismatch
SM-BIAE	Section monitoring – Backward incoming alignment error
ODU-AIS	Optical channel data unit – Alarm indication signal
ODU-OCI	Optical channel data unit – Open connection indication
ODU-LCK	Optical channel data unit – Locked defect
PM-BDI	Path monitoring – Backward defect indicator
PM-TIM	Path monitoring – Trail trace identifier mismatch
ODU-PLM	Optical channel data unit – Payload mismatch

Table 7-2 – Network defects

Name	Meaning
TCMi-BDI	Tandem connection monitoring i – Backward defect indicator
TCMi-IAE	Tandem connection monitoring i – Incoming alignment error
TCMi-TIM	Tandem connection monitoring i – Tail trace identifier mismatch
TCMi-BIAE	Tandem connection monitoring i – Backward incoming alignment error
TCMi-LTC	Tandem connection monitoring i – Loss of tandem connection (i = 1,2,...,6)

7.2 Test signal structure events

Monitoring of network events shall be complemented by the following events which are detected in Out-of-service mode using defined test signal structures:

– Loss of sequence synchronization (LSS): see A.2.20;

– Test sequence error (TSE): see A.1.11.

7.3 Forward error correction (FEC) events

The measuring equipment shall monitor the following events generated by the FEC:

– FEC corrected errors (FCE): see A.1.6;

– FEC uncorrected error blocks (FUEB): see A.1.7.

7.4 Events to be monitored at the OCh layer

The OCh transports a digital client signal between 3R regeneration points. OCh client signals defined in this Recommendation are the OTUk signals.

Measuring equipment shall be able to receive OCh layer signals as defined in [ITU-T G.709].

The following subclauses give the events monitored for OTUk, ODUk, OPUk signals.

7.4.1 OTUk layer

Measuring equipment shall support the capability to disable the FEC decoding process.

Table 7-3 – Events to be monitored at the OTUk layer

Type of event	List of monitored events for this layer
Anomalies	OOF, OOM, SM-BIP-8, SM-BEI, FCE (Note), FUEB (Note)
Defects	LOS, LOF, LOM, OTU-AIS, SM-BDI, SM-IAE, SM-TIM, SM-BIAE
NOTE – In the case of using the FEC code that is defined in ITU-T Rec. G.709.	

7.4.2 ODUk layer

Measuring equipment shall support monitoring for tandem connection monitoring events.

Table 7-4 – Events to be monitored at the ODUk layer

Type of event	List of monitored events for this layer
Anomalies	PM-BIP-8, PM-BEI, TCMi-BIP-8, TCMi-BEI
Defects	ODU-AIS, ODU-OCI, ODU-LCK, ODU-PLM, TCMi-BDI, TCMi-IAE, TCMi-TIM, TCMi-BIAE, TCMi-LTC
NOTE – i = 1,2,...,6.	

7.4.3 OPUk layer

The OPUk layer can only be tested in out-of-service measurement mode.

Measuring equipment shall support monitoring the following OTUk events:

Table 7-5 – Events to be monitored at the OPUk layer

Type of event	List of monitored events for this layer
Anomalies	TSE (Note)
Defects	LSS (Note)
NOTE – In the case of out-of-service mode using TSS4/PRBS.	

7.5 Test signal structure to be used at OPUk layer

The OPUk layer can only be tested in out-of-service measurement mode.

This clause presents the four test signal structures (TSSx) a measurement equipment shall support to monitor OTN performance.

7.5.1 CBR signal: NULL and PRBS test signals

TSS1 to TSS3 shall be used for testing CBR signal (e.g., STM-16/64/256) transported on OTN network.

Two modes are defined for TSS1 to TSS3. One is the asynchronous mode that has justification technique; the other one is the bit synchronous mode that has no justification technique.

7.5.2 TSS4: NULL and PRBS test signals

TSS4 shall be used for NULL and PRBS test signal transport.

At the generator side, the NULL and PRBS signals are not fed from the outside into the OTN equipment, like ordinary payload signals. They are generated inside the applicable atomic functions.

Similarly, at the receiving end, the received NULL and/or PRBS signals are not made available to the outside world. Only some anomalies (like TSE, test sequence errors) and defects (like LSS, loss of PRBS lock) are made available.

OTN measuring equipment shall support the generation of NULL and PRBS test signals according to the specifications in clauses 14.3.4.1 and 14.3.5.1 of [ITU-T G.798].

It shall be possible to insert errors in the PRBS test signal. The number of inserted errors shall be adjustable, such that operation of the pN_TSE detection and of the dLSS defect assertion in the EUT, as specified in clause 14.3.5.2 of [ITU-T G.798], can be tested properly.

OTN Measuring Equipment shall support the reception of NULL and PRBS test signals as specified in clauses 14.3.4.2 and 14.3.5.2 of [ITU-T G.798].

It shall be possible to analyse a received NULL signal for bit errors.

It shall be possible to analyse a received PRBS signal for anomalies and defect conditions as defined in clause 14.3.5.2 of [ITU-T G.798].

8 Generator

The generator is only required for the out-of-service measurement mode.

8.1 Synchronization

It shall be possible to synchronize the measuring equipment to one of the synchronization sources listed below:

– internal clock;

– external clock;

– OTUk layer clock recovered from the receiver part of the measuring equipment.

8.2 Bit rates

The bit rates delivered by the generator of the measuring equipment shall be in accordance with those defined in [ITU-T G.709].

8.3 Test signal structures

The generator of the measuring equipment shall generate different test signal structures in order to simulate real OCh layer signals.

Annex B defines the test signal structure for the different OCh layer signals.

8.4 Digital signal outputs

8.4.1 Digital interfaces

The output signals generated shall conform to the physical interfaces defined in [ITU-T G.709].

8.4.2 Jitter generation

The output jitter of the OCh layer signal delivered by the generator of the measuring equipment shall comply with [ITU-T G.8251].

8.5 Anomaly and defect addition on the output signal

The measuring equipment shall support adding anomalies to the output signal to verify the generator-to-receiver connectivity.

9 Receiver

The receiver must support both in-service and out-of-service measurement modes.

9.1 Digital signal inputs

9.1.1 Digital interfaces

The receiver of the measuring equipment shall accept input signals that conform to the physical interfaces defined in [ITU-T G.709].

9.1.2 Input jitter tolerance

With an OCh layer input signal, the receiver of the measuring equipment shall tolerate an input jitter in accordance with [ITU-T G.8251].

9.2 Test signal structures

The receiver of the measuring equipment shall be capable of analysing the test signal structures defined in Annex B.

9.3 Error performance measurement

This clause is based on [ITU-T G.8201], which defines performance objectives for OTN.

[ITU-T G.8201] contains the definitions of the performance parameters for different measurement configurations and describes the use of the parameters for performance measurements, bringing-into-service and maintenance purposes.

This clause specifies the methods a measurement equipment shall support for measuring the error performance objectives events errored block (EB) and severely errored second (SES) defined in clause 4.4 of [ITU-T G.8201].

9.3.1 In-service monitoring

Each block is monitored by means of an inherent (bit interleaved parity) error detection code (EDC) with a specified EDC usage. The EDC bits are physically separated from the block to which they apply. It is not normally possible to determine whether a block or its controlling EDC bits are in error. If there is a discrepancy between the EDC and its controlled block, it is always assumed that the controlled block is in error.

The EDC for the ODUk path and ODUk TC is BIP-8. The block size for the ODUk path and ODUk TC is equal to the number of bits in the ODUk frame (payload plus overhead). Note that the BIP-8 EDC is computed over the OPUk payload plus OPUk overhead bits, but not the ODUk overhead bits. The EDC usage is $1 \times$ BIP-8.

9.3.2 Relationship between path performance monitoring and the block-based parameters

This clause covers path performance monitoring (PM) and tandem connection monitoring (TCM) as shown in Tables 9-1 and 9-2. ODUk Path and ODUk tandem connection trails are equivalent from a performance perspective. The established rules for ODUk path apply also to ODUk TC. Further details are given in [ITU-T G.709], [ITU-T G.798] and [ITU-T G.872].

9.3.2.1 Anomalies

In-service anomaly conditions are used to determine the error performance of an ODUk path when the path is not in a defect state.

9.3.2.2 Defects

In-service defect conditions defined in [ITU-T G.798] are used to determine the change of performance states which may occur on a path. Tables 9-1 and 9-2 show the defects used in this Recommendation.

Table 9-1 – Defects resulting in a near-end severely errored second (ITU-T Rec. G.8201)

Near-end defects (Notes 2, 3, 4)		
Path termination	**Non-intrusive monitor**	**Tandem connection**
OCI (Note 1)		OCI (Note 1)
AIS		AIS
		IAE
LCK		LCK
		LTC
PLM		
TIM		TIM
NOTE 1 – Paths not actually completed, e.g., during path set-up, will contain the ODUk-OCI (open connection indication) signal. NOTE 2 – The above defects are path defects only. Section defects such as OCh LOS, OTUk LOF, OTUk AIS, OTUk TIM, and OTM LOS give rise to an AIS defect in the path layers. NOTE 3 – When a near-end SES is caused by a near-end defect as defined above, the far-end performance event counters are not incremented, i.e., an error-free period is assumed. When a near-end SES results from ≥15% errored blocks, the far-end performance evaluation continues during the near-end SES. This approach does not allow reliable evaluation of far-end data if the near-end SES is caused by a defect. It should be noted in particular, that the evaluation of far-end events (such as SES or unavailability) can be inaccurate in the case where far-end SESs occur in coincidence with near-end SESs caused by a defect. Such inaccuracies cannot be avoided, but are negligible in practice because of the low probability of the occurrence of such phenomena. NOTE 4 – Refer to [ITU-T G.798] for defects contributing to performance monitoring in each trail termination sink function.		

Table 9-2 – Defects resulting in a far-end severely errored second (ITU-T Rec. G.8201)

Far-end defects		
Path termination	**Non-intrusive monitor**	**Tandem connection**
BDI	BDI	BDI

9.3.3 Error performance events

The following definitions consider that the detection of the performance events is applied after blocks error correction, including FEC correction when FEC is activated.

The following error performance events are defined in accordance with [ITU-T G.8201]:

- **Errored Block (EB)**: A block in which one or more bits are in error;
- **Severely Errored Second (SES)**: A one-second period which contains ≥15% errored blocks or at least one defect (listed in Table 9-3).

Table 9-3 – Threshold for the declaration of a severely errored second (ITU-T Rec. G.8201)

Bit rate (kbit/s)	Path type	Threshold for SES (Number of errored blocks in one second)
2 498 775	ODU1	3 064
10 037 273	ODU2	12 304
40 319 218	ODU3	49 424

- **Background Block Error (BBE):** An errored block not occurring as part of an SES.

[ITU-T G.709] defines BDI and BEI indications. They are available at the near end or at an intermediate point along the path and the tandem connection. They are used by the reverse direction to estimate the performance events occurring at the far end:

- ODUk PM-BEI and TCMi-BEI are anomalies which are used to determine the occurrence of BBE and SES at the far end;
- ODUk PM-BDI and TCMi-BDIs are defects which estimate the occurrence of SES at the far end.

9.4 Defects and anomalies indication

The measuring equipment shall display the applicable anomalies and defects mentioned in Annex A.

9.5 Events time stamping

The measuring equipment shall time stamp the different monitored events during error performance measurements.

10 Forward error correction (FEC) performance verification functions

According to [ITU-T G.709], FEC should detect 16 symbols error or correct 8 symbols errors.

The performance of the FEC is the capability of the EUT receiver to detect or to correct bit errors located in OTU frames.

As performance of FEC decoder shall be verified under a situation similar to a real network, a measuring equipment shall firstly support a capability to generate symbol errors at random by a Poisson Distribution as illustrated by Figure 10-1 and secondly to measure the performance of the FEC decoder in the EUT as presented in clause 10.3.

The method of measurement relies on the assumption that the initial signal contains only errors inserted after FEC computation.

10.1 Parameters and results

The FEC performance is defined as:

FEC performance = Inserted bit error ratio/bit error ratio measured after error correction in the EUT.

The following parameters must be known when comparing two FEC performance measurements:

- Rate λ of the Poisson distribution defined as the average number of error events in a defined observation interval (see clause C.4);
- Measurement duration;
- Inserted bit error ratio;

- Test signal structure x (see Annex A);
- Area of the structure of the OTUk where bits errors are inserted.

10.2 Error generation

Figure 10-1 presents the error generator. To measure the FEC performance, ME sender inserts symbol errors after FEC computation. Errors are inserted following a Poisson distribution law. Annex C defines the parameters and the procedure for testing the goodness of fit of a Poisson distribution.

To verify FEC performance, measuring equipment shall support constructing the OTUk frame based on [ITU-T G.709].

The OTUk forward error correction (FEC) contains the Reed-Solomon RS(255,239) FEC code.

The RS(255,239) FEC code shall be computed as specified in Annex A of [ITU-T G.709].

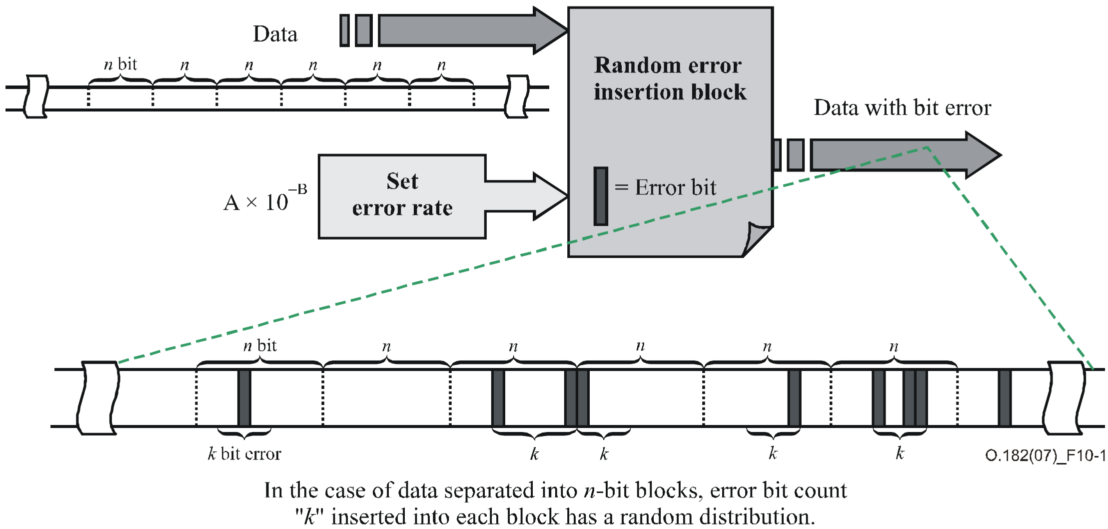

Figure 10-1 – Image of random error insertion

10.3 Measurement of the error correction and detection

Errors are corrected or detected by the EUT receiver according to the configuration of its FEC function. Two separate results are required to measure firstly the FEC errors correction and secondly to measure the FEC errors detection.

10.3.1 Measurement of errors correction

Two methods are identified for measuring the number of errors corrected by the FEC at the receiver side of the EUT.

1) *White box method:*

[ITU-T G.798] says that the number of corrected bits shall be reported by the EUT.

The white box method described by Figure 10-2 consists in reading the number of errors corrected, calculating the number of errors uncorrected and comparing this number with the number of errors inserted initially by the ME generator.

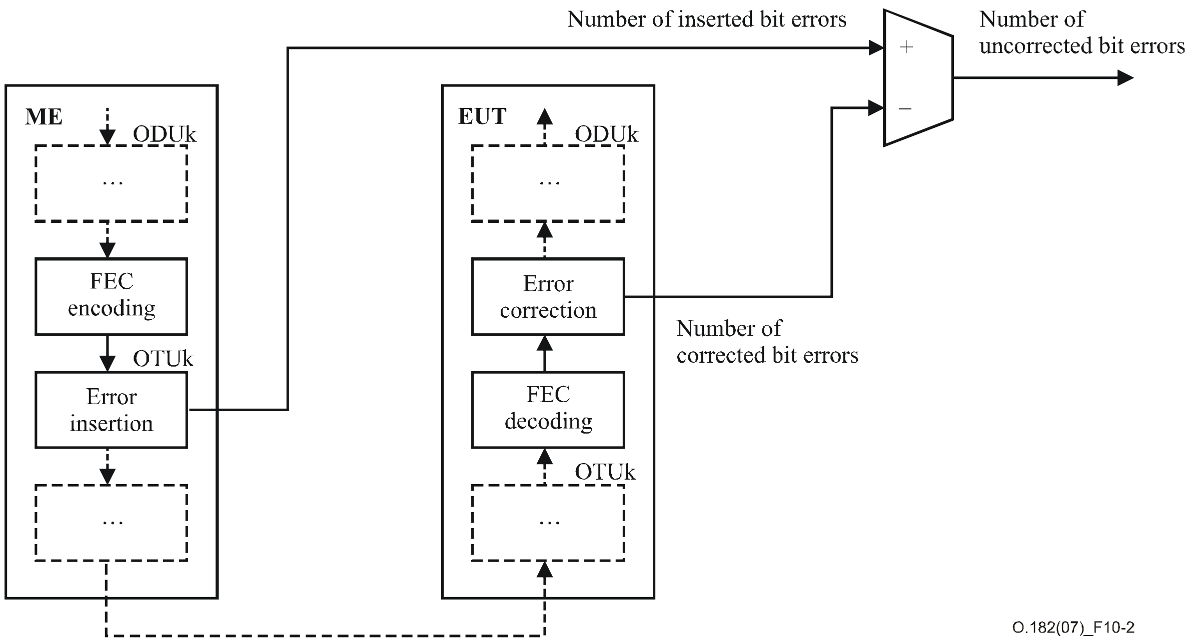

Figure 10-2 – White box method

2) *Black box method:*

The black box method is described by Figure 10-3 where EUT sends the signal to the ME receiver with several errors corrected and with the other errors still present. The result is similar: the ME receiver counts the number of errors still present in the signal received from the EUT sender and compares it with the number of errors inserted initially by the ME generator.

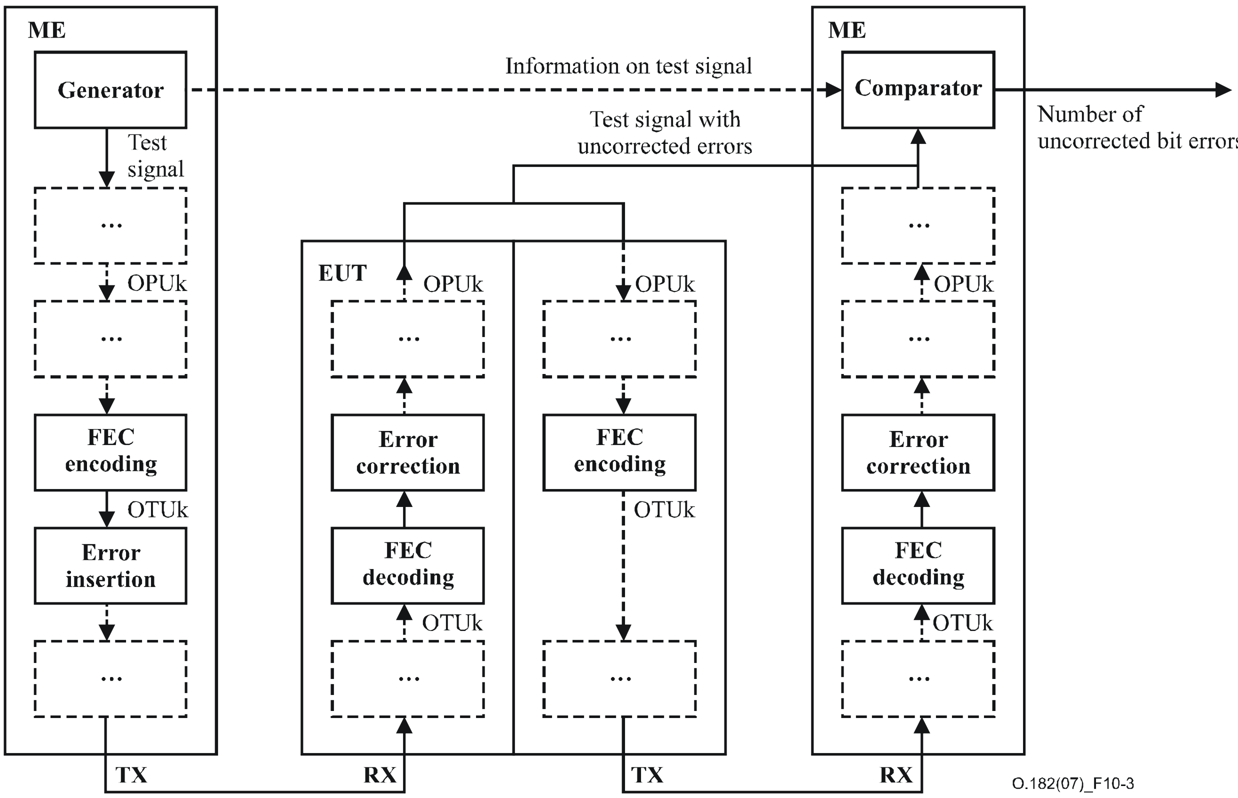

Figure 10-3 – Black box method

10.3.2 Measurement of errors detection

White box method:

As [ITU-T G.798] does not specify the reporting of the number of errors detections, this Recommendation does not propose any method to measure the errors detection of a FEC.

Black box method:

As the signal forwarded to the ME receiver does not carry the number of errors detected on the receiver side of the EUT, this Recommendation does not propose any method to measure the errors detection of a FEC.

11 Miscellaneous functions

The following functions do not directly influence the error performance measurement definitions and shall be considered as optional for the measuring equipment.

11.1 FEC activation

A measurement equipment shall support the capability to disable the FEC encoding/decoding process (ignore the content of the OTUk FEC) for interworking of equipment supporting FEC, with equipment not supporting FEC (inserting fixed stuff all-0s pattern in the OTUk FEC area) or with equipment supporting a different FEC code from RS(255,239) defined by [ITU-T G.709].

11.2 Display

The measuring equipment may include a display to provide easier access to the configuration and measurement parameters.

11.3 Access to overhead bytes

The measuring equipment may display the most important OTU, ODU and OPU overhead bytes of the OCh layer signal analysed by its receiver.

11.4 Output to external recording devices

The measuring equipment may provide the facility to connect an external recording device (e.g., a external storage).

11.5 Remote control port

The measuring equipment may be remotely controlled using standard interfaces (e.g., IEEE 488.1/IEC 60625 specification, ITU-T Recs V.24 and V.28, USB, Ethernet).

12 Operating conditions

12.1 Environmental conditions

To perform the functions listed in this Recommendation, the measuring equipment shall operate under conditions according to [ITU-T O.3].

12.2 Behaviour in case of power failure

The measuring equipment shall indicate a power interruption.

Annex A

Criteria for detecting anomalies and defects

(This annex forms an integral part of this Recommendation)

NOTE 1 – The list of network defects and anomalies found in this annex shall conform to the last agreed versions of the relevant OTN Recommendations such as [ITU-T G.709] and [ITU-T G.798].

NOTE 2 – i = 1,2,...,6 for Tandem Connection Monitoring.

A.1 Anomalies

A.1.1 Out of frame (OOF)

The criteria for the detection of an OOF anomaly shall be in accordance with the criteria defined in [ITU-T G.798].

A.1.2 Out of multiframe (OOM)

The criteria for the detection of an OOM anomaly shall be in accordance with the criteria defined in [ITU-T G.798].

A.1.3 SM-BIP-8 errors

The parity errors evaluated by byte SM-BIP-8 of an OTUk shall be monitored. If any of the eight parity checks fails, the corresponding block is assumed to be in error.

A.1.4 PM-BIP-8 errors

The parity errors evaluated by byte PM-BIP-8 of an ODUk shall be monitored. If any of the eight parity checks fails, the corresponding block is assumed to be in error.

A.1.5 TCMi-BIP-8 errors

The parity errors evaluated by byte TCMi-BIP-8 of an ODUk shall be monitored. If any of the eight parity checks fails, the corresponding block is assumed to be in error.

A.1.6 FEC corrected errors (FCE)

Corrected errors shall be monitored as the indication of FCE.

A.1.7 FEC uncorrected error blocks (FUEB)

The FUEB shall indicate the number of error block that exceeds the error correction ability by RS(255,239) FEC code.

A.1.8 Section monitoring backward error indication (SM-BEI)

The indication of SM-BEI located at "byte 3 / bit 1 to 4" of SM field of OTUk shall be monitored.

A.1.9 Path monitoring backward error indication (PM-BEI)

The indication of PM-BEI located at "byte 3 / bit 1 to 4" of PM field of ODUk shall be monitored.

A.1.10 Tandem connection monitoring backward error indication (TCMi-BEI)

The indication of TCMi-BEI located at "byte 3 / bit 1 to 4" of TCMi field of ODUk shall be monitored.

A.1.11 Test sequence error (TSE)

A TSE occurs when one or more bit errors are detected on the test sequence in the set of consecutive bits.

A.2 Defects

A.2.1 Loss of signal (LOS)

Criteria are defined for optical interfaces.

The criteria for the detection of a LOS defect shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.2 Loss of frame (LOF)

The criteria for the detection of a LOF defect shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.3 Loss of multiframe (LOM)

The criteria for the detection of a LOM defect shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.4 Optical channel transport unit alarm indication signal (OTU-AIS)

The criteria for the detection of an OTU-AIS defect shall be in accordance with the criteria defined in [ITU-T G.798].

OTU-AIS is defined as generic-AIS detection.

A.2.5 Optical channel data unit alarm indication signal (ODU-AIS)

The criteria for the detection of an ODU-AIS defect shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.6 Optical channel data unit open connection indication (ODU-OCI)

The criteria for the detection of an ODU-OCI defect shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.7 Optical channel data unit locked (ODU-LCK)

The criteria for the detection of an ODU-LCK defect shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.8 Section monitoring backward defect indication (SM-BDI)

The criteria for the detection of a SM-BDI defect shall be in accordance with the criteria defined in [ITU-T G.798].

SM-BDI bit is located at "byte 3/bit 5" of SM field of OTUk.

A.2.9 Path monitoring backward defect indication (PM-BDI)

The criteria for the detection of a PM-BDI defect shall be in accordance with the criteria defined in [ITU-T G.798].

PM-BDI bit is located at "byte 3/bit 5" of PM field of ODUk.

A.2.10 Tandem connection monitoring backward defect indication (TCMi-BDI)

The criteria for the detection of a TCMi-BDI defect shall be in accordance with the criteria defined in [ITU-T G.798].

TCMi-BDI bit is located at "byte 3/bit 5" of each TCM field of ODUk.

A.2.11 Section monitoring backward incoming alignment error (SM-BIAE)

The criteria for the detection of a SM-BIAE defect shall be in accordance with the criteria defined in [ITU-T G.798].

SM-BIAE bits are located at "byte 3/bit 1 to 4" of SM field of OTUk.

A.2.12 Tandem connection monitoring backward incoming alignment error (TCMi-BIAE)

The criteria for the detection of a TCMi-BIAE defect shall be in accordance with the criteria defined in [ITU-T G.798].

TCMi-BIAE bits are located at "byte 3/bit 1 to 4" of each TCM field of ODUk.

A.2.13 Section monitoring incoming alignment error (SM-IAE)

The criteria for the detection of a SM-IAE defect shall be in accordance with the criteria defined in [ITU-T G.798].

SM-IAE bit is located at "byte 3/bit 6" of SM field of OTUk.

A.2.14 Tandem connection monitoring incoming alignment error (TCMi-IAE)

The criteria for the detection of a TCMi-IAE defect shall be in accordance with the criteria defined in [ITU-T G.798].

TCMi-IAE is defined as status bit (STAT) at "byte 3/bit 6 to 8" of each TCM field of ODUk.

A.2.15 Section monitoring trail trace identifier mismatch (SM-TIM)

The criteria for the detection of a SM-TIM defect using the information located at "byte 1/bit 1 to 8" of SM field of OTUk shall be in accordance with the criteria defined in [ITU-T G.709] and [ITU-T G.798].

A.2.16 Path monitoring trail trace identifier mismatch (PM-TIM)

The criteria for the detection of a PM-TIM defect using the information located at "byte 1/bit 1 to 8" of PM field of ODUk shall be in accordance with the criteria defined in [ITU-T G.709] and [ITU-T G.798].

A.2.17 Tandem connection monitoring trail trace identifier mismatch (TCMi-TIM)

The criteria for the detection of a TCMi-TIM defect using the information located at "byte 1/bit 1 to 8" of each TCM field of ODUk shall be in accordance with the criteria defined in [ITU-T G.709] and [ITU-T G.798].

A.2.18 Tandem connection monitoring loss of tandem connection (TCMi-LTC)

The criteria for the detection of a TCMi-LTC defect shall be in accordance with the criteria defined in [ITU-T G.798].

TCMi-LTC is defined as status bit (STAT) at "byte 3/bit 6 to 8" of each TCM field of ODUk.

A.2.19 Optical channel data unit payload mismatch (ODU-PLM)

The criteria for the detection of an ODU-PLM defect using the information located at PSI byte in OPUk overhead shall be in accordance with the criteria defined in [ITU-T G.798].

A.2.20 Loss of sequence synchronization (LSS)

The criteria for LSS shall be in accordance with the criteria defined in [ITU-T O.150].

A.3 Other information

This clause lists items that are not yet classified depending on whether they are affecting error performance or not.

A.3.1 Optical channel payload unit justification events (OPU-JE)

The number of positive justification and negative justification shall be monitored.

A.3.2 Loss of timing input

When measuring equipment is synchronized via an external synchronization interface, it shall be capable to detect a loss of timing input.

Annex B

List of test signal structures

(This annex forms an integral part of this Recommendation)

Test signal structures (mapping) of OPUk frames are defined in [ITU-T G.709].

The mapping only changes the OPUk payload area of TSS. A specific TSS is defined for the client signals CBR2G5, CBR10G, CBR40G and NULL/PRBS.

According to [ITU-T G.709], to ensure compatibility with OTN (OCh layer) network equipment, the Recommendation assigns each type of TSS a unique signal label. The label is carried in the PSI field.

STM structures referenced in this clause have to conform to [ITU-T G.707].

B.1 Test signal structure TSS1 applied to all bytes of a CBR2G5

TSS1 is defined to transport CBR2G5 on OTN. A signal of CBR2G5 is accommodated to OPU1 payload area by two mapping methods: Asynchronous mapping method with the positive and negative justification, and Bit synchronous method without the justification (as listed in Figure B.1).

Measuring equipment performing out-of-service measurement shall support "STM16-VC4-16c PRBS23" test sequence to verify a EUT.

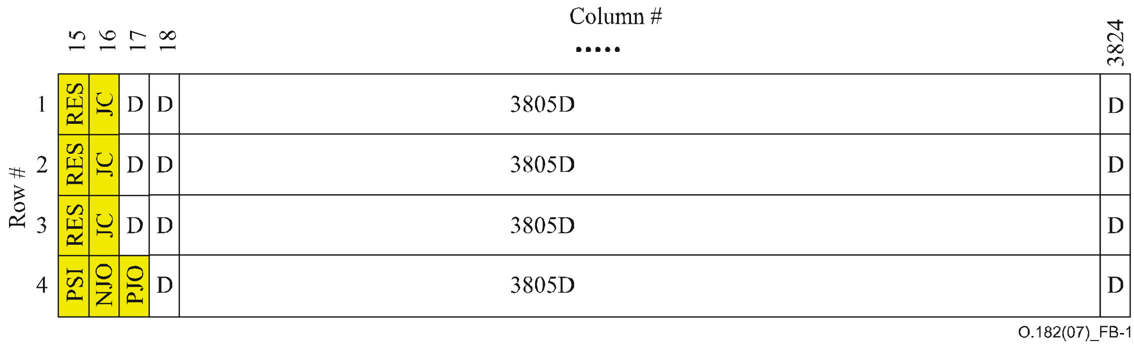

Figure B.1 – TSS1, Frame structure of CBR2G5 signal into OPU1

B.2 Test signal structure TSS2 applied to all bytes of a CBR10G

TSS2 is defined to transport CBR10G on OTN. A signal of CBR10G is accommodated to OPU2 payload area by two mapping methods: Asynchronous mapping method with the positive and negative justification, and Bit synchronous method without the justification (as listed in Figure B.2).

Since the fixed stuff bytes are not part of the CBR10G signal, they must not be included in the measurement.

When justification occurs, stuff bytes are not shifted; only client data are shifted. Measuring equipment does not use the stuff bytes to insert any data.

Measuring equipment performing out-of-service measurement shall support "STM64-VC4-64c PRBS23 or PRBS31" test sequence to verify a EUT.

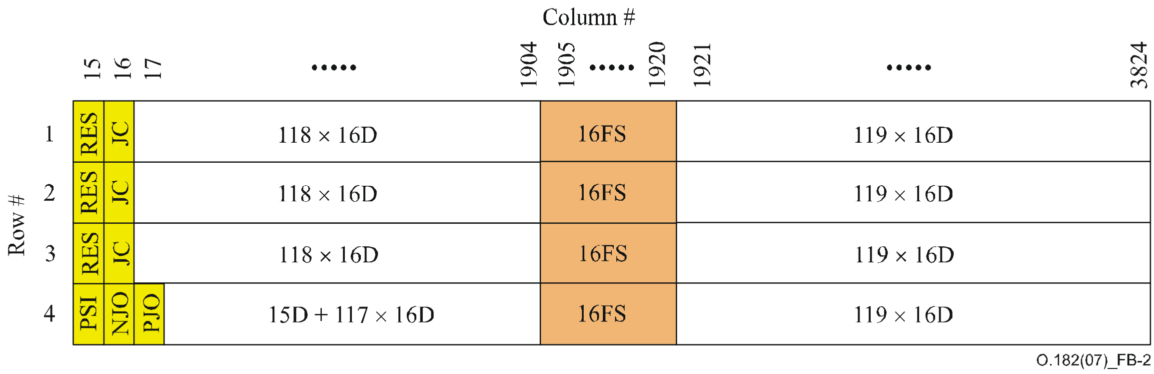

Figure B.2 – TSS2, Frame structure of CBR10G signal into OPU2

B.3 Test signal structure TSS3 applied to all bytes of a CBR40G

TSS3 is defined to transport CBR40G on OTN. A signal of CBR40G is accommodated to OPU3 payload area by two mapping methods: Asynchronous mapping method with the positive and negative justification, and Bit synchronous method without the justification (as listed in Figure B.3).

Since the fixed stuff bytes are not part of the CBR40G signal, they must not be included in the measurement.

When justification occurs, stuff bytes are not shifted: only client data are shifted. Measuring equipment does not use the stuff bytes to insert any data.

Measuring equipment performing out-of-service measurement shall support "STM256-VC4-256c PRBS31" test sequence to verify a EUT.

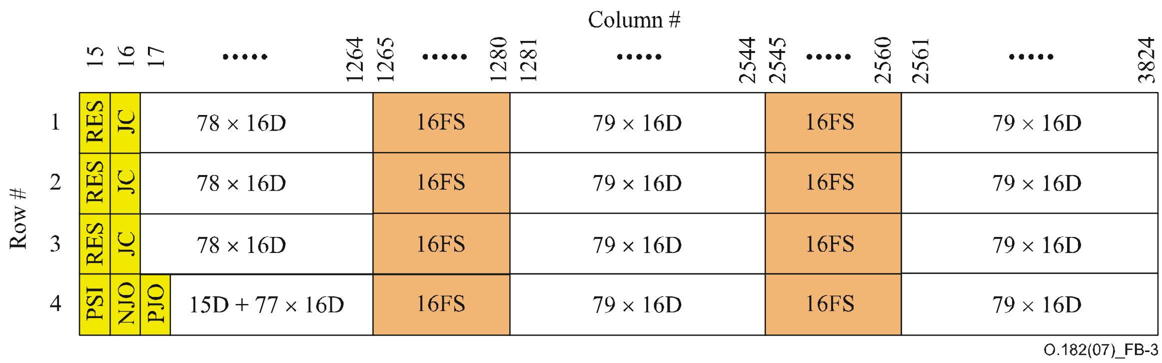

Figure B.3 – TSS3, Frame structure of CBR40G signal into OPU3

B.4 Test signal structure TSS4 applied to NULL/PRBS

TSS4 defines a test signal to carry NULL or PRBS signals.

TSS4 is only applicable to out-of-service mode at any OTN bit rates.

For NULL client mapping (see Figure B.4), the test equipment shall insert an all-0 pattern in the OPUk payload area.

For PRBS client mapping (see Figure B.5), the test equipment shall insert a PRBS31 pattern as defined in [ITU-T O.150] in the OPUk payload area.

Refer to clauses 14.3.4 and 14.3.5 of [ITU-T G.798] for further details.

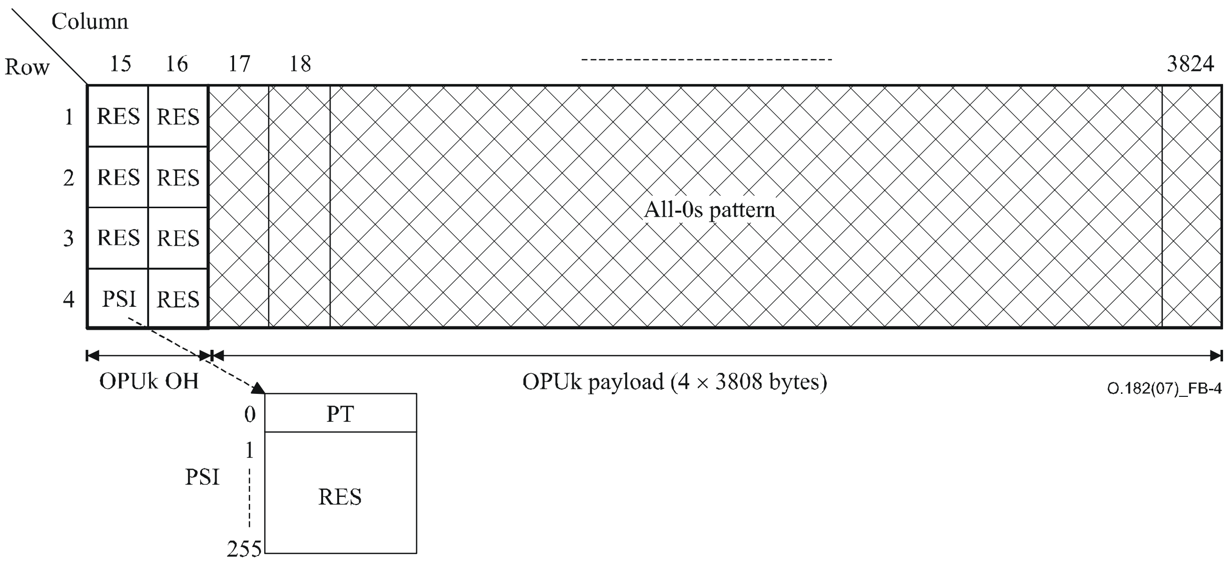

Figure B.4 – TSS4, Frame structure of NULL signal into OPUk

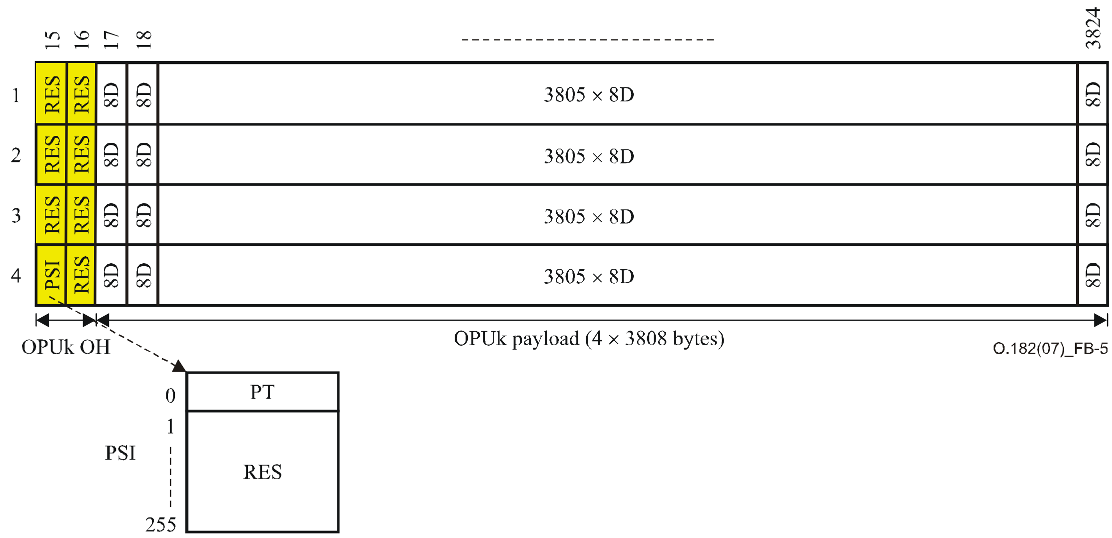

Figure B.5 – TSS4, Frame structure of PRBS31 signal into OPUk

Annex C

Procedure of goodness of fit for Poisson distribution by χ^2 test

(This annex forms an integral part of this Recommendation)

C.1 Introduction

A "Poisson error generator" used for performance tests of digital communications systems should generate random errors satisfying the Poisson distribution. However, the distribution of the random errors generated from such equipment may not necessarily fit a Poisson distribution. Therefore, an objective method to evaluate the distribution characteristic of the random errors is needed. Although there are many methods for testing goodness of fit for Poisson distribution, this annex describes a method using the χ^2 test. Refer to Appendix I for the detailed explanation of this method.

C.2 Fundamental properties

A Poisson process with parameter or rate $p > 0$ is an integer-valued, continuous-time stochastic process $\{X(t),\ t \geq 0\}$ satisfying the following fundamental properties:

a) $X(0) = 0$;

b) For all $t_0 = 0 < t_1 < \cdots < t_k$, the increments $X(t_1) - X(t_0), X(t_2) - X(t_1), \ldots, X(t_k) - X(t_{k-1})$ are independent random variables;

c) For $t \geq 0$, $s > 0$ and non-negative integers k, the increments have the Poisson distribution:

$$P\{X(t+s) - X(s) = k\} = \frac{e^{-pt}}{k!} \times (pt)^k \quad k = 0,1,2,\ldots \tag{C-1}$$

C.3 Other properties

This process satisfies other properties:

i) $$E\{X(t)\} = pt \tag{C-2}$$

ii) $$\mathrm{var}\{X(t)\} = pt \tag{C-3}$$

iii) Let $\{X(t);\ t \geq 0\}$ be a Poisson process with rate $p > 0$ and denote by $t_0 = 0 < t_1 < t_2 < \cdots$ the successive occurrence times of events. Then the interarrival times $\tau_k = t_k - t_{k-1}$ are independent identically distributed (i.i.d.) exponential random variables with mean $1/p$;

The converse of statement iii) also holds:

iv) If the interarrival times $\{\tau_k\}$ of a counting process $\{K(t),\ t \geq 0\}$ are i.i.d. exponential random variables with mean $1/p$, then $\{K(t),\ t \geq 0\}$ is a Poisson process with rate p.

C.4 Poisson distribution parameters

A Poisson distributed error process is defined by the following parameters:

- Bit error ratio p_e;
- Averaged error occurrence λ in an observation interval of 'n' bits;
- Observation interval of 'n' bits.

They are related by the formula $n = \lambda / p_e$.

C.5 Procedure for test of goodness of fit by χ^2 test

The test of goodness of fit for Poisson distribution by χ^2 test is performed by the following steps:

1) Determination of bit error ratio p_e of random error generator under test

   Example: $p_e = 10^{-8}$

2) Determination of averaged error occurrence λ for observation interval of 'n' bits

   Recommended λ: $5 \le \lambda \le 20$

   Example: $\lambda = 16$

3) Determination of observation interval of 'n' bits by using equation $n = \lambda/p_e$

   Example: $n = 16/10^{-8} = 1.6\times10^9$

4) Determination of sample size N

   Recommended N: $N \ge 1000$

   Example: $N = 1024$

5) Creation of histogram table by observation

   The observation block diagram is given in Figure C.1.

6) Estimation of averaged error occurrence $\hat{\lambda}$ using the following equation:

$$\hat{\lambda} = \frac{1}{N}\sum_{i=1}^{j} i \cdot f_i$$

   where f_i is the observed frequency that occurred i times in the observation interval of 'n' bits.

7) Rearrangement of the histogram and calculation of χ^2 by using the following equation:

$$\chi^2 = \sum_{i=1}^{j} \frac{(f_i - e_i)^2}{e_i}$$

   where e_i is the expected frequency that occurred i times in the observation interval of 'n' bits, given by the following equation:

$$e_i = N \cdot p_i$$

   where p_i is i-th probability of Poisson distribution given by the following equation:

$$p_i = e^{-\hat{\lambda}} \frac{\hat{\lambda}^i}{i!}$$

8) Determination of degrees of freedom ν by referencing the rearranged table

   Example: $\nu = k_{max} - k_{min} - 1 = 26 - 7 - 1 = 18$

   where k_{min} and k_{max} are minimum k and maximum k in the rearranged table.

9) Determination of significance level α

   Recommended α: $0.01 \le \alpha \le 0.1$

   Example: $\alpha = 0.05 = 5\%$

10) Calculation of significance point χ_α^2 by looking up χ^2 table

11) Evaluation of goodness of fit test by the following rule:

    Hypothesis is accepted if $\chi^2 \le \chi_\alpha^2$; otherwise it is rejected, where χ^2 is chi-square calculated in Step 7).

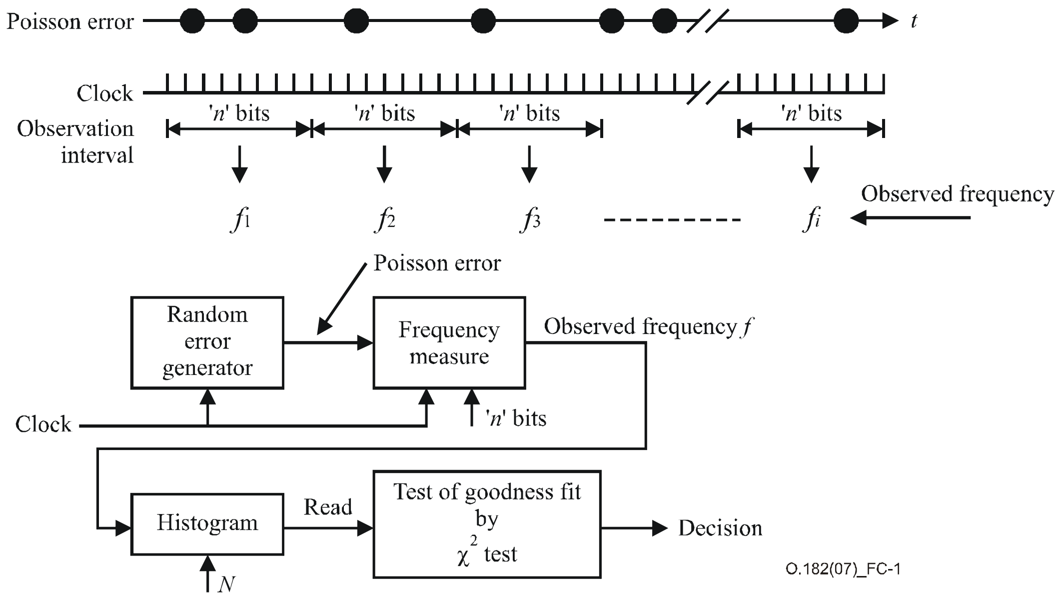

Figure C.1 – Observation block diagram

Appendix I

Test of goodness of fit for Poisson distribution by χ^2 test

(This appendix does not form an integral part of this Recommendation)

This appendix describes the detail of the evaluation method for Poisson distribution described in Annex C. Furthermore, this appendix explains the reasons why this method is used.

I.1 Introduction

The main purpose of the "random error generator" is to evaluate digital communications systems. The principal use is to evaluate error correction functions in digital communications systems. For example, the equipment is used to test the error detection and error correction ability of the error correction functions. The common random error generator used for such purposes is based on the conditions of the bit error ratio $p_e \leq 10^{-2}$. It is clear that random errors generated from the random error generator are distributed as a "Poisson distribution" if the random error generator is used under the above condition. Therefore, such a random error generator is sometimes called a "Poisson error generator".

However, random errors generated by an actual random error generator are often not distributed as a Poisson distribution, depending on the implementation method. Therefore, we need an objective method to evaluate whether the observed samples fit the expected distribution. The "test of goodness of fit" method can perform such an evaluation. There are many methods for testing goodness of fit, but this appendix proposes the "test of goodness of fit by χ^2 test".

Clause I.2 briefly describes the "test of goodness of fit by χ^2 test" method. Clause I.2 also describes why the "test of goodness of fit by the χ^2 test" method is proposed and some precautionary notes.

Clause I.3 briefly describes the theory of the χ^2 test.

Clause I.4 shows two application examples of the goodness of fit test for the Poisson distribution. The first example fits the Poisson distribution at the 5% significance level. The next example does not fit the Poisson distribution at the 5% significance level.

I.2 Test of goodness of fit by χ^2 test

The method for evaluating the characteristics of probabilistic distributions is called the "test of goodness of fit". The "test of goodness of fit" objectively tests whether the distribution of the obtained sample matches the hypothesis. Although there are various methods for testing goodness of fit, this appendix proposes the "test of goodness of fit by χ^2 test". One reason for proposing this method is that it has been used for more than 100 years and is the most conventional method. A second reason is that the procedure is simple and easy.

Performing the test method under the same conditions using the same procedures is more important than choosing the best method. Therefore, the other methods are not described here.

It is called a "hypothesis test" because it assumes a probabilistic distribution, and the hypothesis is tested using finite samples. Although the test for goodness of fit by χ^2 test is one of several hypothesis test methods, it is the most typical hypothesis test.

Like other tests, the test of goodness of fit by χ^2 test is not suitable for all distributions. However, fortunately, it is the best method when the distribution is a Poisson distribution. Actually, the goodness of fit test by χ^2 test is often used when assuming the distribution is a Poisson distribution.

The parameter of the Poisson distribution, $\lambda = p_e \cdot n$, can be set to any value by choosing the bit error ratio p_e and the observation interval of 'n' bits. Incidentally, when carrying out the test of goodness of fit for the Poisson distribution, the values for λ should be chosen in the range given by the following formula:

$$5 \le \lambda \le 20 \tag{I-1}$$

In particular, the lower limit of the formula must be satisfied. On the other hand, the upper limit should be satisfied, because the upper limit is derived from the observation interval n, which should be short. If the situation permits (when increased observation interval does not cause problems), λ does not necessarily need to be restricted by the upper limit of the above formula.

When the λ is small, the distribution of the Poisson distribution will move to the left and the number of observed frequency items will decrease, where, "observed frequency" denotes the frequency of k occurrences, including 0, for the observation interval n. The observed frequency is described in more detail later.

The number of observed frequencies should be greater than or equal to 5. Although this value is based on empirical observation, since the reliability of the test of the hypothesis must be held at a fixed level, it is also a request. Actually, since the χ^2 test is based on a large-sample theory, the test reliability is lowered with small sample sizes.

Conversely, if λ is much too large, the number of observed frequencies will increase and the frequency of each item will become small. Of course, although the frequency of each item becomes unlimited by increasing the number of observation intervals, total observation time increases as a result.

When performing the test of goodness of fit by the χ^2 test, the frequency of each item should be more than 5. This is another restriction on the test of goodness of fit by the χ^2 test to keep the reliability of the test at a sufficiently high level.

I.3 Theory of χ^2 test

Letting $f_1, f_2, \ldots, f_j$ and $e_1, e_2, \ldots, e_j$ be the observed frequency of the sample by N experiments and the expected frequency, where j is the number of possible outcomes, the value given by the equation:

$$\chi^2 = \sum_{i=1}^{j} \frac{(f_i - e_i)^2}{e_i} \tag{I-2}$$

approaches the χ^2 distribution of the degrees of freedom $\nu = j - 1 - t$ with the increase in the number of experiments N, where t is the number of estimated parameters.

Generally, when the test of goodness of fit by χ^2 is performed, the parameter λ is estimated using the experimental sample. In this case, the degrees of freedom ν is given by $j - 2$. From the equation, it is clear that the χ^2 given by the equation becomes small in accordance with fitting of the distribution to the hypothesis distribution. However, usually, this is very rare. If χ^2 becomes 0, then the sample distribution is in perfect coincidence with the hypothetical distribution. The test of goodness of fit by the χ^2 test is just a method of testing the hypothesis, which judges whether the hypothesis should be accepted or rejected by evaluating the value of χ^2 given by the above equation.

Incidentally, the χ^2 distribution corresponds to the gamma distribution with parameters α and β set to $\alpha = \nu/2$, $\beta = 2$. Therefore, the χ^2 distribution is given by the following equation:

$$\chi^2(x) = Ga(x) = \begin{cases} \dfrac{x^{\nu/2-1} e^{-x/2}}{2^{\nu/2}\,\Gamma(\nu/2)}, & x \ge 0 \\ 0, & x < 0 \end{cases} \tag{I-3}$$

where the parameter ν is the degrees of freedom of the χ^2 distribution.

In addition, Γ(α) in the equation is called the gamma function and is given by the following equation:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \quad \text{(I-4)}$$

Since the equation Γ(α+1) = αΓ(α) is satisfied, if α is an integer number, the gamma function can be calculated by Γ(α+1) = α!. Based on this character, the gamma function is sometimes called the "factorial function".

Since the χ^2 distribution function is also the probability density function, the entire area of the function approaches 1. That is:

$$\int_0^\infty \chi^2(x)dx = 1 \quad \text{(I-5)}$$

Now, calculating the right tail area of the χ^2 distribution function from some point, and calling it α:

$$\alpha = \int_{\chi_\alpha^2}^\infty \chi^2(x)dx \quad \text{(I-6)}$$

Since the entire area of the χ^2 distribution function is 1, α given by the equation represents the area ratio, this α is called the "significance level". Generally, the significance level is represented by the percentage as 100 × α.

Since the χ^2 distribution function is monotonous and continuous, χ_α^2 can be calculated from the given significance level 100 × α. This χ_α^2 is called the "significance point", "critical point", or "percent point".

Because it is very difficult to calculate the significance point from the given significance level and degrees of freedom, many statistics books append a table of the χ^2 distribution function. In precise terms, the table is used to determine the significance point from a given significance level and the degrees of freedom.

Figure I.1 shows an example of the χ^2 distribution function with the degrees of freedom ν = 10.

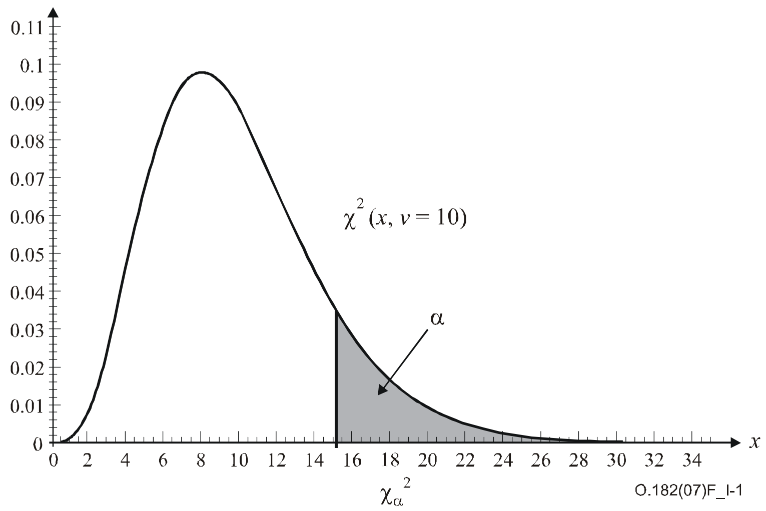

Figure I.1 – χ^2 distribution function

The test of goodness of fit by the χ^2 test evaluates whether the hypothesis should be accepted or rejected by referencing the χ^2 calculated from the obtained sample.

$$H_0 = \begin{cases} accept & if\ \chi^2 \le \chi_\alpha^2 \\ reject & if\ \chi^2 > \chi_\alpha^2 \end{cases} \tag{I-7}$$

where H_0 represents the null hypothesis.

In other words, if χ^2 calculated by using equation (I-2) is less than the significance point determined from the significance level $100 \times \alpha$ and the degrees of freedom ν, the hypothesis is accepted. Otherwise, the hypothesis is rejected.

I.3.1 Estimation of parameter of Poisson distribution

The parameter λ of the Poisson distribution is rarely known when executing the test of goodness of fit by the χ^2 test for the Poisson distribution. Therefore, usually, the parameter λ must be estimated from the obtained sample.

It is known that the estimate of maximum likelihood and the first moment coincide for the Poisson distribution. Therefore, the estimated parameter λ of the Poisson distribution can be calculated using the following equation:

$$\hat{\lambda} = \frac{1}{N} \sum_{i=1}^{j} i \cdot f_i \tag{I-8}$$

where N denotes the sample size.

When the test of goodness of fit is performed, the expected frequency must either be calculated using the given parameters, or estimated.

The k-th probability p_k of the Poisson distribution is calculated by the following equation using the estimated parameter λ:

$$p_k = e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} \tag{I-9}$$

Therefore, the k-th expected frequency is given by the following equation:

$$e_k = N \cdot p_k \tag{I-10}$$

I.4 Application of test of goodness of fit to Poisson distribution

This clause describes two examples of application of the test of goodness of fit by χ^2 for the Poisson distribution. The first shows an example that fits the Poisson distribution at the 5% significance level. The second shows an example that does not fit the Poisson distribution at the 5% significance level.

Both examples are obtained from an actual Poisson error generator under the same conditions with a bit error ratio $p_e = 10^{-8}$ and sample size $N > 1000$. Furthermore, in both examples, the number of average errors occurring in the observation interval n were chosen so that $\lambda = 16$. In other words, the experiment was executed with observation interval $n = \lambda/p_e$.

Incidentally, observation interval means discrete time (number of clocks).

I.4.1 Example fitting Poisson distribution

The sample was obtained from the first Poisson error generator. Table I.1 shows the sample data.

Table I.1 – Sample data of Example 1

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Probability p_k
5	1	1.184	0.000967
6	9	3.114	0.008704
7	7	7.019	0.006770
8	9	13.842	0.008704
9	20	24.267	0.019342
10	32	38.287	0.030948
11	56	54.916	0.054159
12	78	72.203	0.075435
13	93	87.630	0.089942
14	107	98.756	0.103482
15	112	103.875	0.108317
16	83	102.431	0.080271
17	102	95.065	0.098646
18	75	83.328	0.072534
19	70	69.195	0.067698
20	55	54.587	0.053191
21	40	41.012	0.038685
22	26	29.412	0.025145
23	24	20.176	0.023211
24	16	13.264	0.015474
25	4	8.371	0.003868
26	6	5.080	0.005803
27	5	2.968	0.004836
28	3	1.673	0.002901
29	0	0.910	0.000000
30	1	0.479	0.000967
Total	$\Sigma f_k = 1034$	$\Sigma e_k = 1033.040$	$\Sigma p_k = 1.000000$

Figure I.2 is the graph based on the data in Table I.1. The histogram and dotted line curve are probability calculated from the observed frequency and Poisson distribution function, respectively. Although the Poisson distribution is a discrete distribution, to improve the visibility, the Poisson distribution has been drawn as a continuous curve interpolated by the gamma distribution function.

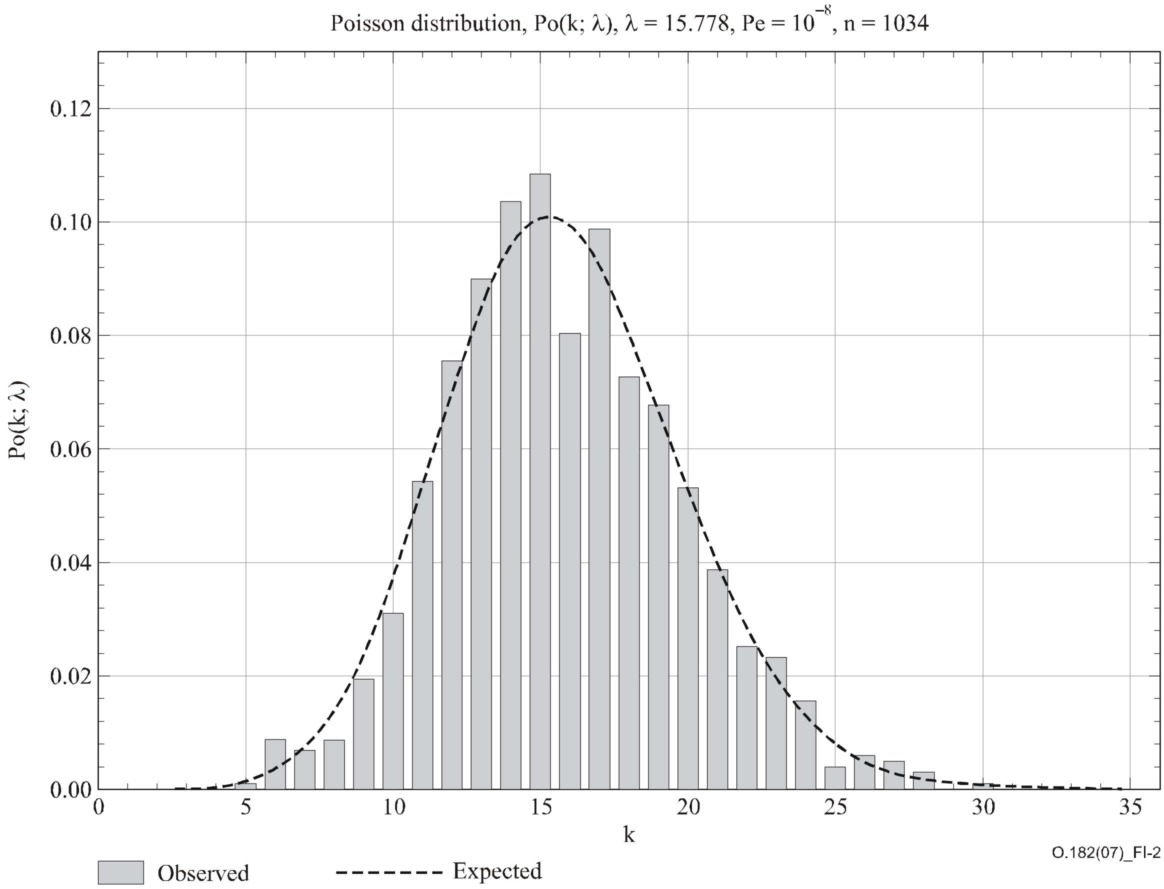

Figure I.2 – Histogram of Example 1

The first column in Table I.1 represents the number k of errors occurring in the observation interval n. The second column represents the count, i.e., observed frequency of k errors occurring in the observation interval n. The third column number represents the expected frequency calculated using the probability density function of the Poisson distribution with the mean error number estimated from the obtained sample. In other words, the expected frequency is calculated by the following equation:

$$e_k = e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} \cdot N \tag{I-11}$$

where N is the sample size. The fourth column in Table I.1 represents the probability of k errors occurring in the observation interval n. This probability is calculated by the following equation:

$$p_k = f_k / N \tag{I-12}$$

The last column in Table I.1 represents the sum of each column. The sum of the second column is sum of the observed frequency, and is also the sample size N. The sum of the third column is the sum of the expected frequency.

Naturally, there are some differences between the totals of observed frequency and expected frequency, because the former is sample size and the latter is the sum of the expected frequencies for the k range $5 \le k \le 30$. The difference is clearly given by the following formula:

$$\sum_{k=5}^{30} f_k - \sum_{k=5}^{30} e_k = \sum_{k=0}^{4} e_k + \sum_{k=31}^{\infty} e_k = N \sum_{k=0}^{4} e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} + N \sum_{k=31}^{\infty} e^{-\hat{\lambda}} \frac{\hat{\lambda}^k}{k!} \tag{I-13}$$

Here, notice that the expected frequencies for $k \leq 6$ and $k \geq 27$ are less than 5. As mentioned previously, to ensure the reliability of the χ^2 test result, the expected frequency should be more than 5 and sometimes more than 10. Therefore, Table I.1 must be re-arranged.

The rearranged Table I.1 merges the rows of $5 \leq k \leq 6$ with the row of $k = 7$. Table I.2 shows the merger of the rows of $27 \leq k \leq 30$ with the row of $k = 26$. The numbers of the fourth column represent the deviations between the observed and expected frequencies. The total of this deviation is χ^2.

Table I.2 – Rearranged table of Example 1

k	**Observed frequency** f_k	**Expected frequency** $e_k = np_k$	**Deviation** $(f_k - e_k)^2/e_k$
≤7	17	11.808	2.283
8	9	13.842	1.694
9	20	24.267	0.750
10	32	38.287	1.032
11	56	54.916	0.021
12	78	72.203	0.465
13	93	87.630	0.329
14	107	98.756	0.688
15	112	103.875	0.636
16	83	102.431	3.686
17	102	95.065	0.506
18	75	8.328	0.832
19	70	69.195	0.009
20	55	54.587	0.003
21	40	41.012	0.025
22	26	29.412	0.396
23	24	20.176	0.725
24	16	13.264	0.565
25	4	8.371	2.282
≥ 26	15	11.578	1.012
Total	$\Sigma f_k = 1034$	$\Sigma e_k = 1034.000$	$\chi^2 = \Sigma(f_k - e_k)^2/e_k = 17.939$

Table I.3 shows the result of the goodness of fit test based on Table I.2. With this sample, the null hypothesis is accepted at the 5% significance level.

Table I.3 – Result of goodness of fit test for Example 1

Item	Symbol	Value
Sample size	N	1034
Estimated λ	$\hat{\lambda}$	15.7776
Degrees of freedom	ν	18 (k = 7, ..., 26)
Chi-square	χ^2	17.9395 (tail-area = 45.96%)
Significance level	α	5.0%
Significance point	χ_α^2	28.8693
Hypothesis	H_0	Accepted

I.4.2 Example not fitting Poisson distribution

The next example is a sample obtained from the second Poisson error generator. Table I.4 shows the sample data. The meanings of the numerical values in the table are the same as in the preceding sample.

Figure I.3 is the graph based on the data in Table I.4. The meaning of the graph is the same as in the preceding sample.

Table I.4 – Sample data of Example 2

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Probability p_k
0	1	0.000	0.000977
2	2	0.016	0.001953
3	9	0.087	0.008789
4	17	0.346	0.016602
5	14	1.099	0.013672
6	20	2.906	0.019531
7	36	6.590	0.035156
8	39	13.076	0.038089
9	51	23.062	0.049805
10	41	36.607	0.040039
11	62	52.824	0.060547
12	51	69.873	0.049805
13	74	85.315	0.072266
14	65	96.730	0.063477
15	65	102.360	0.063477
16	59	101.547	0.057617
17	60	94.816	0.058594
18	39	83.612	0.038089
19	43	69.851	0.041992
20	33	55.438	0.032227
21	37	41.903	0.036133

Table I.4 – Sample data of Example 2

k	**Observed frequency** $\boldsymbol{f_k}$	**Expected frequency** $\boldsymbol{e_k = np_k}$	**Probability** $\boldsymbol{p_k}$
22	29	30.233	0.028320
23	24	20.865	0.023438
24	26	13.800	0.025391
25	20	8.762	0.019531
26	25	5.349	0.024414
27	19	3.145	0.018555
28	12	1.783	0.011719
29	9	0.976	0.008789
30	8	0.516	0.007812
31	10	0.264	0.009766
32	7	0.131	0.006836
33	5	0.063	0.004883
34	1	0.029	0.000977
35	1	0.013	0.000977
36	3	0.006	0.002930
37	3	0.003	0.002930
41	1	0.000	0.000977
42	2	0.000	0.001953
45	1	0.000	0.0009777
Total	$\Sigma f_k = 1024$	$\Sigma e_k = 1024.000$	$\Sigma p_k = 1.000000$

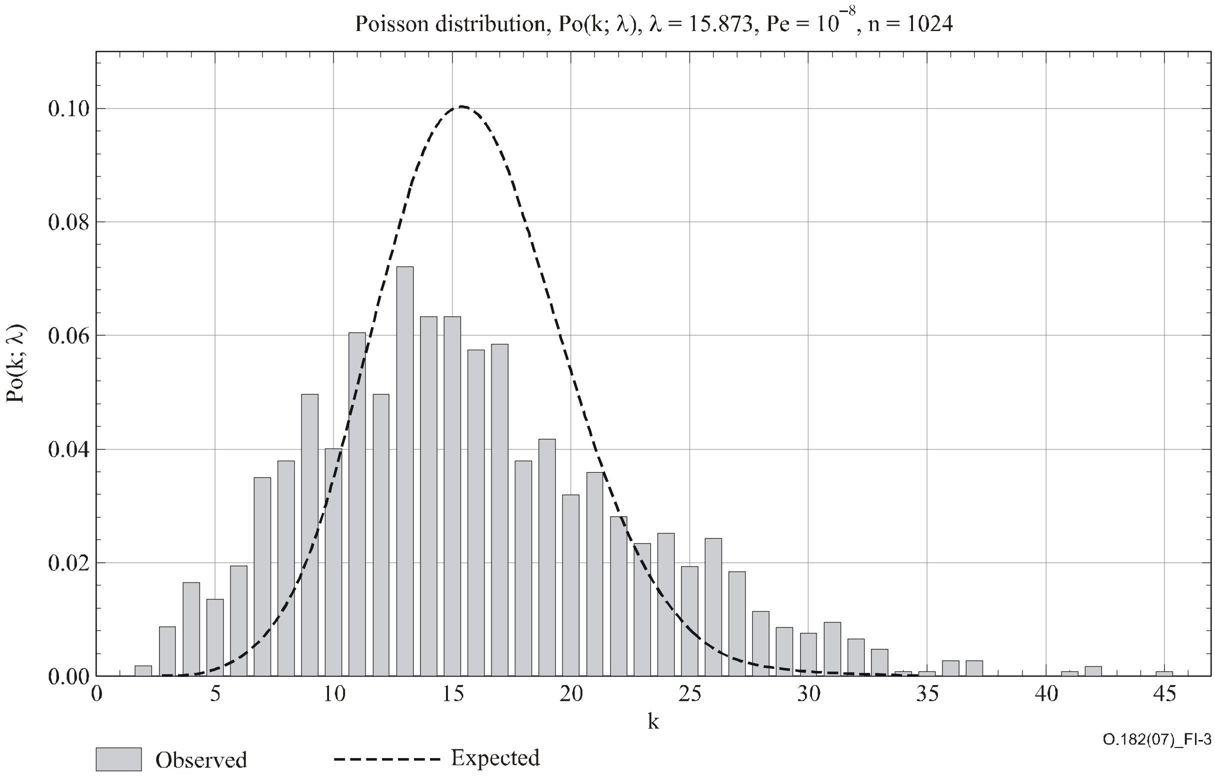

Figure I.3 – Histogram of Example 2

Note that the expected frequencies of rows for $k \leq 6$ and $k \geq 27$ are less than 5. Then, rearranging the table as previously, Table I.4 merges the rows of $0 \leq k \leq 6$ with the row of $k = 7$. Table I.5 shows the merger of the rows of $27 \leq k \leq 45$ with the row of $k = 26$.

Table I.5 – Rearranged table of Example 2

k	**Observed frequency f_k**	**Expected frequency $e_k = np_k$**	**Deviation $(f_k - e_k)^2/e_k$**
≤7	99	11.047	700.225
8	39	13.076	51.394
9	51	23.062	33.844
10	41	36.607	0.527
11	62	52.824	1.594
12	51	69.873	5.098
13	74	85.315	1.501
14	65	96.730	10.408
15	65	102.360	13.636
16	59	101.547	17.827
17	60	94.816	12.784
18	39	83.612	23.803
19	43	69.851	10.322
20	33	55.438	9.081

Table I.5 – Rearranged table of Example 2

k	Observed frequency f_k	Expected frequency $e_k = np_k$	Deviation $(f_k - e_k)^2/e_k$
21	37	41.903	0.574
22	29	30.233	0.050
23	24	20.865	0.471
24	26	13.800	10.787
25	20	8.762	14.415
≥26	107	12.280	730.624
Total	$\Sigma f_k = 1024$	$\Sigma e_k = 1024.000$	$\chi^2 = \Sigma(f_k - e_k)^2/e_k = 1648.963$

Table I.6 shows the result of the goodness of fit test based on Table I.5. With this example, the null hypothesis is rejected at the 5% significance level.

Table I.6 – Result of goodness of fit test for Example 2

Item	Symbol	Value
Sample size	N	1024
Estimated λ	$\hat{\lambda}$	15.873
Degrees of freedom	ν	18 (k = 7, ..., 26)
Chi-square	χ^2	1648.96 (tail area = 0.00%)
Significance level	α	5.0%
Significance point	χ_α^2	28.8693
Hypothesis	H_0	Rejected

Bibliography

[b-ITU-T O.181] ITU-T Recommendation O.181 (2002), *Equipment to assess error performance on STM-N interfaces*.

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