International Telecommunication Union

ITU-T

TELECOMMUNICATION
STANDARDIZATION SECTOR
OF ITU

Y.1541

Amendment 1

(06/2006)

SERIES Y: GLOBAL INFORMATION INFRASTRUCTURE, INTERNET PROTOCOL ASPECTS AND NEXT-GENERATION NETWORKS

Internet protocol aspects – Quality of service and network performance

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Network performance objectives for IP-based services

Amendment 1: New Appendix X – An example showing how to calculate IPDV across multiple sections

ITU-T Recommendation Y.1541 (2006) – Amendment 1

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Network performance objectives for IP-based services

Amendment 1

New Appendix X – An example showing how to calculate IPDV across multiple sections

Summary

This appendix provides an example on how to calculate IPDV when a number of network sections are involved. It builds on the information contained in 8.2.4/Y.1541 and also provides some background information on the method.

Source

Amendment 1 to ITU-T Recommendation Y.1541 (2006) was agreed on 13 June 2006 by ITU-T Study Group 12 (2005-2008).

FOREWORD

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The World Telecommunication Standardization Assembly (WTSA), which meets every four years, establishes the topics for study by the ITU-T study groups which, in turn, produce Recommendations on these topics.

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In some areas of information technology which fall within ITU-T's purview, the necessary standards are prepared on a collaborative basis with ISO and IEC.

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ITU-T Recommendation Y.1541

Network performance objectives for IP-based services

Amendment 1

New Appendix X – An example showing how to calculate IPDV across multiple sections

This appendix provides an example on how to calculate IPDV when a number of network sections are involved. It builds on the information contained in 8.2.4/Y.1541 and also provides some background information on the method.

The definition of IP Delay Variation used here (see discussion in Appendix II/Y.1541) is:

$$\text{IPDV} = \text{IPTD}_{\text{upper}} - \text{IPTD}_{\text{min}}$$

where:

$\text{IPTD}_{\text{upper}}$ is the $1 - 10^{-3}$ quantile (99.9th percentile) of IPTD in the evaluation interval

IPTD_{min} is the minimum IPTD in the evaluation interval.

It assumes that there are a number of network sections S_1, S_2, ... , S_n for which estimates of IPDV_1, IPDV_2, ... , IPDV_n are available. The individual estimates must have been made under comparable network conditions for any end-to-end combination to be meaningful. For example, they might have all been measured during the busiest hour of the month in each of their individual sections. In this case, the resulting combinations will generally not correspond to any real end-to-end measurement that could be made as all of the component sections could not be expected to experience their busiest hours simultaneously. Nevertheless, the result would produce an upper bound that could be used for planning and network monitoring purposes.

The relationship for estimating the UNI-UNI Delay Variation (IPDV) performance from the network section values must recognize their sub-additive nature and is difficult to estimate accurately without considerable information about the individual delay distributions. If, for example, characterizations of independent delay distributions are known or measured, they may be convolved to estimate the combined distribution. This detailed information will seldom be shared among operators, and may not be available in the form of a continuous distribution. As a result, the UNI-UNI IPDV estimation may have accuracy limitations. Since study continues in this area, the estimation relationship given below has been specified on a provisional basis, and this clause may change in the future based on new findings or real operational experience.

X.1 Calculation of Delay Variation

The relationship for combining IPDV values is given below.

The problem under consideration can be stated as follows: estimate the quantile t of the UNI-UNI delay T as defined by the condition:

$$\Pr(T < t) = p$$

We will assume that $p = 0.999$ (99.9th percentile) and for simplicity that all delay measurements have been normalized by removing the measured minimum delay. For the numerical examples below, it is assumed that there are three network sections ($n = 3$) and that all delays are expressed in ms.

Step 1

Measure the mean and variance of the delay in each of the n network sections. For a set of measurements, $D_1, D_2, \dots, D_n$ for the k^{th} section the mean, μ_k, and variance, σ_k^2, are computed as:

$$\mu_k = \frac{1}{n}\sum_{i=1}^{n} D_i$$

$$\sigma_k^2 = \frac{1}{n}\sum_{i=1}^{n}(D_i - \mu_k)^2$$

For our example we suppose that we have found that:

$$\mu_1 = 1.0 \qquad \mu_2 = 2.0 \qquad \mu_3 = 3.0$$

$$\sigma_1^2 = 0.5 \qquad \sigma_2^2 = 1.0 \qquad \sigma_3^2 = 1.5$$

Estimate the mean and variance of the UNI-UNI delay by summing the means and variances of the component distributions.

$$\mu = \sum_{k=1}^{n}\mu_k = 1.0 + 2.0 + 3.0 = 5.0$$

$$\sigma^2 = \sum_{k=1}^{n}\sigma_k^2 = 0.5 + 1.0 + 1.5 = 3.0$$

Step 2

Measure the quantiles, t_k, for each delay section at the probability of interest, $p = 0.999$. These can be determined simply by sorting the measurements, D_i, so that without loss of generality:

$$D_1 \le D_2 \le \dots \le D_n$$

and then selecting as the p^{th} quantile the m^{th} measurement D_m (that is $t_k = D_m$) where m is the smallest integer satisfying $p \le m/n$. If $n = 1000$, then $m = 999$ for $p = 0.999$. Suppose for our example we find that:

$$t_1 = 4.32 \qquad t_2 = 6.02 \qquad t_3 = 7.55$$

Estimate the skewness, γ_k, and third moment, ω_k, for the k^{th} section using the formulas shown below, where $x_{0.999} = 3.090$ is the value satisfying $\Phi(x_{0.999}) = 0.999$ where Φ denotes the standard normal (mean 0, variance 1) distribution function.

$$\gamma_k = 6 \cdot \frac{x_p - \dfrac{t_k - \mu_k}{\sigma_k}}{1 - x_p^2} \qquad \omega_k = \gamma_k \cdot \sigma_k^3$$

$$\gamma_1 = 6 \cdot \frac{3.09 - \dfrac{4.32 - 1}{\sqrt{0.5}}}{1 - 3.090^2} = 1.126 \qquad \omega_1 = 1.126 \cdot \left(\sqrt{0.5}\right)^3 = 0.398$$

$$\gamma_2 = 6 \cdot \frac{3.09 - \dfrac{6.02 - 2}{\sqrt{1.0}}}{1 - 3.090^2} = 0.653 \qquad \omega_2 = 0.653 \cdot \left(\sqrt{1.0}\right)^3 = 0.653$$

$$\gamma_3 = 6 \cdot \frac{3.09 - \frac{7.55 - 3}{\sqrt{1.5}}}{1 - 3.090^2} = 0.439 \qquad \omega_3 = 0.439 \cdot \left(\sqrt{1.5}\right)^3 = 0.806$$

Assuming independence of the delay distributions, the third moment of the UNI-UNI delay is just the sum of the Network Section third moments.

$$\omega = \sum_{k=1}^{n} \omega_k = 0.398 + 0.653 + 0.806 = 1.856$$

The UNI-UNI skewness is computed by dividing by $\sigma^{3/2}$ as shown below.

$$\gamma = \frac{\omega}{\sigma^3} = \frac{1.856}{\left(\sqrt{3}\right)^3} = 0.357$$

Step 3

The estimate of the 99.9-th percentile ($p = 0.999$) of UNI-UNI delay t (in ms) is as follows.

$$t = \mu + \sigma \cdot \left\{ x_p - \frac{\gamma}{6}\left(1 - x_p{}^2\right) \right\} = 6 + \sqrt{3} \cdot \left\{ 3.09 - \frac{0.357}{6}\left(1 - 3.09^2\right) \right\} = 12.23$$

As stated earlier, the nature of the IPDV objective is the upper bound on the $1–10^{-3}$ quantile of IPTD minus the minimum IPTD (i.e., the distribution of IPDV is normalized to the minimum IPTD). In general, units of IPDV values are seconds, with resolution of at least 1 microsecond. If lesser resolution is available in a value, the unused digits shall be set to zero.

X.2 Mathematical background

If the distributions of each of the components T_k were known in detail, the distribution of the end-to-end delay T could be computed using convolutions. Convolutions are challenging in practice: most implementations will rely on Laplace transform techniques including methods to invert transforms numerically to recover the underlying probability distributions. To use this method assumptions would have to be made about the exact nature of the component distributions.

Instead, an alternative method is employed that uses the available information without requiring additional assumptions or complex methods.

The basic idea is to transform a random variable T with known mean μ, variance σ^2, and skewness γ into a symmetric random variable Z which is standard normal (mean 0, variance 1) or nearly so. One such method, called the Normal Power approximation (see [1]) works as follows:

- Define the standardized variable $X = \frac{T - \mu}{\sigma}$
- The Normal Power approximation states that $X \approx Z + \frac{\gamma}{6}(Z^2 - 1)$ where Z is a standard normal (mean 0, variance 1) random variable.

Once the details are worked through, the following approximation is obtained:

$$\Pr(T < t) \approx \Phi\left(\frac{1}{\gamma}\sqrt{9 + 6\gamma\left(\frac{t-\mu}{\sigma}\right) + \gamma^2} - \frac{3}{\gamma}\right)$$

where Φ is the cumulative standard normal distribution function:

$$\Phi(x) = \int_{-\infty}^{x} \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}} dx$$

Although the values of this function are readily available, a more transparent relationship can be derived that eliminates all reference to Φ and allows the quantile t to be directly computed from the component quantiles t_k.

In fact, since the probabilities in all the quantile definitions $\Pr(T_k < t_k) = p$, $\Pr(T < t) = p$ have the common value p, if we define x_p to be the unique value satisfying $\Phi(x_p) = p$, then we have:

$$\frac{1}{\gamma_k}\sqrt{9 + 6\gamma_k\left(\frac{t_k-\mu}{\sigma_k}\right) + \gamma_k{}^2} - \frac{3}{\gamma_k} = x_p \text{ and}$$

$$\frac{1}{\gamma}\sqrt{9 + 6\gamma\left(\frac{t-\mu}{\sigma}\right) + \gamma^2} - \frac{3}{\gamma} = x_p$$

If we multiply the above by σ_k^2 and σ^2 respectively and add over all the components we deduce from the additivity of variances of independent distributions that:

$$\sigma^2 \cdot \frac{\sqrt{1 + 2\delta \cdot \left(\frac{t-\mu}{\sigma}\right) + \delta^2} - 1}{\delta} = \sum_{k=1}^{n} \sigma_k^2 \cdot \frac{\sqrt{1 + 2\delta_k \cdot \left(\frac{t_k - \mu_k}{\sigma_k}\right) + \delta_k^2} - 1}{\delta_k}$$

where we have set $\delta = \frac{\gamma}{3}$ and $\delta_k = \frac{\gamma_k}{3}$. Although this looks complex, it requires only simple algebra to compute the end-to-end quantile t from the components t_k and the available measured quantities.

X.3 Special cases

In the approximation

$$\Pr(T < t) \approx \Phi\left(\frac{1}{\gamma}\sqrt{9 + 6\gamma\left(\frac{t-\mu}{\sigma}\right) + \gamma^2} - \frac{3}{\gamma}\right)$$

if we let $\gamma \to 0$ we produce the result

$$\Pr(T < t) \approx \Phi\left(\frac{t-\mu}{\sigma}\right)$$

corresponding to the case where T has a normal distribution with mean μ, variance σ^2. If we let all the skewness terms $\gamma \to 0$, $\gamma_k \to 0$ the algebraic expression of the previous section reduces to:

$$\sigma \cdot (t - \mu) = \sum_{k=1}^{n} \sigma_k \cdot (t_k - \mu_k)$$

Some further manipulation removes the variances to produce:

$$(t - \mu)^2 = \sum_{k=1}^{n} (t_k - \mu_k)^2$$

This shows that when the component delays T_k are normally distributed with mean μ_k, and variance σ_k^2, then the corresponding quantiles follow a composition law similar to that for variances.

This composition law for normal variates can also be derived directly. The algebraic expression of the previous section can be viewed as a generalization of this particular composition law.

X.4 Estimating skewness from quantiles

Consider a random variable T whose mean μ and variance σ^2 are known and where the quantile t in $\Pr(T < t) = p$ is known but where the skewness γ is not known. Using the Normal Power approximation we have:

$$\Pr(T < t) \approx \Phi\left(\frac{1}{\gamma} \sqrt{9 + 6\gamma\left(\frac{t - \mu}{\sigma} \right) + \gamma^2} - \frac{3}{\gamma} \right) = p$$

From tabulated values of the standard normal distribution function Φ we can find the unique value x_p satisfying $\Phi(x_p) = p$. Therefore:

$$\frac{1}{\gamma} \sqrt{9 + 6\gamma\left(\frac{t - \mu}{\sigma} \right) + \gamma^2} - \frac{3}{\gamma} = x_p$$

This can be solved for γ producing:

$$\gamma = 6 \cdot \frac{x_p - \frac{t - \mu}{\sigma}}{1 - x_p^2}$$

X.5 References

[1] "*A Note on the Normal Power Approximation*", Colin B. Ramsey, ASTIN Bulletin, Vol. 2. No. 1, April 1991.

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